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An Introduction to Modern Logic

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PREFACE

THIS book is intended to be precisely what the title implies—an introduction to modern logic. By “modern” logic is understood that body of logical theories and methods which is usually associated with the names of Lotze, Sigwart, Bradley, Bosanquet, Wundt, Erdmann, and Dewey. The writer has endeavored to place himself in the center of this movement taken as a whole, and to set forth the characteristic doctrines of the modern school, so far as this has seemed possible in a book which is professedly introductory. The traditional or Aristotelian logic, which has played so great a part in the past *history* of thought, is entirely omitted from consideration, as are also symbolic logic and the various attempts at inventing a logical calculus. For all such omissions, as well as for what is included, the sole justification is the nature of an introductory treatise. It has seemed best to avoid polemics on the one hand, and an unmanageable multiplicity of hypotheses on the other, in favor of a certain singleness of purpose and organic unity of thought.

The exercises and suggestions for further reading at the end of the different chapters should be regarded as an integral portion of the book. To attempt to study logic without working one’s way through appropriate exercises is like trying to study mathematics without solving problems, and leads inevitably to a certain superficiality of mental grasp which it is the avowed aim of logical treatises to fight and destroy. So also to confine one’s reading to a single text-book is to shut one’s mind to the interest and infinite variety, as well as to the concreteness and usefulness, of an important branch of modern science.

The writer’s obligations are too numerous to mention. The influence of Plato and Kant, on the one hand, and of Locke, Hume, and Mill, on the other, is sufficiently obvious. But in the case of more recent writers and teachers it would be invidious to single out any one group. The chapter headings of Part III are in large part taken from Wundt’s *Allgemeine Methodenlehre*, but for the contents of the chapters in question Wundt is in no special sense responsible.

UNIVERSITY OF MINNESOTA.

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INTRODUCTION

The Logical Attitude of Mind.—The natural behavior of men, as of other animals, is not logical, but instinctive. We tend to react to most of the concrete situations of life in a way determined by inherited neural dispositions rather than by "reasoning things out." The life of most men appears to be spent in acquiring food and shelter, a family, and money to expend in purchasing houses, automobiles, education and luxuries for the rising generation, and insurance against disaster. That is to say, the main outlines of life—power, love, and protection of the family—are fixed in the main by inherited racial tendencies, and it is around these that almost all our activities are grouped. And it is not only the outlines which are instinctive. Nearly all our more detailed every-day experiences are of the same kind. Taking meals, sleeping, feeling aggressive or timid in our work and play, day-dreaming as we plan for the morrow, feeling elated or depressed at the progress we are making—all these experiences are fundamentally instinctive, and we should be seriously puzzled, if we were called upon to account for them in purely rational terms. To the average man, indeed, it never even occurs to make any such enquiry. "He eats because the food tastes good and makes him want more. If you ask him *why* he should want to eat more of what tastes like that, he will probably laugh at you. The connection between the savory sensation and the act it awakens is for him absolute and *selbstverständlich*, an *a priori* synthesis of the most perfect sort, needing no proof but its own evidence. To the metaphysician alone can such questions occur as: Why do we smile, when pleased, and not scowl? Why are we unable to talk to a crowd as we talk to a single friend? Why does a particular maiden turn our wits so upside down? The common man can only say, '*Of course* we smile, *of course* our heart palpitates at the sight of a crowd, *of course* we love the maiden, that beautiful soul clad in that perfect form, so palpably and flagrantly made from all eternity to be loved!'"¹

¹ William James, *Principles of Psychology*, chapter xxiv, pp. 386–387.

Thus we see that not only the mainsprings of behavior, but also the moulds which shape and direct our multifarious particular activities are, in the end, animal, racial, instinctive. The mind is no réceptive waxen tablet on which any and every element of the physical world can equally impress its sensory image. We see and hear only what attracts our attention; we notice only what awakens our interest and stimulates, directly or indirectly, those dispositions developed in the long course of evolution, which we call instincts. In fact, so large a part do instinct and habit play in our lives, that it is seriously maintained by a great psychologist² that man seldom reasons and that other animals never reason.

But while the behavior of men is naturally instinctive, it is sometimes forced by circumstances to be something more. The insects, it is true, live a life almost purely instinctive; but then, the problems which they have to face are almost unvarying, so that one hard and fast chain of instincts can advantageously direct the honey-comb construction of the Mason bee, or the egg-laying activities of the solitary wasp or Yucca moth. Our human problems, however, are far too unstable and novel. No inherited disposition could keep pace with the enormous changes which have come over civilized life within the last two generations; and even within the brief span of one generation, our habits need to be readjusted and remodeled, again and again. There is thus in our lives something more than instinct or habit, something continuously readjusting, re-shaping our ways, forever solving problems forever new, inventive, creative—in a word, what we call intelligence, thought, or reason.

How is this done? How does intelligence re-shape our lives? How does thought add moral or economic cubits to our stature? How does reason enable us to lead lives more nearly approaching the ideal? Let us consider briefly some of the ways by which we rise above instinct and habit. One of these is *definition*, fixing the meaning of an idea in such a way that, amid all the changes and chances of our fluctuating experience, we hold fast to that one meaning, that one direction of our thought. Another is *analysis*, the splitting up some complex whole into parts so simple that we can readily apprehend their nature, and readily grasp the plan of the whole

² Wilhelm Wundt.

constructed out of such parts. Yet another is *inference* or reasoning, with all its various forms, by which from a given situation we construct mentally the probable consequences or antecedents, and generally enlarge our mental horizon and clear up our ideas. All of these are forms of *scientific method*, the indispensable handmaid of efficiency and success in commerce, in education, and in all the organized institutional activities of the present day.

The logical attitude of mind, then, is concerned with the solution of problems, the bridging of gaps, the removal of inconsistencies and inefficiencies, in cases where instinct and habit alone would be insufficient. There is none of the "of-course-ness" of instinct about this mental attitude. It is reasoned, deliberate, thought-out activity. It has also none of the warmth and immediacy, and none of the vagueness and confusion, of feeling. It is calculating, cool, calm, clear-cut, precise; deals little in promises and much in proofs: little in speed and much in sureness; hesitates in drawing conclusions, but succeeds broadly and inevitably in the end; and is not without its own vision, its uplifting trust in the final rationality of the universe.

The Study of Logic.—The value of cultivating such a mental attitude is beyond question. Without it much of our art and all of our science would disappear; the religion of Apollo would give way to the worship of Pan; and the reign of Chaos and Old Night would come again. But *how* to cultivate this attitude—by studying logic? This has been, and still is, called in question. Plato reasoned sublimely before Aristotle wrote the *Analytics*; and many of our foremost scientists have never opened the pages of our logical manuals. And we must consider a second point: if we study the logic of mathematics, does this make us mathematicians—or logicians? If we study the logic of the emotions, does this teach us to feel—or to reason ourselves out of feeling? If we study the logical principles of any subject, does this give us practical mastery of the subject—or theoretical mastery of the principles? Common sense and a wide experience of men come to the same conclusion. It is well known that theoretical masters in the field of Pedagogy—men like Kant and Pestalozzi—sometimes make poor teachers; and authorities in the field of Aesthetics are seldom themselves creative artists. To put it briefly: if the attitude comes without studying the theory, and if mastery

of the theory is consistent with almost total absence of the practical attitude, what, if any, is the value of studying logic?

To this question there are two answers. The first answer is given by Hegel, Bosanquet, and many other writers in the field of logic. With great modesty they disclaim any practical value for their study, and assign to it only that theoretical value to which any pure science can lay claim. If we desire to know what are the laws of thought, the principles on which the validity or correctness of our thinking depend, we have as much right to raise such questions as we have to study pure physics or pure mathematics. All such studies, like metaphysics, have their sufficient justification in that reaching out after knowledge as such, which is fundamental in our human nature, even apart from practical values. There is a satisfaction in knowing, quite apart from the question of *using* our knowledge; and many logicians are content to leave on one side the question of use, in their certain enjoyment of that satisfaction which comes from finding, discovering, knowing.

It is possible, however, to answer the question differently. While it is true that, in its higher branches, logical study is somewhat remote from every-day concerns, still, even at its deepest, it intensifies and quickens our vision of Truth, and justifies, by the insight which it brings, that confidence in the intelligibility of things, which is fundamental in the logical attitude of mind; and as to its more elementary branches—it is merely self-deception if we suppose that there was ever a great scientist who was not also a good logician. Every scientific text-book and every laboratory course lays especial weight on questions of method, methods of observation, methods of evaluating results, methods of establishing conclusions. But these methods are not peculiar to physics or biology as such. They belong to the general theory of method, which is a branch of logic. Again, the researches of advanced scientists, on which so much of the progress of knowledge depends, are almost always investigations to test the validity of inferences, the theoretical value to be assigned to conclusions, perhaps even to the theoretical assumptions underlying scientific reasoning. All these, however, fall within the province of logic. The simple truth of the matter is this: a little experience, a little common sense, enables each one of us, as we say, to reason correctly, whether we have studied technical logic or no; and so long as we move only within the more elementary

reaches of experience, our common sense standards of truth are, perhaps, sufficient. But if we wish to raise ourselves beyond this elementary and limited sphere; if we wish to think clearly and consecutively in order to reach conclusions above the level of mere common sense; if we wish to acquire business efficiency, or to succeed beyond the average in life or in science, we need to withdraw, for a season, from the more active concerns of mere living, and reflect carefully and systematically on the principles and methods of right thinking. Then, having acquired some grasp of the theory beyond what the average man knows or suspects, we can apply our deeper insight in order to attain a higher level of efficiency and success than is possible without such reflection; and in order to cultivate the best fruits of the logical attitude of mind, it is necessary to make ourselves thoroughly familiar with the principles and methods of right thinking—in a word, to study logical theory.

Preliminary Definition of Logic.—Logic, then, is the study of thought—systematic reflection on the principles and methods of right thinking. This statement is correct, but is not sufficiently precise to serve as a scientific definition of logic. Psychology, for instance, includes a study of the thought-processes, and to some extent deals with the methods of right thinking. Is logic, then, a branch of psychology? If, as its name implies, psychology were studied as the general science of mind, or the general science of behavior, it would include the study of logical behavior or right thinking, and logic would certainly be a department of psychological investigation. But in present day practice, psychologists tend to regard their science, not as the general science of mind or behavior, but as one special mental science; and while they do study logical behavior, they do so in a very restricted way, and from a view-point entirely different from that taken by logic.

Let us consider an instance. Suppose Mr. A to have recently become a member of the Republican party, or of the Congregational Church. Psychological explanation of this change of heart would lay especial emphasis upon the various elements in Mr. A's personal history and environment which had led him to take the step in question: the influence of old and new associations, the weight of social pressure, the greater appeal of the new opinions—what he would himself, perhaps,

call their greater reasonableness. In other words, the viewpoint of psychological explanation is genetic, and traces the steps in his personal evolution which gradually led up to the change in question. This genetic, historical kind of analysis is, however, sharply to be distinguished from logical analysis. The logical analysis of this conversion, for instance, would consist of a clear setting forth of the arguments pro and con, in such a way that we could see the greater comprehensiveness, consistency, and truth of the new opinions. It would be concerned, not at all with the historical side of the thought-processes as such, but wholly with their meaning, with the hanging together of the thoughts in a consistent system, in accordance with the standards of truth. Whether reasoning is consistent or inconsistent, whether conclusions are true or false, no psychology can tell us. Truth and consistency are not mere matters of history or personal evolution, but have their own standards and laws, and the scientific study of these laws is logic. Psychology deals with the process-side, logic with the validity-side of thought.

If, then, we wish to mark out the field of investigation with a preliminary definition, we can say: logic is the scientific study of the laws or principles on which the validity of right thinking depends: or, more briefly, logic is the study of validity.

FOR FURTHER READING

A. E. Avery, *The Present Day Conception of Logic: Philosophical Review*, Vol. XXVII, pp. 405-412. B. Bosanquet, *Essentials of Logic*, pp. 1-34. J. Dewey, *Essays in Experimental Logic*, pp. 1-74. B. Erdmann, *Logik*, (2nd Edit.), pp. 1-33. E. Husserl, *Logische Untersuchungen*, pp. 3-8. H. Lotze, *Logic*, pp. 1-9. Chr. Sigwart, *Logic*, pp. 1-2. W. Wundt, *Logik*, (3rd Edit.), Vol. I, pp. 1-9.

PART I

JUDGMENT ¹

¹ The word *judgment* is used in modern logic, in order to emphasise sharply the difference between logic—which studies what we think or actually judge, *i. e.*, the actual living thought—and grammar, which studies the verbal expression of thought. Historically, the two have frequently been confused, so that in place of an analysis of the living thought, the logician has given a grammatical analysis of the dry bones of verbal expression, *i. e.*, of “propositions” or statements, in place of judgments or thoughts.



CHAPTER I

TYPICAL STAGES OF JUDGMENT.

The Simplest Judgments.—In every scientific study, we start with the simple and proceed gradually to the complex. Logical study also starts with the simple. What, then, is the simplest kind of valid thinking, the unit, as it were, of thought? Let us consider an example. "If the sun is shining, we will go to the woods; but if it is wet, we will stay in and read." We have here a single thought. But though *single*, it is obviously not what we should call simple, but is evidently complex. Let us analyse it, or split up the complex thought into simpler parts. "The sun is shining"; "We will go to the woods"; "It is wet"; "We will stay in"; "We will read." If we compare each one of these brief statements with the original example, we see at once that they are much less complex. Let us further compare them with one another. "The sun is shining" and "It is wet" appear to belong together, to represent two forms of the same kind of judgment. Let us regard them as one group, group A. Similarly "We will go to the woods," "We will stay in," "We will read," seem to express one and the same kind of thought, and thus to belong to a "we will" group, which represents a kind of thought different from that expressed in group A. Let us call the "we will" group, group B. The thoughts in group B are judgments of purpose, and seem more complex than judgments of sense-perception such as "The sun is shining." We shall therefore regard group A, *i. e.*, judgments of sense-perception, as the simplest kind of judgment revealed by a consideration of our example.

Are such perceptual judgments, however, to be considered as ultimate? Are they, in fact, the simplest judgments we can make? Perhaps we should draw a further distinction. "It is wet," for instance, might be thought less complex than "The sun is shining." Perhaps the distinction will become plainer if we compare "It is fine" with "The sun is shining," and "It is wet" with "The rain is heavy." Many logicians

have, in fact, regarded such impersonal judgments at the simplest thoughts we can have.² It seems better, however, to speak of more primitive or less developed forms of the same type of judgment, rather than of greater simplicity or less complexity. "It is fine" and "The sun is shining" have, after all, substantially the same meaning. The only *logical* difference is, that the meaning is more clearly expressed in the more developed form. We shall accordingly take as the simplest and most elementary acts of thought which we can discover, judgments such as "This is red," "It is warm," "There is a noise," "This is heavy," "That tastes bitter," "There is an odor," etc.—*i. e.*, simple judgments of sense-perception.

The Elements of Judgment.—In the above instances we have judgments so simple, that here, if anywhere, we should be able to pick out some of the elements upon which the validity of judgments depends. In cases which were more complex we might be unable to see our way. But in these simple cases, if anywhere, we should be able to discover at least the more obvious elements. Let us therefore consider these judgments of perception closely. Can we discover any simple factors, any constituent elements of such judgments?

Two elements, upon which perhaps the validity of judgment depends, we can discover almost at once. These elements are (a) sensory, and (b) intellectual. (a) All judgments of perception are thoughts about something *present*, something *given* to us in perception, given to us through the senses, sensory. "This-red," "Warm-feeling," "Noise-here," "That-bitter-taste," etc. It may be that all judgments whatever, and not merely perceptual judgments, are similarly thoughts about something which is, in the end, given to us in sensation. In this case, *all* thought would have this sensory element, and the function of sensation would be, to furnish us with a channel by means of which we could come to know objects, or things-which-we-can-experience. (b) On the other hand, so far as judgments of perception are *thoughts* about something present, they undoubtedly contain an element which we should call intellectual. This is in such cases, not so easily noticed as is the sensory element. In order, then, to bring out more

² For a recent study of the whole subject of the impersonal judgment, see S. F. MacLennan, *The Impersonal Judgment*, 1897.

clearly the presence of the intellectual element, we shall proceed to take a number of slightly more developed cases of perceptual judgment. "This *paper* is red," "It is warm *near the stove*," "There is a noise *at the door*," "This *book* is heavy." These instances are all perceptual, but we can at once observe that certain intellectual elements are now more prominent. Instead of reference to the undifferentiated "this" as the sensory given, we refer to the paper, stove, door, book, *etc.*, *i. e.*, to determinate objects, the recognition of which involves distinction, comparison, classification, *etc.*, which are processes predominantly intellectual. Let us further consider yet more developed perceptions. "This paper is a brighter red than that," "It is warmer near the stove than near the door," "This book is heavier than that." We have here much the same general sensory elements as before. But we now have in a very clear and explicit form the intellectual act of comparison. "This paper" and "That paper" are held together in one intellectual act, and are compared in respect of their brightness-value. So too "Near the stove" and "Near the door" in respect of their warmth. In fact, the more highly developed such judgments become, the more plainly can we observe the presence of intellectual elements. But in the very elementary and primitive cases too we can now see the presence of abstraction and recognition at least. The "This" is differentiated into red, warm, heavy, *etc.*, and these are not only clearly apprehended, but are apprehended in distinction from one another. In other words, in these simplest of all judgments there is present, not only a sensory, but also an intellectual element.

Typical Stages of Judgment.—With the sensory element in judgment, logic is not, as a rule, much concerned. It is in epistemology or the theory of knowledge, that the function of sensation as a factor in knowledge is especially studied, and it is still a question whether the theory of knowledge should or should not be regarded as constituting an integral part of modern logical investigation.³ We shall therefore postpone, for the moment, further discussion of the sensory factor in

³Schuppe (*Erkenntnistheoretische Logik*), Wundt, and Windelband (*Encyclopädie der philosophischen Wissenschaften*, Vol. I) regard the theory of knowledge as falling within the field of logic, or vice versa. Erdmann incorporates "corollaries" from the theory of knowledge in his logical *Elementarlehre*. Bradley and Bosanquet are also to be reckoned among the epistemological logicians.

judgment, and shall consider more closely the intellectual factor.

In order, however, to deal with the intellectual element in judgment in any adequate way, we can certainly not confine ourselves to elementary perceptual judgments. It is no longer a question of picking out the least complex type of judgment in order to avoid obscurity, and in order to enable our unaccustomed eyes to see their way in a simple case. We have now to face the more general problem of validity in judging as such, and this involves consideration, not only of the elementary cases, but also of the more highly developed forms or stages of judgment, in order that nothing vital may be overlooked. In order, then, to clear the way for our whole subsequent investigation into the sensory and intellectual factors, we must be able to cover, in some clear and brief way, the whole field of judgment. This we shall do by arranging different kinds or stages of judgment in a table, starting with the more elementary and proceeding to the more advanced—*i. e.*, proceeding from judgments in which the intellectual element is less prominent to cases in which it is more prominent.

TABLE OF JUDGMENTS

1. *Judgments of Perception.*—*E. g.*, "This room is warm," "This oak has less foliage than that beech."

2. *Judgments of Experience.*—*E. g.*, "The freight-trains passing over the bridge grow more troublesome every year," "Noise is usually a compound of tones."

3. *Symbolic Judgments.*—*E. g.*, "Rome was occupied by Caesar," "Strathcona lies on the North Saskatchewan," (where the evidence is not direct experience, but a textbook account of a place we have never seen). So too "The theory of the *synapse* is fundamental for the explanation of conscious behavior" (where we have had no direct experience of *synapses*).

4. *Transcendent Judgments.*—*E. g.*, "God is a substance consisting in infinite attributes, of which each expresses eternal and infinite essentiality," "Things-in-themselves are absolutely unknowable," "True evolution is the progressive self-organization of a system of timeless selves."

Let us explain briefly the above distinctions. A judgment

of experience differs from a perceptual judgment, in that it depends more on memory of previous perceptions than on direct present perception. It is more developed, and sums up many previous experiences, as a composite photograph gives us the result of many direct likenesses of actual persons. For instance, "Most writing-paper is white" sums up many experiences of writing-paper, and while on the one hand less direct and immediate than "This writing-paper before me is white," on the other is more advanced, more representative, more intellectual, than the simple perceptual judgment. Still, even in the perceptual judgment there is *some* appeal to previous experiences. For we know the object before us to be "writing-paper" and "white." The distinction between judgment of perception and judgment of experience is thus not absolute, but is a question of more or less, a matter of degree. Where the sensory element predominates, we have the stage of judgment which we call judgment of perception. Where the intellectual element plays the greater part, we have the judgment of experience.

A symbolic judgment differs from a judgment of experience, in that we are here dealing with a sort of extension of our experience, based indeed on previous experiences, but constructing, on the analogy of these past experiences, new objects of similar type, objects which we might possibly have experienced, or might possibly experience in the future, but which in actual fact we have never experienced. Thus, if we had lived in Rome at the time of its occupation by Caesar, our thought would be a judgment of experience, if not, indeed, a perceptual judgment. But where we have had direct experience of cities differing from Rome, and can only construct for ourselves imperfectly, in the light of very inadequate past experience, what it meant for a city to be occupied in war-time by the forces of Caesar, our judgment is only symbolical. The less directly the case happens to be related to our personal experience, the harder do we find it to spread out our experience so as to make vivid to ourselves scenes so remote from what we have seen and felt, and the further do we tend to fall short of full realisation of the meaning of our judgment. We have the bare skeleton of experience. The flesh and blood, the living elements, are almost totally missing. In such judgments, while the sensory element may be weak, the intellectual element of construction is far more prominent

than in judgments which remain contentedly within the circle of our own experience. And yet, in judgments of experience also there is *some* construction, some putting together of past experiences in order to produce something new, the composite result which—as a composite unity, at any rate—was never actually experienced. We see then that here also the difference is a question of more and less, a matter of degree only. Where the sensory element is relatively greater and the intellectual construction relatively less, we have a judgment of experience. Where the constructive, intellectual element decidedly predominates, we have the symbolic judgment.

The transcendent judgment differs from the symbolic judgment, in that we here transcend or go beyond, not merely our actual experiences, but also even possible human experience. In the symbolic judgment, our subject is always something which might conceivably be experienced, or have been experienced. But in the transcendent judgment, the subject could never be experienced. Where, for example, our experience is finite, any judgment about the infinite transcends the possibility of experience, and we have a transcendent judgment. Such judgments are both natural and common. How natural, or rather inevitable, any attempt to think one's way to a first cause, or to a profound and satisfying standpoint for the conduct of life will show. How common, the slightest reflection on human mental history will sufficiently attest. Consider, for example, the ever recurring interest in mysticism, the medieval search for the philosopher's stone, the inventor's fascination in the case of perpetual motion, the still not uncommon belief that one can read destiny by the lines in the palm of the hand, if not by the conjunctions of the heavenly bodies. The "anticipations" of scientists—intended, no doubt, as symbolic extensions of experience—teem with transcendent judgments. Most of our philosophical theories, much of our moralising, and much of our religious thought is transcendent. In every walk of life the human yearning after some ineffable ideal, some unspeakable perfection—the "vision" (as we call it) of ideal truth, power, love, or happiness—leads us insensibly and inevitably beyond the narrow confines of possible experience.

While in strict theory it might seem as though transcendent judgments were sharply distinguished from symbolic judg-

ments, in definite cases the line cannot be drawn with precision. Transcendent judgments contain some sensory and experiential elements, and symbolic judgments contain a fair portion of that idealising tendency which frees the imagination from the fetters of actual experience. A transcendent judgment is thus only an exaggerated symbolic judgment, and the distinction, here also, is one of more and less. Where the idealising tendency, the intellectual element, is more restrained, we have a symbolic judgment. Where it is almost wholly loosed from its experiential moorings, where the sensory element is distinctly less prominent, we have the transcendent judgment.

The above stages cover the whole field of human thought. They represent four stages of judgment, distinguished from one another only relatively, according as the perceptual or the intellectual element predominates. The simplest judgments of perception exemplify, to *some* extent, the operation of the elaborative, idealising tendency of intellect. And the most transcendent judgments we can make, the finest thought-webs we can spin, are still attached to earth by *some* sensory threads, gilded o'er by the warmth of personal feeling and personal sense-experience. A pure intellect and a pure sensation are equally beyond our human thought. All our thinking moves within these two extremes, and partakes of both principles in varying proportions. It may be that some principle yet more profound remains to be discovered. But the presence of these two principles—in perceptual, experiential, symbolic, and transcendent judgments alike—is certain, and for the present we must look for the conditions, on which the validity of all judgment depends, in the sensory and intellectual elements.

FOR FURTHER READING

J. E. Creighton, *An Introductory Logic*, chapter xxiii. J. Dewey, *Essays in Experimental Logic*, pp. 183-219. B. Erdmann, *Logik*, (2nd Edit.), pp. 426-429. R. C. Lodge, *The Division of Judgments: Journal of Philosophy, etc.*, Vol. XV, pp. 541-550.

EXERCISES

1. Try whether you can discover any judgments simpler than judgments of perception. For instance, are any of the following more simple than perceptual judgments: Remarkable! Fire! Good! Thieves! Dinner!

2. Can you think of any judgments which cannot be classed as perceptual, experiential, symbolic, or transcendent—or at least some transition-form of these? Try to classify the following, arranging them under separate heads, as (1) perceptual, (2) experiential, (3) symbolic, (4) transcendent: Up we go! It takes thirteen days to get from here to Paris. Everywhere you see grain elevators. Nero fiddled while Rome was burning. The prince now possessed the magic sword, the cap of darkness, and the seven-league boots. I am the master of my fate, I am the captain of my soul! This color-mixture of yellow and blue gives gray. A thick rug under the feet tends to prevent chilblains. You can buy a good boat for \$40.00. Humpty Dumpty sat on a wall. Seven plus five equals twelve. O king, live for ever! The hens are laying more eggs than ever before. You are looking pale. These weeds are choking the vegetables. It is sure to rain on Thanksgiving Day.

3. Go over your classified list of the judgments given in the preceding exercise. Can some of the experiential judgments be regarded as perceptual? Can some of the symbolic judgments be regarded as experiential? Can some of the transcendent judgments be regarded as symbolic? And is there an experiential element even in judgments of perception? Is there a symbolic element even in the judgments of experience? Is there a transcendent element in the symbolic judgments? Do all the forms pass into each other by easy, and almost imperceptible, gradations?

CHAPTER II

THE SENSORY ELEMENT IN JUDGMENT.

The Sensory Element.—In order to study the sensory element in judgment, it would, perhaps, be convenient if we could isolate it and examine it in a separate case, uncontaminated by any admixture of foreign elements. But, as we have seen, the sensory element is never found alone; no judgment is found in which the sensory elements are not shot through and through with intelligence, organised and built up into something more than mere blind sensation. The most that we can do, if we wish to discover the validity-value of sensation, is to consider a variety of cases, neglecting the intellectual element, and concentrating our mental microscope on the part played by sensation in giving us a judgment on whose validity we can rely. In order that the cases considered shall cover the whole field of thought, we shall make use of the table of judgments established in the last chapter, and shall begin with judgments of perception.

(A) **In Judgments of Perception.**—What is the sensory element in simple judgments like "This is warm," "This is green," "This is hard," "This is heavy," *etc.*? On the one hand we have a reference to the general sensory continuum which furnishes the background and setting for our more highly specialized experiences; and on the other, in the "warm," "green," "hard," *etc.*, we have the attribution of a special sensory quality, in this setting, to some object ("this") singled out for particular notice. The abstraction which singles out a particular quality from the general sensory setting, is, no doubt, intellectual: it is a mode of articulation or organisation of the sensory side of experience. What remains as the definitely sensory element is (1) the unspecialised feeling of bodily existence. This is composed of the memories, associations, sensations, *etc.*, which together constitute the background of the consciousness of John Smith, as distinct from that of Henry Jones, *etc.* We carry this feeling of ourselves about with us: it prejudices us in various ways, colors

all our thoughts, and spreads itself unnoticed over all our experiences. It is more than merely sensory, but a large part of it is definitely sensory, and requires consideration, as underlying every judgment we make. (2) A further sensory element, differing with different judgments, is that by which we distinguish "warm" from "green," *etc.*, *i. e.*, not the distinguishing itself, which is intellectual, but the positive quality in each, by which we experience warmth as *warmth*, as the specialised mental reaction to a temperature-stimulus, and green as *green*, the specialised color-consciousness, *etc.*, *i. e.*, the positive quality of the special sensation, in virtue of which one sensation can be distinguished from another. This specialised sense-experience is ultimate, and can not be explained except by reference to the conditions of its appearance in consciousness. It has to be experienced to be appreciated as warmth, green-ness, *etc.*, and constitutes the specialised sensory element in simple judgments of perception.

(B) **In Judgments of Experience.**—What is the sensory element in such judgments as "The freight trains crossing the bridge grow more troublesome every year"? What is especially present to sense is *ex hypothesi* no more than a low rumble, which is interpreted as due to a distant freight train; by association former instances of such trains are recalled, and a comparative judgment, based upon such experiences and summing up their result, comes to be formed. That is to say, the actually present sensory element is (1) the general feeling of bodily existence already noticed in the case of perceptual judgments, and (2) the special complex of sense-qualities which constitutes a "rumble." So far there is not much difference from what we found in the perceptual judgment; but in judgments of experience, the weight of the judgment rests less on the actually present sensory elements noted above, and more on the recalled, reinstated, ideally present sensory experiences of the past—*i. e.*, less on the present sensation, and more on its fringe of associations. So too of the inductive generalisations of science, exemplified in such judgments as "All noise is a compound of tones." This represents a summing up of past experiences, and the occasion for making such a judgment of experience is, no doubt, some present instance of "noise" or "tone." The judgment of perception thus rests more immediately on what is present, the judgment of experience more mediately on the given sensation;

but in both cases the sensory element consists of (1) the general background, and (2) a special stimulus which stands out from that background.

(C) **In Symbolic Judgments.**—In judgments such as "Rome was occupied by Caesar," the reference is, of course, to an experience into which we do not enter immediately. We reconstruct for ourselves in idea, so far as our own experience furnishes us with analogous elements, an experience which was never actually ours. What is the sensory element in such judgments? There is certainly present (1) the actually given sensory continuum, the sense of bodily existence which spreads itself over our reconstructions and clothes the dry bones of narrative with our flesh and blood, fusing the living present with a merely imaginary past, so that we can say indifferently, either that we are transported bodily into the past, or that the past is made to live again in the present. In addition to this general sensory element, there is also present (2) some special stimulus which directs our thoughts to the past, to Rome for instance rather than to Athens, and to Caesar rather than to Pompey. Such stimulus is furnished as a rule, either by reading or hearing something about Rome or Caesar, or at least something which by association awakens thoughts of Caesar and Rome. The special sensation—*e. g.*, of visual or auditory signs of words—is here somewhat more remote from the reconstructed experience than we found to be the case in experiential judgments, while the general sensory setting seems to play a relatively greater part, but in the symbolic judgment also the sensory element consists of (1) the general background, and (2) some special stimulus arising against that background.

(D) **In Transcendent Judgments.**—In judgments such as "God is a substance consisting in infinite attributes," or "Things-in-themselves are absolutely unknowable," *i. e.*, in judgments concerning entities which could never be objects of sense-experience, it seems at first sight as though there could be no question of sensation, as though such judgments must be the product of pure thought, unmixed with any element from the sensory side of our nature. And yet, let us consider. The first judgment has meaning for Spinoza's readers, precisely because two of the Divine attributes are definitely known. These are "extension" and "thought," the "essence" of body and mind, much as we experience body and mind. It

is when we extend the attributes of Deity so as to include not only the two which we do experience, but also an infinity of others of which we are unable to form the slightest idea, that we realise the transcendent nature of the judgment. As far as we remain within human experience, so far we feel sure of our ground, so far the judgment has positive significance for us; the "infinity" of divine attributes would be utterly meaningless for us if we were not already acquainted with two of them, and could thus regard the rest as a kind of extension, by analogy, of our experience, could attempt to spread out this experience so as to cover, however thinly, the infinite. So far, however, as the judgment is strictly transcendent, so far as it deals with an infinity of attributes which we cannot conceive, cannot think positively, so far we are attempting to conceive the inconceivable and judge the unjudgeable. In other words, the transcendent judgment, natural and common as it is, really represents a failure to judge. Imagination takes the place of strict thought, and feeling—often, no doubt, sublime, but still, always merely subjective—usurps the place of critical reason.

These observations, however, do not solve our present problem. Because sense-experience is inadequate to extend itself over the infinite, it does not follow that the attempt is not actually made. In every judgment, however transcendent, there is, in fact, always present (1) the general sensory setting of our experience, which colors all our actual thinking, and projects itself to distant spaces and times to cover all objects of our present thinking, such as Rome, Caesar, the synapse, God—and in virtue of so staging them against the sensory background of our own experience, so colors them that they become *ours*, parts of our intimate self-conscious experience, here and now. However inadequate this extension may be in the case of transcendent, metaphysical entities, still, so far as we concern ourselves with them, we think of them in sensuous images, in radiations outward from our present center of sense-experience. Again (2), there is always present some special sensory stimulus which directs our thought into paths which lead beyond the knowable. Usually indirect, as in the case of the written or spoken symbols which furnish the starting-point for symbolic judgment, there is no element in sense-experience which is unable to guide us into thoughts which do often lie too deep for words. Our life rests always

upon a vast ocean of unsolved questionings, and any chance sensory stimulus may suffice to plunge us into the abysses of that ocean. Thus we see that in transcendent judgments also the sensory element consists of (1) the general background to which we have so frequently referred, and (2) a special stimulus arising against that background.

Summary.—If we now cast our eyes over the whole field of judgment, we notice that while in every case there is present a general and a special sensory element, these are present in varying proportions. As we pass from perceptual judgments, in which the intellectual element is relatively less important, to judgments in which the sensory element is slight and the intellectual interpretation almost everything, we observe that the general sensory background of experience plays an increasingly greater part. On the other hand, the special sensory stimulus, so vital in judgments of perception, becomes dwarfed into relative insignificance in the case of transcendent judgments. The stimulus, *e. g.*, of a red surface, which is so all-important for the perceptual judgment "This is red," is of importance for the experiential judgment less as an ultimate sensory quality, and more on account of the associations which it calls to mind. In the symbolic judgment, where the associations are more remote, its direct sensory quality is still less important, and finally, in the transcendent judgment, it is almost a chance affair, almost a matter of indifference: for all roads lead alike to the obstinate questionings which underlie and perplex our whole life. Great, however, as are these variations, in every judgment we find present both general and special elements of a sensory kind.

FOR FURTHER READING

W. James, *Principles of Psychology*, Vol. II, chapter xvii. H. Lotze, *Logic*, sect. 2. A. Riehl, *Der philosophische Kriticismus*, Vol. II, chapter 1. G. F. Stout, *Manual of Psychology*, Bk. II, chapters 1-41. *Proceedings of the Aristotelian Society*, N. S., Vol. VI, pp. 360-362 (quoted in B. Gibson, *The Problem of Logic*, pp. 82-84). W. Wundt, *Logik*, (3rd Edit.), Vol. I, pp. 10-69.

EXERCISES

1. What precisely are the sensory elements in: Here comes Mr. Smith. This bread tastes excellent. I am taller than you?
2. What precisely are the sensory elements in: Electricity is much more efficient than gas. Such requests for subscriptions always suc-

ceed. Eggs placed in water-glass keep for several months. I am always nervous before a large audience?

3. What precisely are the sensory elements in: Not more than three men in a thousand would vote for that program. At least fifty per cent of actions called criminal are due to our social system. For a small house, you will find hot water heating the most satisfactory?

4. What precisely are the sensory elements in: God moves in a mysterious way, His wonders to perform. Time and space are unreal forms of sense, and disguise the Real. I want to be a *great* Artist. I listened to the language of the birds; I knew what the trees whisper to each other?

5. Read Henry James: The Soft Side, pp. 8-9, 18-21, and summarise the essential nature of sensory experience, as there described.

CHAPTER III

VALIDITY AND THE SENSORY ELEMENT.

The Question Concerning Validity.—In the last chapter we have *described* the sensory element. We have treated it as a fact; as a constituent part of every judgment, and have stated what it is, what we find it to be. All this, however, is merely preliminary to a further question, which is the essential question for logic. It remains to ask, not what the fact is, but—what is the *value* of the fact? Being what we have found it to be, does this universally present sensory element in any way contribute to the validity of judgment? Do the correctness, certainty, reliability, truth of the perception of warmth, redness, *etc.*, depend on our bodily senses? Are we to regard sense-perception, the sensory element in judgment, as trustworthy *per se*, or as untrustworthy—or is it possibly indifferent? Is our sense of warmth, for example, perhaps merely a *de facto* condition, a transient event in our embodied experience without which we should not judge a given object to be warm—a mere happening and nothing more—and do we have to look elsewhere for *de jure* conditions, criteria which really test the correctness and trustworthiness of the judgment?¹ Are truth and certainty entirely an affair of “intellect,” or do our bodily senses themselves contribute something by which the validity of judgments can be tested and approved?

(A) **In Judgments of Perception.**—In judgments such as “This room is warm,” there is no doubt that our bodily senses play a great part. But the present enquiry is whether this part which they play is altogether reliable and trustworthy—more particularly, whether any attempt to justify such judgments inevitably ends in an appeal to sensory experience as such. Let us consider. Suppose I feel doubtful as to whether the room *is* warm after all. I can either (1) repeat my judgment, *i. e.*, give myself up to the sensory feeling, and assure

¹ Cf. Wundt, *Logik*, Vol. I, p. 80.

myself, by yielding completely to its guidance, that the room certainly does feel warm, or at least feels warm *to me*. This, in fact, is our usual procedure in testing perceptual judgments. That is to say, we accept as trustworthy the sensory element as such, and only doubt whether we really had allowed it full play, or whether we perhaps were inattentive on the previous occasion. If the second experiment confirms the first judgment, we are usually satisfied, and enquire no further. If, however, our judgment is challenged by someone else, or if we have reason to suspect that perhaps our senses are deranged, as by a fever,—in which case *we* may feel warm, although the temperature of the room may, in fact, be low—in such cases we usually appeal to a thermometer, or some similar objective measure, which represents changes of temperature in a way that appeals, at least immediately, to some sense other than that of warmth, *e. g.*, to the eye. This also, we must notice, is an appeal to sense-experience, and in actual fact, if hard pressed, we are driven to the conclusion that, whatever the thermometer may or may not state, whatever the temperature of the room may or may not be, *we feel warm*. We feel what we feel, and cannot be argued out of our feeling by any reasonings drawn from the reading on the thermometer, *etc.* It is a matter of sensory feeling, and not of reasoning. In other words, our conclusion is, that the sensory element in the judgment is our own feeling of warmth, and if our judgment that the *room* is warm turns out to be false, the falsity depends not on the sensory element, which is what it is independently of reasonings about it, but on the intellectual element which, given the feeling of warmth, interprets this experience as due to the temperature of the room. The same is the case with all the “illusions of sense” produced by mirrors, prisms, or cunningly devised appeals to misleading associations. The error depends in all such cases upon some misinterpretation which leads us away from the direct apprehension of the sensory elements. Where we are not so led astray, where we directly apprehend those elements, the simple sensory experiences are, as we have seen, “ultimate.” In perceptual judgments, all that we can do to test their validity is, avoiding interpretations and associations as far as possible, to analyse the given case and break it up into parts so simple, that an unbiased appeal to sensory apprehension can be made, and then—trust absolutely to our direct apprehen-

sion of the sensory elements. Direct, simple, sensory apprehension thus furnishes the ultimate basis for testing and approving the validity of judgments of perception.

(B) **In Judgments of Experience.**—In perceptual judgments it is, after all, not difficult to see that the sensory element must condition the validity of our thinking. We have merely to free ourselves from misleading interpretations and associations, and make an unbiased sensory judgment. But in judgments of experience the element of interpretation seems more essential. Such judgments represent a summing up of sensory experiences, and the associations are vital to the conclusion. Here, at any rate, there can be no question of “freeing ourselves from interpretations and associations”; for this would be to deprive us of the experiential judgment altogether. When, for instance, on the sensory basis of a low rumble, I interpret my experience as due to a freight train crossing the bridge, surely the case is parallel to the interpretation of a feeling of warmth as due to the temperature of the room; and when I further compare the disturbance due to freight trains in recent years with the similar experiences of the past, my judgment is surely based, at least in large part, on associations rather than directly apprehended sensations. Judgments of experience are thus more complex, and seem to require a different kind of explanation. And yet, if we wish to discover whether the sensory element here also is a conditioning factor in determining the validity of the judgments in question, we must ask, precisely as we did before, whether the attempt to justify such judgments inevitably ends in an appeal to sensory experience.

Let us consider. If such a judgment were challenged, how should we attempt to justify it? If it were doubted, for instance, whether the rumble was really due to a train, we should justify our statement by leading the doubter to the railway bridge, and letting him see as well as hear the train. We should then withdraw him gradually from the bridge, until he sufficiently realised that the roar of the train gradually changed into the low rumble in proportion as he drew further away from the bridge. This gradual transition would assure him of the continuity of the sensory experience, and he would come to see that what in the distance was a rumble was in fact one with what was experienced as a “roar of the train” in the neighborhood of the bridge. In other words, the proof

of the validity of our judgment consists in an appeal to actual sensory experience. In this proof we show that the rumble sensation is continuous with, and in fact *is*, the sense-perception of a distant-train-crossing-bridge. The intellectual element consists chiefly in so ordering the experience that the doubter could give himself up to the sensory side of the experience, and realise for himself its thorough-going continuity. This appeal to unbiased sensory experience is accepted as final.

"Yes," it might be objected, "but this is, after all, a perceptual judgment. You have an auditory perception of a distant train. This is a little more elaborate than the perception of bodily warmth, but in the last analysis is much the same kind of judgment, and in such judgments we have already granted that the ultimate appeal is to sense-experience. But how about a judgment in which the reference is not to present perception, but to the distant past? How could we justify such a thought as that the freight trains in recent years are, on the whole, more troublesome than the freight trains in the remoter past?" —Well, we answer, how *do* we justify such judgments? The usual method is to appeal to the memory of a number of the local inhabitants who have experienced the disturbance in question during a sufficient period of years. If their conclusions agree with ours, reasonable doubt is usually satisfied. In what, then, does this justification by appeal to memory consist? It is necessary to inquire, for the appeal to memory is certainly an appeal to associations, and associations, as we have already seen, are not always to be trusted. What, then, constitutes the essential difference between a trustworthy recollection, and an association which we cannot trust?

Let us compare the former case—in which uncertainty as to the distant train is removed by our realising the spatial continuity of (1) the rumble and (2) the train-actually-seen-on-the-bridge. In that case, the validity of our judgment rested on the spatial continuity of our sensory experience; the experience of the "rumble" actually was an experience of the spatially distant train. In the present case, can we show that the rumble-experience is actually an experience of trains distant in time rather than in space? In other words, does our sensory experience possess not merely spatial, but also temporal continuity? Let us examine this. The actual sensory present, according to psychologists, embraces a period of time

covering approximately from two to four seconds. Our sensory consciousness is at its best at this brightly illuminated focal point, and it is what is brought into the center of focus that we apprehend most clearly. But our sensory consciousness has also a margin, and from the brightness of the focal center to the darkness of the extreme margin there is unbroken continuity, a twilight which deepens by imperceptible gradations. So it is with memory, with our associations. The nearer they are to the present, the more closely connected with the focus of sensory experience, the more clearly can we apprehend their value. They are always continuous, in the stream of consciousness, with present sensation. What makes us trust the memories of yesterday, and mistrust the memories of many years ago, is not absence of continuity, but some difficulty in so ordering our mental vision that we can directly apprehend that continuity. Where we leap to conclusions without establishing that direct apprehension, we have untrustworthy, uncritically accepted, associations. In order to test critically the trustworthiness of memory, we experiment with our associations until we are in a position to apprehend directly, without bias, and in the present, some sensory element which extends with clearly unbroken continuity into the past. In such cases, just as the rumble-sensation is directly continuous with the spatially distant train-roar of two seconds ago, so also it is directly continuous with the temporally distant train-roar of ten years ago. In other words, the justification, in both cases, depends on establishing clearly the continuity of the sensory experience in space and time, in such a way that we directly apprehend the distant (whether in space or in time) as an extension, towards the margin, of the present focal sensory consciousness.

Finally, in the case of the inductive generalisations of empirical science, such as "All noise is a compound of tones," justification—or, as it is termed in science, "verification"—consists in an appeal to "demonstration," *i. e.*, to the direct sensory experience of a typical case. Thus we see that the judgment of experience is merely a complex and elaborate case of the judgment of perception, and, like the perceptual judgments examined previously, is dependent, for its validity, upon a direct apprehension of the sensory element. The associations which are misleading, and the interpretations which withdraw our attention from this sensory apprehension, must

first be removed; we can then realise the full force of the sensory elements, with their spatial and temporal continuity, and see how they underlie and make verifiable the judgments of experience.

(C) **In Symbolic Judgments.**—Based as it is on direct past experience, it is fairly intelligible that the experiential judgment can be verified in an extended perception, which fuses together past and present in a living unity. It is in fact an extended perceptual judgment. But when we come to the symbolic judgment, a striking difference seems to confront us. In the symbolic judgment we attempt to reconstruct, not a distant experience of our own, but—an experience which was never ours, an experience which may have belonged to someone else in the past, or may some day come to us—or to someone else—in the future. Surely in such cases there can be no question of extending some ray of focal illumination into the marginal twilight of our own consciousness—surely here there can be no resting upon the living continuity of our own sensory experience. In studying, for instance, the history of Rome, we note its occupation by Caesar. Except metaphorically, we cannot, of course, transport ourselves into the first century before Christ, and consequently there can be no question of “apprehending directly one continuous experience” which extends back to the occupation of Rome in one complex perceptual judgment. It would seem, then, that symbolic judgments require a different kind of explanation, a different kind of answer to the question which, here again, we must ask:—In attempting to justify such judgments, do we inevitably appeal, in the end, to sensory experience?

It must at once be admitted that no personal experience can possibly assure us of the certainty and truth of events which lie beyond that experience. What Caesar did when he entered Rome, we do not exactly know; as we should say, we were not there to see. Accordingly we have to trust to tradition rather than to personal experience, and to indirect, in place of direct, evidence. Furthermore, familiarity with the errors of history books, as well as with works of professed fiction, will convince us that we can not infer with certainty from the printed statement to the actual occurrence of events. The certainty and truth, then, of symbolic judgments is and must be, in the nature of the case, largely a matter of interpretation, of intellectual rather than sensory elements. In this

respect, symbolic judgments resemble the perceptual judgment "It is warm *in the room*"; *i. e.*, the immediate sensory element is something less than the total judgment, to which intellectual elements also have contributed their part. As in the perceptual judgment, in examining into its validity we abstracted from intellectual elements, and laid bare the sensory element in order to apprehend it directly in its full force, so here in the symbolic judgment, let us, for the present, leave out of consideration the intellectual, interpretative element, and try to discover what the sensory element is, and what part it plays in the something less than the total judgment.²

If, then, we leave out of consideration the question as to whether, in actual fact, Caesar did or did not occupy Rome in the manner indicated by our history book, what remains of the complex judgment? There remain two parts which we still think:—(1) we still think that the history book states that Rome was occupied by Caesar; (2) we still try to experience, as well as we can, the wider meaning of this statement, *i. e.*, what a Roman citizen must have felt, how Caesar himself must have felt, *etc.* In the first case, if our judgment is challenged—suppose it maintained, *e. g.*, that we have misread the book, that there is no literary evidence for Caesar's having occupied Rome—in such a case, the only possible appeal is to direct sensory experience of the printed page. The doubter must read for himself, and have direct sensory experience of the printed symbols, in order to reconstruct for himself their meaning. The final appeal is certainly here to the sensory element in experience, and the thought appears to be a complex perceptual judgment. "The book has black marks in it" is obviously a judgment of perception. If the black marks are regarded as symbols, "The book contains symbols" is still a perceptual judgment. So too with the thought that the book contains letters, words, English words, the definite English words "Rome" "Was" "Occupied" "By" "Caesar." In short, the judgment "The book states that Rome was occupied by Caesar" is a complex perceptual judgment, a legitimate extension of the direct sensory apprehension of the letters in the book. Or if it is the associations that we emphasise, by means of which we are able to read printed symbols,

² What in a history book is less than the total judgment is equivalent to what, in a work of fiction, is the entire judgment—*i. e.*, where no claim to represent the external course of actual events is made.

in that case we have a judgment of experience. In both cases, however, the appeal is certainly to direct sensory apprehension, and that is the final sensory evidence for the validity of our judgment.

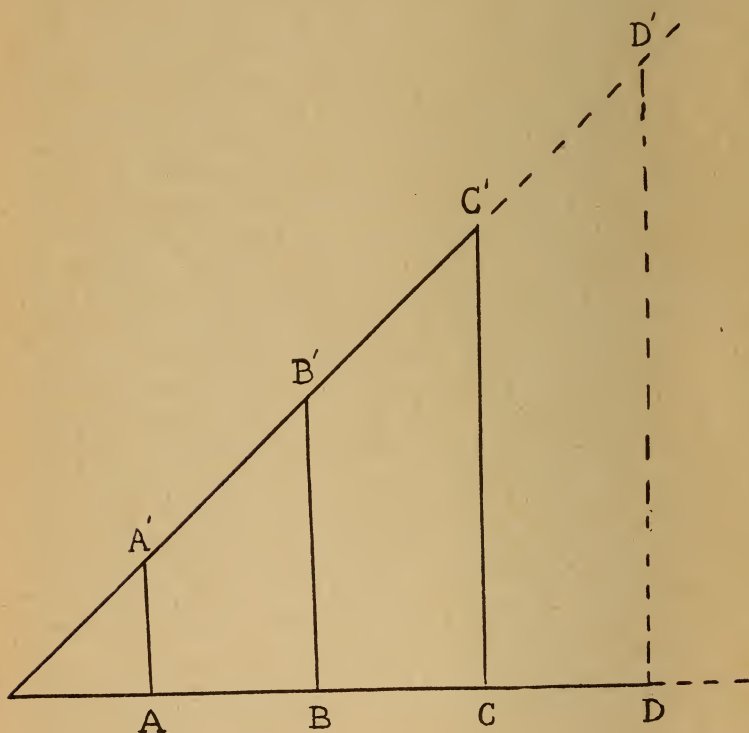
The second part of our judgment is an extension of the meaning which we apprehend in reading the book. If asked with what right we build up some approximation to the experience of Caesar and the Roman citizens, we can only state that it is a legitimate extension of the meaning of what we have before us. As we read the printed symbols, the sensory experience of the meaning of the words "Rome," "Occupied by," "Caesar," *etc.*, expands so as to include appropriate elements from our own experience in one continuous whole, so that, so far as this personal experience permits, our reading is no neutral exercise in spelling, but is a fuller, richer, more adequate experience which approximates to the actual life about which we are reading. We cannot feel the actual glow of triumph which animated Caesar's veterans, or the confusion and blind panic of the partisans of Pompey. We do not know the Italian sky, the Roman crowds. But we have all had *some* experience of triumph, or panic; we have all experienced warm suns, blue skies, jostling crowds. And out of such experiences we can piece together something which comes to us with the warmth and intimacy of personal experience, and at the same time represents for us, by analogy, a pale copy of the experiences symbolised, the Roman scenes about which we are reading. The validity of this reconstruction rests wholly upon the meaning of the documentary evidence before us. We must abstract from misleading associations and uncritical interpretations, and confine ourselves to legitimate expansion—the extension authorised by what we read. In other words, the validity of the symbolic judgment rests on the same evidence as the experiential judgment; as far as it goes, it is to the sensory element that we must at last appeal, as the touchstone of its validity.

(D) **In Transcendent Judgments.**—Transcendent judgments represent, in extreme form, an element which has been gradually forcing itself upon our attention in the preceding cases. In the inductive generalisations of empirical science, *i. e.*, in judgments of experience, the utmost we can do is to approximate to certain knowledge. There is always some slight gap between the law we wish to establish, and the evidence on

which it rests. Inductive evidence can come as close to complete proof as we please; but there is always some falling short, some inability to establish perfect certainty. In the symbolic judgment, this gap is still more noticeable. Reorganise and reconstruct our personal experience as we may, we can never quite enter into the experiences of others, or into experiences which have never yet been ours. Analogy, similarity, even partial identity—but we always fall short of full identity, always fail to pass completely the gulf which separates desire from performance. In other words, our actual experience always falls short of the ideal which, all unconsciously, drives us onward, and is the hidden root of that dissatisfaction with the not-quite-perfect, which fastens upon all of us at times. In the transcendent judgment, this ideal is more insistent, and the gap between what we actually attain and the perfection, the consummation of infinite desire, is at its greatest. We can never know Things-in-themselves, never experience the First Cause, the underlying Substance, the Divine, perfect, infinite experience in which all the illusions and weaknesses of our finite, pitifully thwarted efforts are transcended and made perfect in harmony. If empirical and symbolic judgments cannot be completely verified, how much more is this the case with transcendent judgments! It is of their very essence to pass beyond the realm of experience, of the definitely verifiable. They are not given in sense-experience, but are constructed by idealising intelligence. "Things-in-themselves" mean things beyond that aspect which alone we experience; "God" is infinitely greater than the ideas we can form of Him; the "Vision Splendid" which guides our efforts towards a better, finer life, is broader, deeper, sublimer than anything we have known. How then can we, in the present case, ask whether the validity of such judgments depends on the sensory element in our thought?

The case is, perhaps, not as hopeless as it appears. Transcendent judgments, as we have already seen, contain *some* sensory elements. The metaphysical concept of God does not stand out of all relation to human life: the sphere of ideal Divinity, though larger than mere humanity, includes all our experiences as a portion of itself; the finite is not only transcended by, but is contained in, the infinite. All ideals, in short, which can have tangible meaning for us, stand in *some* connection with our sensory experiences. Let us consider a

concrete case, for instance, the ideal of personality. We form our concept by putting together appropriate experiences of our own, and then transcending them in the direction of a more inclusive, or rather an *all*-inclusive concept. We can take, for example, the personality of the grammar-school graduate, of the high-school graduate, of the university graduate, and link them up with our present focal consciousness in a curve representing the development in personality, somewhat thus:—



So far, we have a judgment of experience, a complex summing up of many perceptions. But we can go a little further; we can produce the line of the curve a little beyond the point we have actually reached; we can look ahead, and, so far as we remain within reasonable limits, we transcend

indeed actual experience, but remain within the sphere of possible experience. That is to say, we form a symbolic judgment, and the concept of a not-yet-experienced personality which we thus form is justified precisely so far as it rests upon, and is a strictly continuous expansion of, the sensory elements linked together in the experiential judgment. It is in fact an extended judgment of experience, and its validity depends on the accuracy with which we have avoided misinterpretation and have remained faithful to the direct perceptions of that experience. What then is the transcendent judgment? It is an extended symbolic judgment, a judgment which extends beyond the limits of possible human experience—a judgment which includes, indeed, all that we have experienced, but expands to infinity, to *all-inclusiveness*. All our thoughts, feelings, and wishes, in a word all our experiences, so far as they contain inklings of an ideal—point beyond themselves to this infinite, in which every thought finds its ideal completion, every feeling its full expansion, and every wish its perfect satisfaction. Our whole life is lived in the midst of this infinite ocean, and there is no sensation, no element in our living experience, which does not naturally extend itself without limit, seeking to transcend its own finiteness and to attain the full completeness of absolute Reality.

That such metaphysical constructions go beyond the immediate sensory elements of experience, we have already seen. And yet, if we wish to justify the validity of some such extension, to what can we appeal, if not to the experience which is thus to receive its ideal completion and satisfaction? The concept of God must satisfy our ideal aspirations, the hopes embodied in our concrete, living experience. The ideal of Personality is surely a concept in which we find—developed, transformed, our baser parts transmuted—our own selves, the “true” self, at present hidden, thwarted, dwarfed by extrinsic circumstances, but to the eye of thought revealed in that full perfection which alone could completely satisfy us. In other words, like the symbolic judgment of which it is an extension, the transcendent judgment is dependent, for its validity, so far as we can here speak of validity, on the accurate and direct apprehension of the sensory elements which extend continuously from the focal sensory consciousness.

Conclusion.—Let us try to put together the results reached in considering the different types of judgment. In the pre-

vious chapter we saw that, in passing from the simpler to the more complex types of judgment, the intellectual element plays a greater part, while the sensory element plays at any rate a less direct part. We saw further that the direct sensory stimulus plays a greater part in the simpler judgments, while in the symbolic and transcendent types the stimulus is relatively overshadowed by elements from the sensory background. In the present chapter we have deliberately neglected, so far as possible, the intellectual element. We have noted in passing that to judge, on the basis of a warmth-sensation, that "*The room is warm*" requires intellectual elements, and that in experiential, symbolic, and transcendent judgments the gulf between direct sensation and full judgment has grown increasingly wider, *i. e.*, that the intellectual element has played an increasingly greater part. But leaving to subsequent chapters all consideration of the part played by intelligence, we have confined ourselves to studying the importance, for the validity of the judgment, of the directly apprehended sensory elements.

In estimating the validity of a perceptual judgment, which asserts the warmth, heaviness, redness, *etc.*, of an object—*i. e.*, which deals in sense-qualities—it is impossible to test its accuracy without appealing to sense-perception, in which alone such qualities can be directly and satisfactorily experienced. Similarly in judgments of a more complex type, so far as these are based upon sensory experiences, any test of validity must, in the end, appeal to direct sense-experience; for complete satisfaction of the desire for validity we must be able to apprehend in one continuous connection the sensory expansion from the stimulus which includes the relevant elements drawn from the general sensory background of experience. The same is true of symbolic and transcendent judgments; so far as these contain sensory elements, we can pass upon their validity only so far as we come to apprehend clearly and without bias their continuous connection with the focus of sensory consciousness. So far as symbolic and transcendent judgments really follow without deviation the directions approved by the judgments of experience—*i. e.*, so far as these extensions of the focal consciousness are governed strictly by elements verifiable in sense-experience, so far they are valid. So far as, without violating this condition, they transcend sensory veri-

fiability, they may or may not, perhaps, be valid, but such a point cannot be decided by an appeal to sensory experience.

To sum up:—So far as judgment contains sensory elements, our thought depends on sensory apprehension, and so far as the sensory apprehension to which we appeal is valid, so far the judgment, at least on its sensory side, is valid. In its simplest and most direct forms, sensory apprehension is ultimate and its validity beyond reasonable question. Consequently, so far as the sensory element in judgment can be reduced to these simplest and most direct forms of sensory apprehension, or to some legitimate and continuous extension of these,³ so far the judgment itself, on its sensory side, is valid. In other words, so far as judgment contains sensory elements, its validity depends wholly upon direct apprehension of those sensory elements. So far as it contains elements other than sensory, nothing has been as yet determined as to its validity, though a suspicion has been expressed that the validity of other elements also will be found, in the end, to be verifiable only by reference to sense-experience. The consideration, however, of such non-sensory elements must be deferred to the succeeding chapters.

³ Cf. Bosanquet, *Logic or the Morphology of Knowledge*, Vol. I, p. 77: "Reality is given for me in present sensuous perception . . . The real world, as a definite organised system, is for me an extension of this present sensation and self feeling . . . and it is the essence of judgment to effect and sustain such an extension. . . . The given and its extension . . . are continuous with each other . . . It is the character and quality of being directly in contact with sense-perception . . . that forms the constantly shifting center of the individual's real world, and spreads from that center over every extension which the system of reality receives from judgment."

FOR FURTHER READING

H. Lotze, *Logic*, sect. 267, pp. 337-339. Chr. Sigwart, *Logic*, Vol. I, pp. 262-264. W. Wundt, *Logik*, (3rd Edit.), Vol. I, pp. 78-83.

EXERCISES

1. Is it necessary to appeal to sense-experience in order to justify the following: The sky is blue. This water is hot. This book is heavier than that?
2. Is it necessary to appeal to sense-experience in order to justify the following: Fire burns. Water quenches thirst. Peaches are in season in September?
3. Is it necessary to appeal to sensory experience in order to justify the following: This student has been below grade so often that it is mathematically impossible for him to reach a passing grade

this term. Nothing venture, nothing win. It's sure to be cold in January?

4. Is it necessary to appeal to sense-experience in order to justify the following: It is impossible for a thing both to be and not to be. *Ex nihilo, nihil fit*. The soul is a simple, uncompounded substance?

5. Do novelists find it necessary to appeal to sensory experience, even when they describe such entities as we have never experienced, and probably never could experience? Take as an instance, any one of H. G. Wells' romances, such *The Angel*, *The Sea-Lady*, *The First Men in the Moon*, *The War of the Worlds*, *The Invisible Man*, *The Food of the Gods*.

CHAPTER IV

THE INTELLECTUAL ELEMENT IN JUDGMENT.

The Intellectual Element.—What we understand by “validity of judgment” has not been exhausted in our preceding discussion. We have seen that a judgment is valid, so far as it really forms part of the continuous stream of sensory experience. But by “validity” we certainly mean more than “forming part of a continuous sensory stream.” The very notion of validity, correctness, accuracy, truth, or however we name it, seems to go beyond this idea that it is “given” to us. We contrast a judgment which is valid, not with one which is non-sensory or discontinuous, but with a judgment which is invalid, unsound, false. For the great majority of logicians, judgment is concerned not so much with the sensory side of experience, as with intellectual truth and falsity. For sensory continuity they would substitute intellectual consistency, and would throughout appeal, not to sensory, but to intellectual standards of validity. From continuity as such, for example, we should never obtain fixity, definiteness, clear-cut outlines. Where everything is fluid, as in a process characterised by unbroken continuity,—in such a case sameness, identity, sharp distinction, *i. e.*, the necessary instruments for the erection of complex thought-structures, are absent. Most logicians, accordingly, emphasise the intellectual side of judgment, and regard sensation as furnishing the material for thought, while intellect introduces its standards into this otherwise formless material, gives it definite form and outline, identity and distinctness, and on the basis so provided elevates structures in which intellectual instruments have so far transformed the original material that a great thinker like Kant can assert that reason only apprehends what itself has constructed in accordance with its own laws.

It is not quite correct to regard sensation as merely furnishing the bare material for thought. The sensory consciousness is no pure receptivity, no *tabula rasa* open to impressions from any and every kind of object alike. As we have seen, it is

already, in its simplest stages, shot through with instinct, and presents a rudimentary organisation. The animal consciousness, for instance, is not a chaos of unrelated sensory elements, but is an organised whole in which even abstract general ideas may play a part.¹ We cannot, therefore, regard all organisation of the sensory elements as exclusively the work of intellect. And yet, what the central nervous system secures for the animal at one level of development, the intellect—perhaps here also by means of the nervous system—secures at a higher level. The function of intellect is undoubtedly a kind of organisation, but an organisation which, for full comprehension, must be regarded from the view-point, not so much of its survival value for the organism, as of its conforming to its own peculiar standards, which are intellectual rather than biological.

In order to obtain a clear idea of the nature of this intellectual element in thought, we shall trace its function in judgments, starting with the more simple, and proceeding gradually to the more elaborate.

(A) **In Judgments of Perception.**—In “This is red,” “The room is warm,” and similar judgments, the impulse which induces us to single out for special attention the redness or warmth of some object which stands out against the general sensory background, is probably not intellectual interest. It is usually some feeling which, at bottom, is instinctive. Such feelings lead us to distinguish between background and stimulus, and thus to organise in a rudimentary perceptual relation “This” and “red,” or “The room” and “warm.” But at this level of consciousness the distinction is not clear-cut. The one is not yet a “background” and the other not yet a “special stimulus.” Neither has as yet a precise identity in virtue of which they are distinguished sharply and clearly from one another and deliberately set over against each other in an intellectually self-conscious act of thought. The introduction of such standards as “identity,” “difference,” “distinction within a wider whole which unifies,” *etc.*, in a word, the raising of the vaguely felt organisation of sensory consciousness to a higher power, is the work of intellect. Furthermore, if we have any reason to mistrust our judgment, as

¹ Cf. Wundt, *Logik*, 3d edition, Vol. I, chapter 1, sects. 2–4. Cf. also Erdmann, *Logik*, 2d edition, chapters 11–14, pp. 65 and 71.

when the object is only dimly seen, or when our conclusion is denied by a second person, in such cases the critical self-consciousness with which we take especial notice and make a deliberate judgment, is intellectual, and involves reference to intellectual standards, which must be conformed to before we are satisfied. In such cases the judgment is accepted, not merely as conforming to sensory apprehension, but as fitting into a system, an organised whole, of judgments of the kind in question. In perceptual judgments this reference to a wider system of perceptual thought is not clearly present, unless our conclusion is challenged. But it is always a part of the intellectual background, and if asked to justify our judgment, there is always some direct reference made to this systematic, self-consistent unity of our thought.

Let us emphasise the difference still further. Sensory apprehension of "This," "red," "warm," *etc.*, is vague, not precise, without outlines, not clear-cut, and forms part of a living stream of consciousness which extends continuously from the focal present in many directions without limit. Intellectual apprehension of the same qualities is sharply defined, an apprehension of entities taken out of the stream, and delimited against each other in such a way that their natures, that which makes them what they are known to be and distinguishable from what they are known not to be—stand out as clearly envisaged identities. Sensory apprehension is an awareness of qualities in the living kaleidoscope of flowing consciousness. Intellectual apprehension is the result of an analysis which takes the constituent elements out of their movement in the kaleidoscope and subjects them, one by one, to the scrutiny of our mental microscope, and studies them in their own individual natures, as well as in their sharply defined relations to other contrasted elements. For sensory apprehension, every moment of our changing life is *new*, a variegated texture of experience in which no element is ever apprehended a second time in quite the same setting. Just as we cannot step into the same stream twice, because the water into which we stepped before is now washed away down-stream, so the red which we experience today is not the same as the red which we experienced yesterday. External conditions have changed; we also have changed. *Varium et mutabile semper homo*. For intellectual apprehension, on the contrary, all is fixed and definite. The redness of a sen-

sory experience is abstracted from its sensory context, is cut off and fixed by the mind; and for the intellectual elaboration of perceptual experiences we use, not directly apprehended sensory elements, but a kind of mental counters, not a particular red or warm, but universalised qualities, *red-ness*, *warm-ness*, qualities that never were on sea or land, artificial entities, abstractions, non-living, discontinuous, conceptual identities. The function of intellect in judgments of perception is thus to introduce, into the continuity of sensory experience, ideals of identity, difference, and organisation. The elements thus hewn out with our mental hatchet are so transformed that they constitute portions of an intellectual system, amenable to standards of consistency, systematic unity, and identity.

(B) **In Judgments of Experience.**—In “The freight trains crossing the bridge grow more troublesome every year,” and similar judgments, the sensory element consists, as we have seen, in so extending the focal consciousness of the low rumble as to include in one continuous experience, not only the spatially distant train of the present, but also the temporally distant trains of the past. The associations from past experience, by the aid of which this is accomplished, are fused with the focally present rumble-sensation, and constitute an integral portion of the unbrokenly continuous sensory consciousness, the complex perceptual judgment which we call a judgment of experience. Certain elements in the judgment appeared to us to be definitely intellectual, especially the “comparison” of the past trains with the present rumble. For the rest, the interpretation of the rumble as a present train, and the recall of past trains, together with associated feelings of annoyance at the disturbance so occasioned—all this might well take place on the sensory level of consciousness. In the usual run of things, such a judgment, expressing, as it does, mere general annoyance at being disturbed, would not tend to rise much above the sensory level. But where it is thought advisable to establish such judgments of experience more firmly, or where we are not content with a vague general impression but insist on investigating further the phenomenon in question—in such cases we raise the judgment to a higher power by introducing exact intellectual standards.

How is this accomplished? We analyse or split up the continuous experience into a number of special cases of trains-on-

the-bridge, and pass these special cases in review, one by one, noticing in each case, as well as we can, the comparative amount of disturbance thus caused. The distinction of these special cases from one another, and the careful attention to the disturbance occasioned by each one, considered by itself, gives us a far clearer basis for our comparative judgment as to the increase of the disturbance in question. The intellectual standards thus introduced are (1) identity—the introduction of which gives us units sufficiently equal or identical for comparative purposes, *viz.*, the individual train-disturbances; (2) difference—the introduction of which enables us to separate out and distinguish, both from one another and from experiences which are irrelevant, the various unit-experiences on which the judgment is based; and (3) organisation—the introduction of which enables us so to classify and arrange these disturbance-units within the wider system of our general experience, that we can conclude as to the increasing annoyance, within that system taken as a whole, of the rumble due to the passing trains. In place of the continuity and vague flow of actual sensory experience, we thus have sharply differentiated, isolated train-disturbance experiences, compared with one another in the light of general principles of organisation. The clearness thus introduced is due to a definite transformation of the original experiences, and the intellectual side of judgment is based always upon elements thus torn from their living context and fixed, universalised, organised on principles, not of sensory continuity, but of strict relevance, consistency, conformity to intellectual standards of systematic unity and truth. It is in this way that bare sensory experience is taken up into the structure of an empirical science based on judgments of experience.

(C) In Symbolic Judgments.—In “Rome was occupied by Caesar” and similar judgments, the sensory side consists of elements taken from different strata of our conscious life and so re-arranged as to present, with something of the warmth and intimacy of personal experience, an analogy to the experiences actually lived through by Caesar and the citizens of Rome at the time of which we are reading. The nature of the part played by intellect is here conspicuously plain. In the taking elements from different experiential strata, *i. e.*, in the tearing from their living contexts what we need for our purposes of construction, we clearly see the introduction of

the standard of identity. With the idea of one identical experience-type in mind, we pass in review the scenes from our past which association presents, and single out only what passes the test and conforms to our standard. By introducing the standard of difference, we are able to distinguish and omit from the reconstruction everything in the experiential context which is irrelevant to the demands of identity. Finally the standard of organisation enables us so to reorganise these mutilated experiential fragments, as to reconstruct something consistent with the wider system of life which extends from the present to the time of Caesar, and is guided in detail by the text of our history book.

In symbolic judgments there is, however, yet further evidence of intellectual transformation of the original experiences. The unit here is not merely the isolated triumph and panic-experiences, but is something still more artificial, still more conventional. So far as our judgment is an expansion on the basis of the printed symbols before us, the ultimate units are these symbols themselves. It is on the apprehension of these devitalised, conventionalised, universalised elements that our reconstruction of the meaning of what we read is ultimately based. Perhaps we can realise the part played by such conventional symbols more clearly in a different instance. Take the case of a problem about ducks, or workmen, or time-pieces, solved by simultaneous equations; take the case of the distance of a ship at sea, discovered by an appeal to trigonometry; take any case of the movement of bodies, worked out in mathematical physics. In all such cases, the raising the problem from bare sense-experience to the intellectual level means the substitution of identical, conventionalised elements in place of the original continuity of life, and it is on the organisation of such symbols, strictly treated as such, that the certainty of the conclusion depends. In taking up our sensory experiences into the structure of empirical science, an enormous part is played by the introduction of such conventional identities, and it is hardly too much to say that our experiences admit of scientific manipulation only so far as they can be organised and formulated in terms of such symbols.

(D) In Transcendent Judgments.—Transcendent judgments, as we have seen, constitute a kind of extended symbolic judgment. The elements, for instance, from which we construct

our concept of the "infinite attributes of God," are taken from our own experience, and the resulting ideal represents man writ large—so large as to extend to infinity, *i. e.*, infinite knowledge, infinite power, infinite justice, *etc.* In symbolic judgments the conventional identities separated out and organised still retain *some* relation to our experience, and it is from their fringe of associations that they have meaning for us. That is to say, the conceptual identities on which the judgment is based are not entirely cut loose from experience; it is mutilated fragments which we put together, torn, if we will, from the living experiential context, but still retaining some semblance of life, some clinging strands which bind them to our personality and make them ours. In that extension of the symbolic judgment which we call transcendent, however, this relation is reversed. *Our* experience is here regarded as an isolated fragment torn from its true place in the Infinite Experience; to consider it as our own, as constituting a finite unity in itself is, for transcendent thinking, a fundamental error. We must learn, on the contrary, to view everything *sub specie aeternitatis*. It is the Divine Experience, of which we can construct the outlines, which is real and concrete; it is our sensory experience, with its continuity in space and time, which is fragmentary, riddled with contradictions, unreal, abstract. To arrive at metaphysical truth we must start, indeed, with our human experience; but by strictly introducing the standards of identity, difference, and organisation—far more strictly than in experiential and symbolic judgments—we pass, step by step, from the ideas of human goodness, human knowledge, and human power, to greater-than-human, and finally to ideas of absolute, infinite, Divine goodness, Divine knowledge, Divine power. It is by the strict introduction of these intellectual standards that we not merely reach the extreme limit of possible human development in goodness, *etc.*, but ultimately cut ourselves loose from the remaining strands which bind us to humanity, and arrive at ideas which far transcend these in dignity and power. We use these intellectual standards as a kind of tower by which to climb the steep ascent of heaven. After the top of the tower is reached, in physical reality we could mount no higher: but knowledge is not a physical tower, and has no such limitations. From the purely human standpoint, it would seem as though the more strictly we introduce the standards of

identity, difference, and organisation, the more attenuated becomes the living, sensory experience with which we start, and the more formal, artificial, and devitalised become the concepts which we thus construct, until finally the symbol is cut loose from life as we know it, and sense is lost in intellectual vision. For the metaphysician, on the contrary, this death to sense is the beginning of intellectual life, and in the ultimate constructions of transcendent thought we shake off the fetters which bind us to the earth below, and become one with the ideal which is also the only real, and by way of the intellectual love of God, enter directly into the Divine Experience.

Conclusion. The Function of Intellect.—If we now put together the results of our enquiry in the present chapter, we realise that, as in other spheres of life, so also in the realm of thinking, the function of intellect is to organise our life-processes and make them significant and rational. The context of living sensory experience contains much which is irrelevant and accidental, due to considerations of place and time. Our intellectual analysis leaves us with sharply differentiated identities, taken out of this context and stripped of all irrelevancies—timeless and placeless entities, mental counters which can be put together in accordance with the demands of systematic unity. Out of these elements we proceed to build up an edifice of thought, a structure based on principles which are intellectual rather than sensory. It is like introducing the card-index system into our business—in this case, the business of thinking. We can now take hold of our experiences, sort them out and handle them, shuffle and manipulate them in such a way as to gain all the advantages of scientific efficiency; and the resulting clarity and distinctness are undoubtedly a real gain.

But there is another side to this process of intellectual analysis. Something has been lost which can not be replaced. The elements into which we have analysed our experience are no longer fluid and living. They are discontinuous, fixed, rigid, lifeless. They are like the separate pictures which a moving-picture artist puts together to represent some drama of real life. The utmost which the scientist can do in order to visualise intellectually the life which he has split up into fragments, is to put together the separate pieces in such a way as to simulate the original movement and continuity. But

the result is always jerky, unnatural, interrupted by flashes, by sudden transitions, by small flaws in the film. When all is done, it remains artificial, mechanical, a photographic imitation of life. The scientist is like a child with a toy which he has taken apart. He understands now how it works, but not all the king's horses and all the king's men can put together that unsightly heap of torn flesh and dissected organs which was once a living frog or embryo chicken. Thus we see that the introduction of intellectual standards of identity, difference, and organisation gives us clearness, certainty, science; but at the same time we realise that this clearness has been bought with a price: and it is a question how far the transformations wrought by intellect are legitimate. Before, however, entering upon this question—the general question of validity—we must study more in detail what is meant by identity, difference, and organisation.

FOR FURTHER READING

J. Dewey, *Essays in Experimental Logic*, pp. 220–229. H. Lotze, *Logic*, pp. 10–18. A. Riehl, *Der philosophische Kriticismus*, Vol. II chapter i. W. Wundt, *Logik*, (3rd Edit.), pp. 74–90.

EXERCISES

1. What precisely are the intellectual elements in the following: That suit looks old. These nasturtiums have an unpleasant odor. The corn grows larger here in the sun than what I see there in the shade?

2. What precisely are the intellectual elements in the following judgments: What we call "robins" are usually starlings. *Mr. X's* hand-writing is nearly always illegible. In the more northerly States, a closed sun-porch is of more use than an open screened-in porch?

3. What precisely are the intellectual elements in: There is no Royal Road to success. Income: \$100.00; expenditure: \$99.99; result: happiness. Expenditure: \$100.01; result: misery. According to results based upon statistics, I should expect three members of this class to receive the grade A?

4. What precisely are the intellectual elements in the following: Whether it falls within the sphere of possible experience or not, the world *must* be rational, through and through. I solved the riddle of the universe. Virtue, though chained to earth, will still live free!?

CHAPTER V

IDENTITY.

Identity or Sameness.—What do we mean by “identity”? We should usually say that any one thing or idea is identical with itself, is the self-same thing that it is: beauty is beauty, $x = x$, etc. Where we have what appear to be two entities, we should call them identical if they were the same in all respects; *e. g.*, two triangles on the same base and with the same apex would coincide or fall together absolutely. They would be identical, and taken strictly we might maintain that there was only one triangle present. Identity thus excludes diversity or difference; so far as entities are different, they may be equal, but can never be strictly identical. *E. g.*, $x^2 - y^2 = (x+y)(x-y)$. Both sides of this equation lead to the same result: but the intellectual operations of squaring and subtracting, on the one side, are balanced by *different* operations on the other, *viz.*, addition within brackets, subtraction within brackets, and multiplication of complexes. The two sides of the equation are thus equal, but not absolutely the same, not identical.

Let us apply this to logic. We have stated that intellect introduces identity as a kind of standard for organising our otherwise somewhat chaotic sensory experience. This means that intellect singles out from the heterogeneous mass of sensory elements those which are of one and the same kind, and regards these as units, as identical points with reference to which our thought builds up its characteristic chains of judgments and inferences, the elaborate structures of science and art. Let us consider in detail how this introduction of identical points of reference takes place.

(A) **In Judgments of Perception.**—Take such a case as “This room is warm.” We are seated in the room, and direct our thought to the temperature, neglecting the geometrical, social, and other features of the room. Omitting from consideration all other elements in the sensory environment, we single out only such stimuli as have one and the same refer-

ence, *i. e.*, stimuli which possess temperature value. In this way we come to judge that the room *is warm*. The warmth of the room is thus one unit within the judgment, an identical point of reference around which our thought, which was previously vague and fluid, crystalises and becomes clear and precise. Another such unit within the judgment is undoubtedly "the room." This unit is constituted by direction of our thought to those features of our sensory environment which all alike are connected with the place where we are seated, rather than to the hundred and one other elements which might similarly have been selected as starting-points for judgment: *e. g.*, our work, our furniture, other people in the room.

In this way, in place of the vague fluctuating mass of relatively unorganised sensations, we come to have a new organised unity, our judgment, and within this judgment two special unities, the starting-point and the end-point of our thought. These two special units, "The room," with which we start, and the "Is warm," which represents our further determination of "The room,"—are recognised by logicians under the names "logical subject" or *S*, and "logical predicate" or *P*. *S* means the subject of discourse, what we are thinking about, and *P* means what we judge or think about *S*. They are thought with reference to one another; the room is thought of as being warm, and warmth is thought of, not in a general way, by itself, but in definite relation to the room.

The *logical* distinction between *S* and *P* should not be confused with the *grammatical* distinction between subject and predicate of a sentence. One and the same thought can have a hundred modes of grammatical or rhetorical expression, and it is easily possible for *S* to be expressed by the predicate of a grammatical sentence, and for *P* to be expressed by the grammatical subject. Usually, indeed, there is a tendency for *S* and the grammatical subject to coincide; but there is no *necessary* connection between the order of thought and the order of verbal expression. Consider, for example, "It is warm in the room," "Warmth is the most pronounced feature of the room," "Warm ist das Zimmer." These are three different ways of expressing one and the same thought, and if "The room" is the subject of discourse, it remains the subject of discourse whatever the grammatical or linguistic vehicle of expression. Or again, if I am discussing the subject of warmth, so that "warm" or "warmth" is *S*, the subject of discourse,

I might say, "Warm is . . . why, *the room* is warm, *the fire* is warm, *the water from this faucet* is warm, *etc.*" In this case, "warm" still represents the logical subject, while "the room," "the fire," "the water from the faucet," *etc.*, represent new logical predicates, units singled out of the sensory environment in order to explain or determine further what we are thinking about, *viz.*, "warm." Thus we see that *S* and *P* are independent of grammatical distinctions,¹ and in the case of perceptual judgments, we can conclude generally that our thought organises the vague flow of sensory consciousness by crystalising around two points of reference, a starting-point and an end-point of judgment, *S* and *P*, respectively. This introduction of the standard of identity gives us clearness, definiteness, and precision, an intellectualised basis for the further organisation of our thought.

(B) **In Judgments of Experience.**—In such a judgment as "The freight-trains crossing the bridge grow more troublesome every year," we are seated at work, and are disturbed by the rumble of a passing train. Our reaction to this disturbance expresses itself in a feeling of annoyance, and with this feeling in mind, we omit from consideration all other elements in the environment, and single out from our experiences only such as have one and the same reference, *viz.*, comparative annoyance-value of this type. In this way we come to judge that the disturbances are growing more annoying every year. The increasing annoyance of such disturbances thus constitutes one unit within the judgment—an identical point of reference around which our experiences, which were previously fluid, vague, and somewhat chaotic, become organised, determinate, fixed. In place of the vague continuity of annoyed feeling, we have reached a definite conclusion, in which we can rest. Another such point of reference is undoubtedly the disturbance-due-to-freight-trains. This furnishes the starting-point of our judgment, and it is of this freight-train disturbance that we judge, that it "is growing more troublesome." As a unit of reference it is constituted

¹ We may further suspect that in reading or hearing a grammatical sentence taken out of its context, it is impossible to say with certainty what is the subject of discourse. From such a statement as "That is a tree," taken by itself, it is impossible, apart from the context, to decide whether it is "trees" which are being spoken of, in which case "Tree" is *S*, and "That" is *P*, or whether "that object" is being discussed, in which case "That" is *S*, and "is a tree" is *P*.

by concentration of our thought in the one direction, and by omitting from consideration our work, the room we are in, and the hundred and one other features of the sensory environment in which we might have been especially interested.

In this way, in place of the general attitude of attention to our work, with vaguely felt impressions coming in through all the avenues of sense in an unorganised manner, we come to have a new organised unity, our judgment concerning the increasing annoyance of train-disturbances. Within this judgment we have two subordinate unities, (1) the starting-point of our thought, *viz.*, the disturbance, and (2) the end-point, *viz.*, that it is growing more troublesome. The disturbance due to freight-trains is thus the subject of discourse, or *S*, and the thought that such disturbances "are growing more troublesome every year" is what we judge of *S*, *i. e.*, is the logical predicate, *P*. The introduction of the standard of identity thus gives us, in place of the irrelevant and chaotic elements which form part of our continuous experience merely because they have happened to us, for the most part as mere events without rhyme or reason—in place of this relatively unmeaning and unorganised continuity, the introduction of identity gives us clearness, relevance, definiteness, an intellectualised basis for the further organisation of our thought.

(C) In Symbolic Judgments.—In such a judgment as "Rome was occupied by Caesar," we are reading a history book, and, in order to realise and make vivid to ourselves the meaning of what we read, we summon from our personal experiences all which have one and the same reference, all which in some way bring to our mind feelings of panic and triumph suitable to the occasion, the alarm of the Pompeians being balanced by the triumph of Caesar's friends. In this way we come to realise, as well as we can, what "occupation" means, and thus to think fully, or to judge, with full understanding of the meaning of our judgment, that Rome "was occupied by Caesar." Was-occupied-by-Caesar thus furnishes one unit, one point of reference within the judgment, around which the rough mass of associated experiences becomes gradually organised, and crystallises in definite clear-cut form. Another such nucleus of organisation is furnished by "Rome," the general scene and background against which the various episodes of Roman history successively stand out. It is the starting-point of our judgment, and is constituted by the or-

ganisation of all of our experiences which relate in any way to the Eternal City. It arises in our minds as a unit of reference when we direct our thought only to such elements of our experience as are connected with the great city about which we are reading, to the exclusion of other subjects of possible interest. "Rome" is thus the subject of discourse, the *S* of our judgment; "was occupied by Caesar" is what we judge about Rome, *i. e.*, is the logical predicate, *P*.²

In making this judgment, what have we effected? In place of the general attitude of mind consequent upon reading, with vaguely continuous sensory impressions coming in through eye, ear, *etc.*, and vaguely aroused associations from past experiences, unorganised and irrelevant but all forming part of the cross-section of our conscious life, we have now a sharply distinguished, clear-cut unity, the judgment that "Rome was occupied by Caesar." Within this unity we have two definite unities, (1) Rome, the background of our history and starting-point of our judgment, and (2) "was occupied by Caesar," the new determination of our thought of "Rome," the end-point or logical predicate of our judgment. These two units, *Rome* and *was-occupied-by-Caesar*, are clear fixation-points introduced as identical points of reference by the intellect, and furnish an intellectualised basis for the further organisation of our thoughts, as we build up for ourselves an adequate conception of History.

(D) In Transcendent Judgments.—If we compare the logical subjects in the various types of judgment considered hitherto, "The room," "Freight-train-disturbances," "Rome," we notice at once that, while all alike are units or identical points of reference within their respective judgments, yet the later ones are increasingly complex. "The room" is a matter chiefly of present sensory experience; "Freight-trains" include not only present sensory experience, but also the relevant experiences which go back for some years; and "Rome" furnishes a centralising nucleus for very many and very complex experiences. So too when we compare the logical predicates. "Warm" is a relatively simple experience as compared with "grow more troublesome every year," and

² It is assumed in the text that it is a "History of Rome" which we are reading. If, however, the sentence occurred in a "Life of Caesar," so that Caesar and his actions constituted the main subject of discourse, then "Caesar" would be *S*, and "Rome-occupied-by-him" would be the further determination of this subject, which we call *P*.

this again is less complex than the innumerable series of experiences which are organised with reference to the idea of "was occupied by Caesar." As we have seen before, judgments of perception are organisations chiefly of immediate sensory experiences; judgments of experience include also associations from past actual experience of the subject of discourse; while symbolic judgments go beyond this, and include all possible experience, *i. e.*, all combinations of actual experiences which the mind can construct suitably to the occasion. This is more complex than what we find in the experiential judgment, for there we are confined to those experiences which have actually occurred in the combination in question, whereas in the symbolic judgment we rearrange and re-combine elements originally experienced in other combinations, and thus have a much larger stock of experiences on which to draw. If, then, the same rule is exemplified in transcendent judgments also, we shall expect to find that the identical points of reference introduced into this class of judgments are the most complex and far-reaching of all.

Consider such a case as "God is a substance consisting in infinite attributes." The logical subject here is, presumably, "God," and the logical predicate "is a substance consisting in infinite attributes." The term "God" includes all our experience, both actual and possible, and is thus the most complex, all-inclusive idea that we can form. And if we go still further, and include in the idea not only all possible human experience, but also the thought of experiences infinitely wider and deeper than anything which any human being can know, we include not only all possible human knowledge, but go beyond this, and launch out upon the infinite ocean of the barely thinkable and imaginable. So too with the logical predicate of this subject. Not only spatial and mental qualities are ascribed to this substance, but they are attributed to it in an infinite degree, including all we could ever know, and more. And further, an infinity of other attributes, each one of which lies in the great Beyond, beyond anything of which we finite human creatures can even frame a concept, are also assigned to this substance. Thus we see that in the transcendent judgment, both *S* and *P* reach the limit of inclusiveness, the extreme limit of complexity.

It remains to ask whether it is by using the principle of identity that we form these transcendent concepts. So far

as a symbolic judgment is concerned, so far, that is, as out of the shreds and patches of our experience we construct an ideal man, a man writ large, if we will, but still humanly possible, still man—so far, we have already seen, it is by selecting from our experience only such elements as have one and the same reference, *i. e.*, by introducing the standard of identity, that we construct the symbolic concept in question. But when we go further, when we transcend the humanly possible and construct the idea of something beyond what we could experience, something infinitely, Divinely perfect, not man-writ-large, but GOD—do we reach this transcendent ideal also by holding fast to some identical point of reference, or do we adopt some other method?

The answer is that here also, beyond experience as within it, both *S* and *P* are formed by the aid of this identity-standard. The point of reference remains one and the same throughout. It is the idea of the "absolutely perfect." So long as we confine ourselves to human experience, we can only attain to rough approximations to this standard, and the resulting concept falls short of our ideal, remaining all too human, and limited by human imperfections. When we cut ourselves loose from the limitations of experience, we do not in any way change the main direction of our thought: we still proceed to build up the ideal of the absolutely perfect. All that is new is that we select strictly and exclusively what conforms to this standard. We leave out of the picture those aspects of human nature which are finite and imperfect, and retain only the formal abstract essence of goodness, truth, power, *etc.*, and even add the purely formal idea of other attributes not exemplified in our human experience at all. Thus we see that in the realm of complex feelings which spread out over the whole of our experience, the introduction of the standard of identity enables us to organise our experiences in a way which gives us at least clearness and precision, and furnishes a definite basis for the further organisation of our thought.

Conclusion: The Standard of Identity.—In all types of judgment, then, one function of intellect consists in selecting from our experiences all such as have one and the same identical reference, all which are strictly relevant to some definite subject of discourse, whether the field is sensory, experiential, or some symbolic or transcendent extension of

the field of direct experience. By introducing this standard of identity-of-reference we come to have, in place of the vague general flow of sensory consciousness (1) a sharply demarcated unity, which constitutes the judgment as a whole, *e. g.*, "It is warm here," "This disturbance is excessive," and (2) within this new field two subordinate unities, (a) a clearly envisaged subject of discourse, or logical subject, and (b) a definite determination of that subject, *viz.*, the logical predicate, "is warm," "is excessive." In this way, then, in place of an experience determined by the order in which things have happened to us—a chance medley of sensations, wishes, and feelings—we introduce an order determined by unity of meaning, relevance, and identical reference.

FOR FURTHER READING

B. Bosanquet, *Logic*, Vol. II, pp. 206-208. F. H. Bradley, *Principles of Logic*, pp. 131-135. B. Erdmann, *Logik*, (2nd Edit.), pp. 237-253. Chr. Sigwart, *Logic*, Vol. II, pp. 24-30, 80-92. W. Wundt, *Logik*, (3rd Edit.), Vol. I, pp. 552-553.

EXERCISES

1. How far does the standard of identity enter into the following: This mirror is dusty. I like this picture. That hurts?
2. How far does the standard of identity enter into the following: Children are a joy, so far as my experience goes. The card-index system has increased the efficiency of my business. The cost of living has steadily gone up for the last three years?
3. How far does the standard of identity enter into the following: Sea-sickness depends upon the functioning of the semi-circular canals. Bleriot was the first man to cross the English Channel in an air-plane. Little Bo-Peep has lost her sheep?
4. How far does the standard of identity enter into the following: Things in themselves are the underlying ground of sense-perception. Oh, for a mansion in the skies! A form of unutterable beauty appeared to me?

CHAPTER VI

DIFFERENCE.

Introduction of Difference.—What do we mean by difference? “A is different from B” means that A is distinguishable from, other than, B; A and B are diverse; A and B can be separated¹ from one another, can be set over against and opposed to one another, can be contrasted with and denied of one another. A is not B. A and B are not the same, do not constitute an absolute unity, an identity; on the contrary, they constitute a duality, a plurality. They are not one, but at least two, and can be sharply delimited against one another. Stated negatively, each is not what the other is; stated positively, each is what the other is not. Stated negatively, an electric bulb is different from a typewriter, because each fails to possess qualities essential to the functioning of the other. The bulb has no keys, no ribbon, no place for inserting paper, and thus cannot be used for the same purpose as a typewriter. The typewriter, on the other hand, has no transparent surface, no wires, no connection with an electric battery, and thus cannot be used to illuminate a room. They are thus sharply distinguishable or different from one another. Stated positively, in order to emphasise the positive element on which the differentiation depends, each has its own identity, its own nature, that which makes it what it is. The bulb has the transparent surface and platinum wire, the machine has the keys and ribbon. Each has the qualities which fit it for the performance of its own special function, and it is in virtue of specialisation or differentiation of function that each cannot perform the other’s function, and is thus different from it. If we apply this to logic, we see that “introduction of the standard of difference” will mean splitting up the stream of consciousness into

¹ Separated, *i. e.*, in thought, not necessarily in reality. The concave and convex aspects of one and the same curve, for instance, cannot be separated in actuality, but can be separated in thought, and regarded as “different.”

identities which are sharply distinguished from and contrasted with one another. Let us consider more in detail what this means.

(A) *In Judgments of Perception.*—Let us consider such a judgment as "This room is warm." Before we introduce intellectual elements into our thought and frame a judgment, our consciousness flows evenly at the sensory level. We are seated quietly at work, and a cross-section of our consciousness reveals, beside the complex work-interest, only a vague general background of feelings, incipient impulses, and half-apprehended sensations, all equally without distinction contributing to swell the fluid undifferentiated mass, in which no difference, no clear-cut outlines, have as yet been introduced.

What change occurs when we introduce difference? In place of a continuous, undifferentiated mass of sensations, feelings, and impulses, we have the distinct sensation of "warmth" standing out in sharp contrast over against our previous state. This sensation forces itself upon our attention by excluding other conscious elements, and the more clearly we become aware of it, the more sharply do we distinguish it from the general sensory background. It constitutes a nucleus for introducing still further distinctions. We split up the stream of consciousness into elements, among which "the room" is also prominent. "The room" is "warm." Perhaps it ought not to be warm. The furnace needs attention, or there is a fire. In our judgment we thus have two elements, "the room" and "warm," analysed out from the general conscious flow, separated, cut off and fixed by the mind in sharp distinction from one another. These two elements are what we have come to know as *S* and *P*. In the previous chapter we considered them as "identities" constituted by the positive concentration of thought in some one direction. In the present chapter, we are considering them from a more negative aspect, as differentiated or distinguished from one another: *S* is not *P*, and *P* is different from *S*.

Let us consider what this means. Suppose, if possible, that *S* and *P* were not distinct from one another. Take, *e. g.*, such a statement as "The room is the room," "warm is warm." Such "identical propositions"—as they are called—are almost without meaning. As we approximate more and more to complete absence of difference, so do we approximate to com-

plete absence of thought, of judgment. If the end-point is in no respect different from the starting-point, then we have never really started: there has been no movement of thought, no mental operation. It is, in fact, only where *S* and *P* are sharply differentiated, that a genuine judgment, in which something is actually thought, some problem solved, some conclusion reached—takes place. When Pilate says "What I have written, I have written," he means "What I have written stands, is unalterable, must and shall remain what I have made it": and it is precisely this difference between *S* and *P* which gives us the movement, the living element essential to meaningful thought. Thus we see that introduction of the standard of difference into our thinking is vital to judgment, and results in giving us the logical *S* and *P*, no longer as mere identities, but as sharply differentiated identities, clear-cut elements whose distinctness is essential to the intellectual organisation of our experience.

These differences hold within the judgment itself. But we should also note that the judgment as a whole has been carved out of our sensory experience. The mental hatchet of difference has thus separated off not merely the *S* and *P* of logical thought, but also the judgment as a whole, as an intellectualised complex, sharply distinguished from the previous state of mind.

(B) In Judgments of Experience.—Consider such a case as "The freight-trains crossing the bridge are growing more troublesome every year." We are seated at work, when the distant rumble forces itself upon our attention. At once, in the place of the steady, even flow of consciousness in the direction of our work-interest, we have opposition, contrast, interruption. The in-breaking sensation interferes with and thwarts, for the moment, our work-interest, and is, so far, annoying. If our consciousness remains at the sensory level, we experience only a vague general sound-annoyance, which comes and goes. But if the stimulus persists, or is very loud, the experience tends to rise to the level of an intellectualised thought, a judgment in which difference is clearly present. As we attend more and more to the rumble, the two elements, (1) of what the annoying rumble means, *viz.*, freight-train-disturbances, and (2) the increasing troublesomeness of such disturbances, become clearly separated out and differentiated in our minds. The train-disturbances are becoming too

troublesome; they ought not to be allowed to grow worse; some protest must be made. The steady, even flow of consciousness has disappeared, and in its place we have sharply opposed and clearly outlined elements, (1) train-disturbances in the past and present, and (2) the thought that these are growing increasingly troublesome.

We thus have a judgment in which the two elements which we have come to know as *S* and *P* stand out prominently. In the preceding chapter, these were considered positively, as identities introduced by the concentration of our thought in definite directions. At present, they are being considered more negatively, as different from, distinguished from, and opposed to, or contrasted with, one another. They are contrasted identities. Each has its own nature, but these natures are different, and it is the meaning of this difference for judgment, which we are trying to realise. Suppose there were no such difference. Suppose *S* and *P* coincided, as in "The freight-trains crossing the bridge are the freight-trains crossing the bridge." Is there any valuable element of meaning about such a statement? Would any one seriously regard such a form of words as expressing a judgment, an act of thought which might, *e. g.*, cause the thinker to be treated as intelligent rather than imbecile? If there were really no shade of differences introduced between *S* and *P*, if they strictly coincided, clearly there would be no act, no movement of thought, no discovery, no new step, no solution of a problem, no intelligent behavior at all. In other words, in order to form a judgment, to think something which is meaningful, significant, to advance instead of to mark time, it is necessary to introduce the standard of difference into our thought. Only by so doing do we attain to that sharp differentiation of elements, that clear distinction of *S* and *P* from one another, which is essential to the intellectual organisation of our experiences. In place of the even flow of sensory consciousness, the buzz and hum of unreflective life, we now have, taken out of the stream of consciousness, carved out and separated off from one another, elements which have lost the continuity of their sensory character, and have become loosed from their context and intellectualised, fixed by the mind negatively as well as positively—the differentiated identities, *S* and *P*. It is on the distinction, as well as the identity, that we depend for the

new combination of these elements which gives us intellectually organised experience, in a word, empirical science.

Here also, as in the perceptual judgment, we may further note that, as within the judgment *S* and *P* are distinguished from one another, so also the standard of difference distinguishes the judgments as a whole, as an intellectualised complex, from the sensory consciousness out of which it has been cut out and put together by the mind.

(C) In Symbolic Judgments.—Let us consider such a judgment as “Rome was occupied by Caesar.” To form a clear idea of the sensory level of the reading-consciousness, as distinct from the intellectual level of clear-cut judgment, we must call to mind those states of fatigue or distraction, in which our eye has followed the symbols on the printed page, but our mind—as we eventually discover—has not taken in the meaning at all. It is possible for the eye to have made every adjustment necessary for distinct vision, it is possible for the hands to have turned over page after page in due course, and yet, when we look back on what we are supposed to have read, we discover, only too clearly, that our experience has remained at the sensory level, and that we have no grasp on the subject-matter, no intellectual organisation of our visual sensations sufficient to give us an understanding of this chapter in Roman History.

What happens, then, when we do rise to the intellectual level, and not merely attend to the printed symbols, but read into them the full meaning which they will bear? As we attend, *e. g.*, to the word “Rome,” the steady even flow of consciousness is arrested. We look before and after. We pause and think. The word we read serves as a nucleus for associations from our past experience. As we continue to attend, and think of Rome as “occupied by Caesar,” we split up still further the massive stream of consciousness, sensory and associative. We analyse it out into minute elements, and select from our variegated experiences only such elements of panic and triumph as are suitable to the occasion. We exclude all which are irrelevant, different from what we are seeking. The concepts of “Rome” and “occupied by Caesar” which we thus construct are organised complexes, into which difference, as well as identity, has entered. The elements out of which they have been put together have been carefully selected, in such a way as to exclude all which are irrelevant,

inappropriate, other than what is required. The "Rome" of our judgment is different from the Rome of Romulus, the Rome of Scipio Africanus, the Rome of Sulla. It is the Rome of Pompey and Caesar, faction-torn and fearful of proscriptions, in short, Rome as "occupied by Caesar."² So too the predicate-concept "occupied by Caesar" is not constructed out of panic-and-triumph elements in general, but of such elements torn from their context in my own experience, and so split up, altered, re-organised, that they now represent, not my actual experience, but—something different, an experience such as I might have had, had I been a citizen of Rome at the time of its occupation by Caesar. The sensory consciousness has thus become intellectualised by the introduction of difference, as well as identity.

A second way in which difference enters into the judgment is seen if we consider the subject and predicate concepts. "Rome" and "occupied by Caesar" are contrasted, set over against and opposed to one another. Each has its own nature, its own identity. But they are diverse natures, opposed identities, and it is out of the clash and conflict between them that the dramatic significance of the judgment arises. A mother-city ought not to be "occupied" by one of her own sons. It is an outrage, none the less tragic because it was, perhaps, inevitable. *S* and *P* are thus different, distinct. We can, perhaps best realise the importance of this distinction, if we compare with the above judgment such statements as "Rome is Rome," "Caesar is Caesar," "Occupation is occupation." Such "identical propositions" show colorless and lifeless in the comparison. If we look still closer, they are even meaningless. If *S* and *P* are wholly without difference, if they coincide absolutely so as to be one identity, and not two, then there is no distinction between end-point and starting-point of our thought: which is as much as to say, there has been no act or movement of thought at all. Nothing has been accomplished, no problem has been solved, no meaning read into our sensations. In other words, difference is essential to the symbolic judgment, and its function is, to split up the stream of sensory consciousness into sharply differentiated elements, out of which clear-cut, cleanly outlined complexes, *S* and *P*, can be constructed and set over against each

² Cf. Lotze, *Logic*, sec. 58 (E. T., p. 63b).

other in such a way as to produce meaning, significance, intellectual life.

Finally, as in the preceding types of judgment, so here, we may note that difference enters not only into the elements out of which *S* or *P* is constructed, not only into *S* and *P* as wholes contrasted with one another in the judgment, but into the judgment as a whole. As an intellectualised complex, this judgment can not be too sharply differentiated from the even-flowing sensory consciousness out of which it has been constructed.

(D) **In Transcendent Judgments.**—Take such a judgment as “God is a substance with infinite attributes”. Transcendent judgments of this type are so far removed from the sensory level of consciousness, that it is difficult to determine with certainty the sensory background, and especially the particular sensory stimulus which induces us to dare the great venture, and seek a firm foothold for our faith beyond the possibilities of human experience. But, since any stimulus whatever may lead to pursue a train of reasoning beyond the bounds of experience, we may suppose something of the following kind. We are in a “brown study,” with a tendency in the direction of religious feeling. The most prominent factor in consciousness is simply vague feeling, at the undifferentiated sensory level. This feeling extends vaguely, *i. e.*, without precise limits, over the whole background of experience, and thus, as being without clear-cut limits, presents a kind of sensory analogon to infinity. If now the thought of some personal failure, for example, or the loss of a loved friend, leads us to think of our own helplessness, the aspiration after something better, finer, greater, less limited than ourselves arises as a correlative idea, and as we follow up this line of thought, we can hardly fail to pass beyond the bounds of space and time, beyond the border-line which separates the humanly possible from what we can never experience—to the idea of an absolutely perfect, ideal, Divine Being, in Whom we live and move, and from Whose viewpoint all our finite, human problems find their completely satisfying solution. This is the thought of God, and we have reached it by introducing a distinction between our own imperfect selves and a perfect, ideal self. It is thus a complex construction into whose organisation difference, as well as identity, has largely entered. Though at this level of thought

sensory feeling, perhaps, still plays a great part, it is feeling organised around the thoughts of (1) human self, (2) Divine Self, and (3) the vast difference, or rather impassable gulf, between them.

From an intellectual feeling of this kind to the judgment that God (as experienced in this way) is a "substance with infinite attributes," is perhaps a far cry for one who may not have been educated in the technicalities of Aristotelian or Spinozistic terminology. We do not, perhaps, quite naturally interpret our feeling after an ideal Self in terms of "substance" and "attributes." The terms are unfamiliar to us. But the thought behind this terminology, the idea which expresses itself in this kind of way, is familiar enough, and for our present purpose it is sufficient to note that such an idea as "substance with infinite attributes" is a complex idea into which difference largely enters. There is (1) the distinction between substance and attribute, (2) the difference between the many distinct attributes united in this one infinite substance, and (3) the vast difference between these attributes as we human beings know some of them, and these attributes as extended to infinity, as they are ascribed to Divinity.

Thus we see that, as in the symbolic judgment, so here, both subject-term and predicate-term are complexes into whose organisation difference, as well as identity, largely enters. The fact that in symbolic judgment we remain within the field of possible experience, whereas in transcendent judgment we pass beyond this to absolute infinity, does not seem to involve anything new in respect to the standard of difference; for example, it does not involve our not introducing this standard. On the contrary, the intellectual element of difference seems to be introduced more sharply, in proportion as our thought travels further and further away from direct sensory experience. Within the field of experience, for instance, all differences are merely relative, and it is frequently possible for us to learn to pass over the intervening distances. But between Human and Divine, between finite and infinite, between relative and absolute, there is no ratio. Here the difference is such that we human beings can never learn to pass it. In fact, the whole point of the transcendent judgment is that it deals with extreme differences, with difference made absolute.

It remains to notice another way in which difference enters

into the transcendent judgment. Not only does difference enter into the organisation of elements which constitutes the complex *S* or the complex *P*, but *S* and *P* as wholes, as units, are different from one another. Both "God" and "substance with infinite attributes" may, it is true, be ways of naming that feeling after a perfect Self which we have briefly described above. But they are *different* ways of describing that experience, and we must try to realise the importance of this difference for the transcendent judgment. Consider, for instance, such a statement as "God is God," or "Substance is substance," or "Infinite attributes are infinite attributes." We see at once that, by comparison with "God is an infinite substance," such "identical propositions" (as they are called) are relatively colorless, and almost without meaning. In fact, if *S* and *P* are really intended in precisely the same sense, if there is no change or development of meaning, then no operation of thought has taken place, nothing has been judged. Difference between *S* and *P* is, then, vital to our transcendent thought, and serves to make possible that movement and development in our thinking, in which something (*P*) is definitely judged of something else (*S*).

Finally, yet another way in which difference enters into our transcendent thinking is seen when we compare, not *S* with *P*, but the judgment as a whole with the feeling-consciousness at the sensory level, from which our clearly articulated thought that "God is a substance with infinite attributes" has arisen. In place of the steady, even flow of this sensory consciousness we have elements torn out of their sensory contexts and placed together in a new order, in such a way as to contribute to that highly articulated organisation of intellectualised elements which aims not merely at summing up actual experience, or to express symbolically the meaning-values of an experience thought of as only possible—but which seeks to sum up and state the meaning of an experience which goes infinitely beyond what is possible for human beings, and transcends the range even of what we can clearly and consistently symbolise.

Thus we see that the function of difference in the transcendent judgment is, to split up the sensory consciousness into intellectualised elements, on the basis of which a complex *S* and a complex *P*, sharply distinguished from one another, are constructed, in such a way as to produce a single, highly

organised judgment. This judgment, in its turn, is clear-cut, sharply outlined, and very different from the relatively unorganised mass of sensory feeling from which it originated. Each step of this construction is a function of difference no less than identity, and, so far as the transcendent differs from the symbolic judgment, so much the more strictly and rigidly has the standard of difference been introduced.

Conclusion—The Function of Difference.—If we put together the results reached in considering each of the above types of judgment, we find that the introduction of difference into our thought splits up the even-flowing, continuous sensory stream into intellectualised entities. Out of these, by a process of selection and re-arrangement, the intellectually organised complexes, *S* and *P*, are constructed, in such a way as to produce a significant judgment. Difference thus enters into judgment in three ways: (1) The elements out of which the complex *S* and the complex *P* are constructed, are sharply differentiated from one another; (2) *S* and *P* as wholes are clearly outlined and delimited against each other; (3) finally, the judgment as a whole is distinct from the sensory consciousness from which it has arisen. Without difference at each of these stages, there would be no distinctness in our thought, and if there were no distinctness in our thought, that intellectual organisation of sensory consciousness in which judgment consists, could not come into being.

In the last chapter, we observed that the introduction of the standard of identity made itself noticeable in precisely these same three ways. What, then, is new in the present chapter? What is new is chiefly another kind of emphasis. The elements out of which *S* and *P* are constructed are identities; *S* and *P* are themselves identities; and finally the judgment itself, whether sensory, experiential, symbolic, or transcendent, is an identity. This represents a positive aspect of that intellectual organisation which gives us judgment. In the present chapter, we have been emphasising a more negative aspect of the same mental operation. The identity which is *S* and the identity which is *P* are distinct identities, held over against and contrasted with each other in one act of thought: and it is on this difference, as well as on identity, that the significance of judgment depends. Similarly, if the different elements out of which *S* and *P* have been constructed were not sharply distinguished, we should have no clear-cut,

distinctly apprehended, intellectualised complexes, but merely vague masses of sensory consciousness. Finally, if the identity which is the judgment as a whole were not very different from the relatively unorganised sensory consciousness, if it were not an articulated whole in which difference, as well as identity, played a large part, it would not be what we call a judgment. From this we conclude that identity and difference are two correlative aspects of one and the same process. We can only differentiate entities which have natures of their own, such as the electric bulb and the typewriter; and similarly, if we are to apprehend clearly the identical nature of anything, we can do so only by at the same time distinguishing it from other identities which it is not. What, then, is this process, of which identity and difference are two correlative aspects? The answer to this question must be left to the following chapters.

FOR FURTHER READING

B. Bosanquet, *Logic*, Vol. II, pp. 208–210. F. H. Bradley, *Principles of Logic*, pp. 135–142. B. Erdmann, *Logik*, (2nd Edit.), pp. 247–253. W. Wundt, *Logik*, (3rd Edit.), pp. 553–555.

EXERCISES

1. How far does the standard of difference enter into the following: Here we are. This typewriter is heavier than that. I am thirsty?
2. How far does the standard of difference enter into the following: Nobody loves me—everybody hates me. There is usually an enormous difference between a freshman and a senior. Radishes tend to do well in spring?
3. How does the standard of difference enter into the following: A straight line is the shortest distance between two points. Little Jack Horner sat in a corner. I can do tomorrow what I am leaving unfinished today?
4. How does the standard of difference enter into the following: We ought to devote our lives to the service of the highest ideals. God loves us. Our love can never die?

CHAPTER VII.

ORGANISATION, (a) INTERNAL

The Meaning of Organisation.—Perhaps we can best understand what is meant by organisation, if we compare an aggregate with a totality. A chance heap of stones is an aggregate, a juxtaposition without inner principle, without coherence, without unity. Each stone in such a heap preserves its own individuality, and does not unite with the others in realising some common purpose. On the other hand, imagine suitable stones withdrawn from the heap and put together in such a way as to construct an arch-way over a door. Such an arch-way is a totality, and possesses a larger individuality of its own. If one of the stones is withdrawn the arch is weakened, and, on the other hand, by belonging to such a totality, each stone partakes in a higher individuality than it possessed by itself, an individuality, *e. g.*, with social and aesthetic, as well as physical, values. An organised totality is thus a systematic complex of elements, each of which has its own identity, and is different from each of its fellow-elements; but the differences are not absolute, they are transcended so far as all elements are pervaded by one and the same common purpose or meaning, some principle which unites them all in the service of the whole, and thus gives them a value different from, and higher than, that which they possessed apart from such organisation.

Let us apply this to judgment. Take the case $x^2 - y^2 = (x+y)(x-y)$. This is a complex totality, a unity of elements such that if one were withdrawn or altered, the equation would be radically changed. Exchange, for instance, a plus for a minus sign, remove a pair of brackets or the sign of squaring, and the equation simply vanishes. Consider further the elements x and y . Apart from its place in such a judgment, x , for instance, has perhaps, an individuality of its own, as a letter of the alphabet, which may conceivably be used as an algebraical symbol. Place it in the judgment, and it is at once altered by the fringe of relations into which it enters. These

give it a new meaning and value. On the one side of the equation it is squared, and y^2 is subtracted from it; on the other side, y is added to it in one bracket, and subtracted from it in another, and the complex results are multiplied together. Further, both sides of the equation have a certain unity: by diverse paths they both lead to one and the same result. Our element has thus ceased to be the mere alphabetical x , and has become a symbol transferred to do algebraic service, and transmuted in the process. It is now steeped in a new coloring derived from its relations to y within the totality which is the whole judgment of equality. Furthermore, while x by itself is different from y by itself, the unity which is the total judgment overcomes and is constituted by the special differences of x and y in relation to one another within the equation.

So much for a general example. Let us now turn to the special types of judgment, in order to consider the part played by organisation somewhat more in detail.

(A) In Judgments of Perception.—Such a judgment as “This room is warm” is a complex totality in which no part could be altered without changing the meaning. Let us consider the elements out of which it is constructed. As for S and P , we have already seen that “The room” and “warm” are identities, and that they are different identities. It remains to see how they are organised with reference to one another, how the differences are overcome, and how, within the judgment, both “The room” and “warm” acquire new and higher shades of meaning by uniting to constitute the higher individuality which is the total judgment. Consider “The room” apart from the judgment. It is a space enclosed by four walls, a ceiling, and a floor, and possesses two windows and a door. It might be thought of as usable for studying, or for entertaining friends, or for a thousand other purposes. That is to say, it might enter into a thousand judgments. But apart from judgments of definite kinds, it is not so thought of. It is merely “This room.” Within the judgment it at once receives a new meaning. It is thought of as “warm,” *i. e.*, in relation to temperature-values as sensed by me, temperature-values of which, perhaps, I disapprove, and which it is in my power to alter. Similarly “warm” is not thought of as warmth-in-general, mere physical warmth, but as the genial or unpleasant warmth of this room of mine, a degree of tem-

perature which may make my work comfortable or uncomfortable, may leave me contented, or may lead me to take measures to change the temperature. It is plain, then, that in the judgment, both "The room" and "warm" have become pervaded with new shades of meaning. *S* is more than mere *S*, and *P* is more than mere *P*. They are now the *S* and *P* thought of in relation to one another in a special judgment which I make. The organisation of *S* and *P* which is the judgment, on the one hand gives them their new meaning. On the other, the relations of identity and difference of *S* and *P* within the new totality constitute my judgment of perception.

Let us push our analysis more into detail, and examine further the elements which compose *S*. For *S*, within the judgment, is itself no bare identity, but is a complex, constructed out of a number of different identities. What, then, *are* these elements? Let us consider. In such a judgment as "This room is *x* feet long, *y* feet broad, and *z* feet high," "The room," as the subject-concept, is constructed out of spatial elements. In such a judgment, on the other hand, as "This room is thoroughly suitable for entertaining friends," the subject-concept is constructed out of social elements, or at least out of elements which have an especially social reference. In the judgment that "This room is warm," again, it is constructed chiefly out of temperature-elements. In other words, that which decides just which out of the countless possible elements shall be utilised in constructing *S*, is the meaning of the judgment as a whole. Is the judgment concerned with spatial values? Then the *S*-elements are spatial. Is it with temperature-values that we have to do? Then it is temperature-elements which constitute the main feature of *S*.

Similarly of the elements which compose the predicate. Apart from the judgment, "warm" is usually understood as a sensation consequent upon the rapid motion of physical particles. The concept "warm" can thus be built up out of motion-elements. But in such a judgment as "It is warm at noon," time-elements enter into the concept; and in such a judgment as "The bath is warm," elements connected with water, with washing, *etc.*, enter in. In different judgments, then, our predicate-concept is constructed differently. We have *noonday*-warmth, *bath*-warmth, *this-room*-warmth. Just precisely which, out of the innumerable possible elements, shall be used in constructing our predicate-concept, is thus seen to

depend upon the meaning of the judgment as a whole. In this way we come to realise that the judgment as a whole, as an organisation, is penetrated with one and the same general meaning, through and through, down to the minutest details. Not only *S* and *P* as wholes, but also the elements out of which *S* and *P* are constructed, are organised in accordance with the principle which gives us a single judgment, an act of thought which is *one*.

It remains to compare the sensory consciousness with what we find when our thought is organised so as to reach the intellectual level. At the sensory level, as we have already seen, there is a continuous flow of consciousness, without distinction or unity, not in any way cut into lengths. Elements are placed in this stream in the chance order in which they happen to us, without any reference to relevance or congruity. Thus, if we arbitrarily cut off a section of this stream and examine it, we find, *e. g.*, desires for something to eat, a wish that it were not so cold, a half-stifled suggestion of conscience that we ought to be working, some reverberations in consciousness of the last book we have been reading, and a thousand other heterogeneous elements of experience, all jumbled up together in what, for the intellectual judgment, would be hopeless confusion. At the sensory level, however, this is appreciated only as a vague richness of life-feeling. When this mass of sensory feeling is intellectualised, organisation, with its tools of identity and difference, has analysed out of the mass only such elements as fit in with the plan of the judgment, has excluded every element which fails to cohere in a single complex meaning, and leaves us with a closely organised, coherent system of intellectualised elements, each of which is permeated with one thought, and contributes to the construction of a complex thought-totality, a judgment which is not many, but *one*.¹

(B) In Judgments of Experience.—Such a judgment as “The freight-train disturbances are growing more troublesome every year” is a complex totality, in which, if any element were omitted or added, some change to the whole would cer-

¹ In the above case, and throughout the chapter, we are assuming that the intellectual process of organisation is carried out completely, and thus really does penetrate down to the details. In actual fact, in the rush and hurry of practical life, it is seldom that we have time for such patient analysis, and the details tend to be slurred over, to an extent which frequently leads us into serious error.

tainly result. Let us consider the elements which constitute this complex, in order to discover what new increments of meaning, if any, they acquire by entering into the judgment. *S* in this judgment is "the freight-train disturbances." Apart from the judgment this might be used in summing up impersonal, neutral occurrences, a fit subject, *e. g.*, for statistics. But the moment such a subject enters into the judgment, the moment that such "disturbances" are disturbances of *my work*, and not only so, but are *growing worse*, so that I decide that something will really have to be done—*S* has ceased to be an impersonal, neutral entity, and has acquired new shades of meaning of a very pronounced kind. Similarly with *P*. Within the judgment, *P* is no increasing-troublesomeness-in-general but is a very specific kind of increasing troublesomeness, a *freight-train* troublesomeness which interferes with my work to such an extent that I am goaded into action. *S* is no mere subject, but the subject-of-this-predicate: and *P* is no mere predicate, but the predicate-of-this-subject. Apart from the experiential judgment which brings them together in this way, each would, no doubt, possess its own individuality, but they would not enter into that larger individuality of the judgment, in which they participate in a wider and deeper significance, and constitute integral portions of a meaningful thought-structure.

Let us now consider *S* and *P*, not so much in their relation to the total judgment, as in their character as complexes. For the experiential judgment is a summing up of many more elementary perceptual experiences, and this characteristic of complexity is reflected in the constitution of *S* and *P*. In the case of *S*, it is not difficult to realise what elementary perceptual experiences are therein summed up: the elements are clearly single sense-experiences of freight-train disturbances. Just what principle governs the selection of these experiences, we can, perhaps, best realise by comparing a number of judgments with the same subject. (1) "The freight-train disturbances are, on the whole, a help, a stimulus to better work"; (2) "The freight-train disturbances are things of the past," (3) "The freight-train disturbances are amply compensated for by the ease with which we send and receive shipments." In each of these cases, *S* is the disturbance of my work due to freight-trains. But in each case it is composed of slightly different elements, and has a distinctly different

coloring. In the first case, the disturbance is looked on, not as a blank evil, which must be stopped, but as a positive help, a "most favorable pause," as the psychologists call it. *S* is in this case a complex composed of a series of "most favorable pauses." In the second case, the disturbances to my work are over and done with, things of the past, out of practical politics once for all. The elements of which *S* is composed are here single disturbances, to each of which is appended the clearly written label, "Past-and-done-with." In the third case, the disturbances to my work are real enough, but the advantage of having my shipments carried is so much in the foreground of my consciousness, that I am willing to put up with a little disturbance. In this case, the complex *S* is composed of a number of freight-train noises, each of which has two sides, (a) a disturbing side, (b) an advantageous side which at least counterbalances the disturbance. In the case which we are especially considering, *S* is composed of a number of single train-disturbances, which are thought of as without redeeming qualities, as troublesome, and increasingly troublesome, as goading me into action of some sort. On what does the difference in the composition of *S* depend, in these various cases? It is not hard, after such comparison, to realise that it depends on the judgment as a whole. In making a judgment with an eye to "favorable pause," we select from the mass of experiences only such as are thoroughly attuned to this chord, and can be regarded strictly from that viewpoint. In making a judgment in the light of irritated feelings of increasing troublesomeness, we naturally pick out for our complex *S*, only such elements as are to the point, strictly relevant to the issue. In the same way, the elements out of which *P* is constructed depend on the principle of organisation which gives us the judgment. Consider the following cases: (1) "My increasing correspondence is growing more troublesome"; (2) "The management of my increasing income is growing more troublesome"; (3) "To have so many attentive and devoted friends is growing more troublesome." We can see at once that the elements out of which the "troublesomeness" is constructed, must be distinct in such cases as correspondence, management of income, and numerous friends. In fact, without further analysis we can, perhaps, sufficiently realise that the difference in coloring which thus spreads over the elements out of which *P* is constructed,

comes from the judgment as a whole. In this way we see that, both in the case of *S* and in the case of *P*, the new coloring, the new shade of meaning, passes over, transforms, and permeates every detail, in exact proportion as the judgment is thoroughly organised on one principle.

We must now compare the judgment as organised, with the sensory level of consciousness. At the sensory level, as we have seen, our present sense of the train-rumble expands in space and time, so as to include in one continuum all similar train-rumbles in the past. But it remains to point out that this continuum differs sharply from the intellectual organisation of those same train-rumble experiences. The sensory continuum contains these experiences in the same order in which they first happened to us. They are not compared and summed up. And further: they do not stand out from their context, but each is embedded in a tissue of associated experience, which clings to the rumble-experience merely because it occurred to us at the same place, or in immediate sequence. Association by contiguity or succession, as this is technically called, is mechanical, unintelligent, blind, a matter of chance happening, rather than of rational organisation. Rational organisation analyses this mass of associated experiences, selects only what is appropriate to the purposes of the judgment, excludes everything which is irrelevant, and out of elements thus chosen for their suitability constructs the complexes *S* and *P*, in such a way that their relation to one another contributes to the unity and significance of the judgment as a whole. Intellectual organisation, then, penetrates down to the minutest details of the experiential judgment, and rearranges these in the light of its own principle, or main purpose. Only thus do we construct a complex thought-totality which has unity, a judgment or act of thought which is *one*.

(C) In Symbolic Judgments.—Take such a judgment as “Rome was occupied by Caesar.” That this is a totality, a complex which has its own principle of unity, we can at once realise if we try to change any of the component elements. Let us consider these elements, and see what new shades of meaning they acquire by entering into this totality. *S* for instance, apart from this judgment is the city of the seven hills, the subject of the various changes recorded in the History of Rome. How this differs from the “Rome” of our

judgment, we can perhaps best realise if we compare a number of judgments, in each of which "Rome" is the subject. *E. g.*, (1) "Rome is the city of the seven hills"; (2) "Rome was the chief seat of the medieval Christian Popes," (3) "Rome is the seat of the modern Italian government." In none of these cases do we find any of that excitement of panic and triumph characteristic of the Rome occupied by Caesar. Geographical Rome, Christian or Papal Rome, the Italian capital—we at once see that as the judgments differ, into which the concept "Rome" enters, so does the concept itself differ. In the case especially before us, it is the panic and triumph, the flight of Pompey and the in-march of Caesar's veterans, which lend their shades of meaning to the city of the seven hills. By entering into such a judgment, the concept has acquired a distinctive meaning, new shades of significance which it never possessed before. So too of the predicate-concept. "Occupied by Caesar" has a different significance when predicated of Rome, than when predicated of *e. g.*, the various cities of Gaul or Spain. These latter-mentioned "occupations" were, in a way, legitimate enough: incidents of minor importance in wars sanctioned by the conditions of the age. But the forcible occupation of Rome by one of her own citizens was of major importance, a matter of especial daring, and—in spite of precedent—was felt by many to be without possible justification. Thus we see that "occupied by Caesar" has a different significance in different judgments. In other words, the predicate-concept, as well as the subject-concept, receives a new coloring, peculiar shades of meaning, from the judgment into which it enters.

If we carry our analysis further, and consider not merely *S* and *P* as wholes, but the elements out of which *S* and *P* are made, here again it is not difficult to realise that these two are permeated with the meaning of the judgment into which they enter. In every symbolic judgment, as we have seen, the elements out of which we construct, for example, our *S*-concept, are drawn from our own experience, and selected and arranged in order to meet the requirements of the special case. In Rome as the city of the seven hills, the elements from which we construct the *S*-concept are experiences of hills, geographical experiences. In the second case, the elements are peaceful, Christian experiences, suited to the milder, more religious and venerable aspects of Church history. In

the third case, it is elements connected with the aspiration for liberty and unification—associated with the names of Garibaldi, Victor Immanuel, and Humbert,—on which we draw. In the Rome occupied by Caesar, on the other hand, it is, as we have already seen, elements of panic and triumph, which we tear from their contexts in our personal history and reconstruct in such a way as to feel something as a Roman citizen might have felt under the circumstances. That is to say, in selecting the elements out of which to construct our *S*-concept, we are governed strictly by the judgment as a whole, and accept only such elements as are relevant to the general meaning of that judgment.

So too with the elements out of which *P* is constructed. "Occupied by Caesar" has, as we have seen, different shades of meaning in different judgments. From our present standpoint, this means that the elements which we take from our own experiences and re-arrange so as to realise the meaning of our concept, differ when the meaning is different. When it is some Gallic town which is occupied by Caesar, we do not select feelings appropriate to the occupation of the greatest city of the then world, which was at the same time Caesar's own mother-city. Thus we see that, in every detail of the judgment, in the elements of which *S* and *P* are made up, as well as in the case of *S* and *P* taken as wholes, as units, the principle which governs the selection of materials, and decides what shall be used and what rejected, is the meaning of the judgment as a whole, the principle of organisation which makes it one complex, a totality.

Finally, we must compare the organised symbolic judgment with the sensory level from which it has arisen, the level at which our eye appreciates the form and position of the letters, and our hand turns over the pages, but our minds fail to grasp and hold fast the further meaning of the symbols in our printed book. At the sensory level, our consciousness flows evenly along, without a ripple disturbing the serenity of its surface, blissfully unaware of the tragedies of Roman life and the ascendancy of Caesar's star. Intellectual organisation of this stream of consciousness occurs when we arrest the flow of this stream in order to stop and think. We look before and after, combine the letters into words, the words into the unity of apprehended sentences, of judgments in which the symbols are realised in terms of re-organisations

of associated experiences. It is like what happens in the experiential judgment, but with this difference, that the organised result represents no mere summing up of our own experiences, but goes further. It consists of an organisation of experienced elements which approximate to, and are used to stand for, an experience we have never had, the experience of a citizen of Rome during the time of its occupation by Caesar. Our intellectual organisation penetrates down to the minutest elements of our thought, the fragmentary experiences which we put together in order to realise what is meant by *S* and *P*, and every element in the judgment, so far as it is intellectually organised, is penetrated through and through with one and the same complex meaning, in such a way as to give us an act of thought which, though complex, is single, a judgment which is *one*.

(D) In Transcendent Judgments.—Such a judgment as “God is a substance with infinite attributes” is an organised totality, *i. e.*, is such that if a single element (*e. g.*, one of the Divine attributes) were taken away, the meaning would be radically altered. Let us consider these elements. As belonging to an organised whole, they must be altered by participating in it. Consider *S*. Apart from the judgment, “God” might mean, *e. g.*, the ideal of a Perfect Self to which we aspire. God as the goal of aspiration differs sharply from God as a “substance with infinite attributes.” The warm, personal shades of meaning associated with human aspiration are lost in the impersonal, mathematical relation of attributes so many as to transcend all human qualities whatever. The concept of God has now acquired the “eternal” or timeless aspect which we attribute to mathematical entities, and the chill remoteness of this highly intellectual fringe of meaning has altered our concept almost beyond recognition—at least, for all who have not yet learnt to consider ideas *sub specie aeternitatis*. Similarly the predicate-concept “substance with infinite attributes”—in itself a wholly colorless quasi-mathematical entity—acquires by being associated in one and the same act of thought with “God”—the object of reverence and aspiration—something at least of the warmth and intimacy of personal feeling, a shade of meaning which connects it with human life. Thus we see that both *S* and *P* receive new and valuable elements of meaning by entering

into the intellectual organisation which is the transcendent judgment.

Let us consider further the elements which together constitute the complex *S* and the complex *P*. These elements are, of course, selected as being suitable to form *S* and *P*. But if the standard of organisation has been thoroughly introduced into the judgment, we should expect only those elements to have been selected which are appropriate to the special meaning of this *S* and this *P* in our organised judgment. Let us consider whether this is, in fact, the case with *S*. Are the elements which constitute the concept of God the same in such judgments as "God is the all-knowing, all-powerful creator of the world," "God is the ideal spiritual life in which we live and move and have our being," and "God is a substance with infinite attributes?" A little attention suffices to convince us that the component elements of the God-concept are different in each of these judgments. In the first, God is thought of as composed of elements of knowledge and power suitable to the world-creator, magnificent, wonderful, the subject of admiration and perhaps fear. In the second, God is thought of as life at its best, the kind of life of which we catch faint glimpses in our most exalted moments, and the component elements of the concept are precisely these moments of life in which we are at our best. In the third, the elements which together constitute the concept are the different attributes, extension, thought, and an infinity of others, each one of which is magnified to infinity. As far as *S* is concerned, then, it appears that its constituent elements have been selected in accordance with the meaning of the judgment as a whole.

So too of the elements which together make up the predicate-concept. Extension and thought, as attributes within our human experience, are imperfect and finite. But as attributes of a substance which is not human but Divine, they are at once altered to fit the new case. We piece out their imperfections with our thoughts, and try to conceive them as perfect, or, as Spinoza expresses it, "infinite." Further, we human beings know only the two attributes of extension and thought. But when we wish to build up the idea of a substance which is God, we postulate an infinity of other attributes, of which we have not the faintest positive idea, but only the persuasion that they must be added as necessary

elements in the absolutely perfect Being. For this is to be without limitations or wants, and transcends the best that we human beings know, not only in quality, but also in quantity. In this way, then, we see that the introduction of the standard of complete intellectual organisation affects not only the *S* and *P* of our transcendent judgment considered as wholes, but also the minutest details among their component elements.

It remains to compare the transcendent judgment as a whole with the sensory consciousness from which it has arisen. At the sensory level we have a vague general awareness, without precise limits, which contains in the germ the feeling of helplessness and dependence, as well as countless impulses, sensations, and feelings of all sorts. On the limpid surface of this broad stream, larger ripples come and go; but they are lost in the general motion of the stream, and nowhere do we find sharp outlines, clear-cut distinctions. All is continuous, even, placid. In comparison with this sensory level, the introduction of organisation which results in the transcendent judgment stands out in the greatest possible contrast. It is all lines and angles, discontinuities, sharply differentiated identities. The stream of consciousness has been split up into innumerable distinct elements. The hundred and one elements which are irrelevant to the conception of God as a substance—*e. g.*, the sensations and impulses arising from the chance sounds and sights of the external world—are sifted out and excluded from the judgment. On the other hand, all elements which are strictly relevant—such as feelings of helplessness and human dependence—are retained, and not merely retained, but retained in a very special form. They are differentiated and identified, cleared of every vestige of irrelevance, purified of the particular accidents of their sensory contexts, brightly polished until they are fit to take their place in the new system—a system characterised by intellectual clearness, coherence of meaning, unity of plan. In place, then, of a stream of sensory waves bound together merely by the continuity of happening, we have a system of intellectualised elements distinguished and held over against one another in a unity which is bound together by identity of reference, relevance, meaning.

Conclusion—The Function of Organisation.—If we now put together the results reached in considering the various types

of judgment, perceptual, experiential, symbolic, and transcendent—we see that organisation makes itself felt by uniting diverse identities in the service of one common purpose or meaning. We see this especially in three ways: (1) With reference to *S* and *P*. These are diverse identities. Apart from the organisation which brings them together, each has its own individuality and meaning. But when organised, with reference to each other, in a single act of thought, each acquires a new significance by partaking of a common meaning, which is wider and deeper than either possessed by itself. (2) The elements which together make up *S* and *P* are similarly influenced and transformed, down to their minutest details, by entering into the organised judgment. According to the meaning of the judgment, some of the possible elements are sifted out and rejected, while others are taken up and joined together in the service of the new judgment which makes them partakers of its own meaning. That is to say, only such elements are selected as are suitable to form, not *S*-in-general or *P*-in-general, but the *S*-in-relation-to-*P*, and the *P*-in-relation-to-*S*, in the unity of the new act of thought. (3) Finally, by comparison with the sensory consciousness, we discover that, while at the sensory level the different component elements of the conscious stream are held together by continuity, in the order in which they have happened to us—at the intellectual level, the articulate system of elements in which the organised judgment consists is held together by identity of reference and unity of meaning.

FOR FURTHER READING

J. G. Hibben, *Logic*, Part I, chapter xi. Chr. Sigwart, *Logic*, Vol. II, pp. 144–158.

EXERCISES

1. What are *S*, *P*, and their respective elements, and how are they affected by being brought together in the following judgments: These beets taste excellent. This overcoat is too heavy. That dark patch is slippery?

2. What are *S*, *P*, and their respective elements, and how are they affected by being brought together in the following judgments: I have found shorthand useful in my work. The business section of the town has been expanding in the last few years. Spinach has proved unsatisfactory as a garden vegetable—at least in my experience.

3. What are *S*, *P*, and their respective elements, and how are they

affected by being brought together in the following judgments: Honesty is the best policy. The sources upon which Tacitus relied in writing his *Annals* were prejudiced. A life spent without reflection—without taking stock of one's powers and critically deciding upon a plan of action—is no life at all?

4. What are *S*, *P*, and their respective elements, and how are they affected by being brought together in the following judgments: What *must* be, and *may* be, assuredly *is*. With God, all things are possible. I have invented a motor which will generate its own motive power, and thus go on for ever.

CHAPTER VIII

ORGANISATION, (B) EXTERNAL

The Introduction of External Organisation.—Let us consider what we already know about organisation. The minutest elements of our thought are organised from the viewpoint of some wider unity, which is *S* or *P*. *S* and *P* in turn are organised from the viewpoint of some wider unity, which is the judgment. Can we continue, can we regard judgments also as organised from the viewpoint of some yet wider unity? *S* and *P*, for instance, have two kinds of organisation. (1) In reference to their constituent elements, they are *internally* organised. (2) In reference to the judgment as a whole, they are *externally* organised. In the same way, the minutest elements are externally organised in reference to *S* and *P*, or to the judgment as a whole. The judgment as a whole, however, has been so far considered only in reference to its constituent elements, *i. e.*, as internally organised. In the present chapter we must attempt to discover whether it has external organisation also, and, if so, what part this plays in our thought.

Take, for example, the judgment " $7+5=12$." This is internally organised in reference to the units which constitute the judgment. But the matter does not stop here. Each arithmetical judgment is not a unity with internal organisation only, standing, in splendid isolation, aloof from all other judgments. Arithmetical judgments hang together, cohere in one system of meaning. 12, for instance, can be reached by other equations, such as $20-8$, 4×3 , $48\div 4$, *etc.*, and these different judgments belong together in such a way that we can say *e. g.*, " $7+5=20-8=4\times 3=48\div 4=12$." They belong together in virtue of the fact that they are externally organised in reference to one and the same arithmetical series, 1, 2, 3, . . . and represent the internal organisation of this series. So too geometrical judgments cohere in one system, being externally organised in reference to space, plane geometry to a space of two dimensions, solid geometry to a

space of three dimensions, and "metageometry" to a space of n dimensions. Similarly algebraical judgments belong to a single system, and if we pass to the more empirical sciences of physics, chemistry, biology, psychology, *etc.*, we must recognise here also, that each of these sciences consists of a group of more or less coherent judgments.

But the matter does not stop even here. These various groups, arithmetic, psychology, physics, *etc.*, are organised still further in reference to one another, or perhaps to a wider unity to which all alike belong. Many problems, for instance, can be solved indifferently by arithmetic, algebra, or geometry. New discoveries in physics or chemistry shed light on dark places in botany, psychology, *etc.*, and all the natural sciences make much use of mathematical equations. This universal use of mathematics shows that the various scientific thought-structures have at least one common factor. When we further reflect that biology, psychology, physics, *etc.*, deal with relations of cause and effect, while all sciences whatever deal with relations of ground and consequent and other logical relations—*i. e.*, where all sciences make use of logic and mathematics, they must be, to that extent at least, interrelated, must form parts of a wider whole which is at least partially organised. From this we can, perhaps, realise that all judgments whatever are, at least ideally, interrelated, and all belong to the vast body of organised, or ideally organisable, knowledge.

The ideal of knowledge is thus a vast system in which all possible discoveries in the departmental sciences might be completely organised in reference to one another. The system is internally organised in the form of the special sciences, and conversely, the various judgments which compose the special sciences can be regarded as externally organised in reference to this system of possible knowledge, and thus as being subject to the demands of consistency within a system which is *one*. The unity of the thinkable is thus the ultimate intellectual principle, in reference to which all judgments are externally organised, or at least externally organisable. In actual practice, in the hurry and rush of our every-day concerns, few, even scientists, push their researches to this length. They tend to remain content with an external organisation which merely gives the fringe or general setting of their special science; but it is always understood that such results

are provisional merely, until they have been worked over and transformed from a deeper viewpoint.¹ Then only are they fit to take their position in the ideal system which is Truth.

So far, then, we have seen that judgments are externally organised (a) in reference to the special department of knowledge within which they fall, and (b) in reference to the ideal unity of what can be thought consistently. Let us consider the meaning and value of this for the special types of judgment.

(A) **In Judgments of Perception.**—Take such a judgment as "This room is warm." In the sense in which we have hitherto understood it, this judgment falls into at least the following departments of knowledge: (1) temperature-judgments, (2) sense-judgments, (3) practical judgments, (4) psycho-physical judgments with a background of nervous physiology, leading to (5) chemical and (6) physical judgments, with all which these, in their turn, also imply. Ultimately, it belongs to (7) the class of "thinkables," *i. e.*, judgments intellectually organised in the systematic unity which contains every possible thought.

Let us consider, then, for the perceptual judgment, what is the meaning and value of its external organisation in respect of these various classes. For example, the temperature class consists of judgments like "This is cool," "This is warm," "This is just right." That is, it is internally organised into a number of precise judgments which, taken together, contain the whole meaning of the temperature class. If any one of these judgments is taken apart from its membership in such a class, it has, no doubt, some faint meaning of its own, but it loses all connection and contrast with the other judgments within the group, and it is just the extra fringe of meaning given by membership within the group which makes the judgment significant. "This room is warm." Taken as an isolated fragment of thinking, apart from the class of temperature-judgments, *i. e.*, apart from any contrast with the temperature which is "cool" and the temperature which is "just right," such a judgment has almost no significance. It is only by becoming a member of such a class that

¹ Cf. *e. g.*, Groos, *The Play of Animals*, E. T. 1911, pp. 30-31, and A. E. Taylor, *Elements of Metaphysics*, 1903, pp. 3-5.

it ceases to be an abstraction, an isolated fragment, and constitutes a vital portion of our concrete thinking. Thus we see that the external organisation of "This room is warm" in terms of temperature-judgments is an integral part of the wider meaning of our thought.

So too with the other departments of knowledge within which our perceptual judgment falls. *E. g.*, the class of "practical" judgments consists of thoughts which have a clear connection with advantageous action, *i. e.*, which lead to such action. If our judgment concerning the warmth of the room is not a member of this class, it can have only a contemplative significance, divorced from action. It is only so far as my thought is externally organised with reference to action that I rise and attend to the furnace, for example, or take precautions against an outbreak of fire; and the practical value of such connection with such action speaks for itself. The practical element, then, constitutes an integral portion of the wider meaning of our perceptual judgment.

It remains to ask what difference membership in this external context makes to the judgment itself. Are the elements, for instance, out of which the judgment is constructed, in any way altered as our judgment enters into some wider class? Does the organisation which we have called external remain merely external, or does it penetrate even into the internal construction of the judgment, and into the elements out of which *S* and *P* are built up?

There can be no doubt as to our answer. We have already seen that the internal organisation is dependent on the general meaning of the judgment, and that the meaning of the judgment alters according as we regard it on the one hand as an isolated fragment of thinking, or on the other as a member of some definite class, *i. e.*, as externally organised in some wider intellectual context. For instance, if "This room is warm" be regarded (1) as a member of the class of practical judgments, and (2) as not a member of such a class, there can be no doubt that the connection with action—or severance from action, as the case may be—enters into the selection of elements out of which *S*, *P*, and the whole judgment are built up. In the first case, most of the temperature-values selected will be connected with well defined actions. Elements of "just-rightness," for instance, are associated with sitting still; elements of "coolness" or "warmth" are asso-

ciated with diverse operations upon the thermostat or directly upon the furnace. In the second case, on the other hand, every element is carefully divorced from its customary association with action, and is regarded in a rigidly speculative light. In this way we see that the selection of elements out of which the judgment is composed, is governed not only by (1) the meaning of *S* and *P* and (2) the meaning of the judgment as a whole, but also by (3) the wider meaning of the intellectual context into which the judgment enters. Hence we conclude that, just as *S* and *P* are externally organised in the wider totality which is the judgment, so the perceptual judgment is externally organised in the wider totality which is its intellectual context.

(B) **In Judgments of Experience.**—Take such a judgment as “The freight-trains crossing the bridge are becoming yearly more troublesome.” In the sense in which we have understood it previously, this judgment falls into at least the following departments of knowledge:—(1) judgments based on sense-perception, (2) practical judgments, and—in the wider field of implication—(3) social judgments, (4) physical judgments, *etc.* Finally it falls into the class of “thinkables” or intelligent judgments thought of as forming a single coherent system. Experiential judgments are thus externally organised in much the same way as we have found to be the case with perceptual judgments.

Let us proceed to ask what difference it makes to an experiential judgment, to be externally organised in this kind of way. Hitherto we have regarded our example of an experiential judgment as falling predominantly within the class of sense-judgments. It is emphatically the *noise* made by the trains which is such a disturbing element, and the judgment as a whole is a summing up of such noise-disturbances, such offences to our ear. The sensory element is thus very pronounced. If, now, we think of it apart from membership in such a class, if we think away the *noisiness* of the freight-trains, they have also lost their disturbing character—in other words, the characteristic meaning of the judgment has vanished. Our thought is thus dependent on such external organisation for the significance which it has for us, and membership in this class is a legitimate part of the wider meaning of the experiential judgment.

So too with the other classes in reference to which our judg-

ment is externally organised. Consider, for example, the practical and social classes. In the sense in which we have taken it hitherto, our judgment about the train-disturbances is no impersonal, contemplative summing up of evidence. The disturbances interfere with our work, and to such an extent that we are impelled to do something about them, to write to someone, to organise social pressure and bring it to bear, *etc.* Deprive the judgment of its external organisation in these classes, and you destroy a large part of its significance for us. Thus we realise that here also, the practical and social elements constitute a legitimate portion of the wider meaning of our judgment of experience.

The same is the case with all the classes into which our judgments of experience undoubtedly fall, and our general conclusion inevitably is, that such judgments are not complete, if we regard them as units to be taken by themselves; on the contrary, each judgment of this kind enters into a wider intellectual context in which it obtains most of the elements of meaning which make it valuable and significant for us.

It remains to ask whether this external context is merely external, or whether, as we found to be the case with perceptual judgments, it enters also into the internal structure of the judgment. Do *S* and *P*, and the elements out of which these are put together, remain constant, unaltered, however their intellectual context may vary, or are some elements sifted out and rejected, while others are selected and retained, according as the guiding-thread of the external organisation directs? There can be no doubt about our answer. What governs the selection is, as we have seen, the meaning of the judgment as a whole, and the meaning of the judgment as a whole varies in accordance with the intellectual context into which it enters. Hence the elements selected in the construction of *S* and *P*, and of the judgment as a whole, will vary as the wider intellectual context varies. For example, if the wider context demands practical and social action in order to put a stop to these noises which interfere with my work, then that aspect of each of the recalled train-disturbances becomes selected in which the practical and social importance of putting a stop to such disturbances is prominent. If, on the other hand, the wider context assures us that steps have been taken which will prevent a repetition of the disturbances, then each of the

recalled train-noises comes before us with the special label "over-and-done-with." Thus we see that, in the judgment of experience also, the internal organisation is through and through dependent on the external organisation.

(C) In Symbolic Judgments.—Take such a judgment as "Rome was occupied by Caesar." In the sense in which we have understood it hitherto, this judgment falls into at least the following classes:—(1) dramatic, even tragic, judgments, (2) historical judgments, with all which these, in turn, imply, —*e. g.*, (3) archeological judgments, (4) epigraphical judgments, (5) linguistic judgments, *etc.* Ultimately, as history occupies a certain place among the sciences, it belongs to the system of knowledge, or what we have called "thinkables," regarded as an organic totality.²

What difference does such external organisation make to the symbolic judgment? Let us consider. In the sense in which we have always taken it, the occupation of Rome by Caesar is a dramatic event, an event of even tragical significance. On the one side we have the Pompeians, the Senatorial party, representing law and order, the majesty of Rome. On the other side we have the "populares," a mixed set of ruined and turbulent citizens, led by Caesar at the head of his Gallic War veterans, invading their Mother-country. It looks like the clash of Might against Right, and the uncertainty as to which party and which leader is in the right adds to the complex dramatic nature of the situation. Deprive the judgment of the external organisation in virtue of which it becomes a member of this class, and it pales into insignificance. Remove the dramatic element, and the meaning dwindles away to a mere nothing, a common-place event of no interest-compelling importance. In this way we realise that external organisation in terms of the dramatic is part of the wider meaning of our judgment. It is only when we envisage it as a great step in a fateful drama that we appreciate its full significance.

As with the class of dramatic judgments, so also with the other elements in the external organisation which constitutes the intellectual context of our judgment. Abstract from this context *e. g.*, the historical significance of the judgment, cut off our judgment from the evidence of archeology, from its dependence on ancient manuscripts, *etc.*, and it becomes at

² A. L. Jones, *Logic*, pp. 266 ff.

once a very different thing. Events conceived as unhistorical, *e. g.*, in works of professed fiction, may have dramatic significance, but our attitude towards fictitious events is sharply distinguished from our attitude towards events regarded as historical, and to relegate our judgment to the fictitious class would be seriously to curtail its legitimate meaning. In this way we realise that the symbolic judgment cannot be treated as a self-sufficient unit, existing by itself in splendid isolation from all other judgments, but that it is essentially an organic portion of a wide intellectual context which endows it with the dramatic, historical, and other elements of meaning which make it valuable and significant for us. It is thus externally organised in reference to wider universes of meaning, and ultimately in reference to, and dependence on, the totality of meanings, the system of thinkables, within which historical and dramatic judgments have their specific places.

As in the case of perceptual and experiential judgments, we must now ask whether this external organisation remains *merely* external. Does it merely affect the judgment as a whole, or does it enter into the internal organisation also, and modify the details of the judgment? As we have seen, *S*, *P*, and the judgment as a whole are built up out of elementary experiences; our question thus means, Are these elements independent of the external organisation, do they remain constant, however the external context may vary—or does the external context modify them down to their minutest details? Let us consider. The elements used in constructing our judgment are such stuff as dreams are made of, as well as sober history:—experiences of sunny skies, of crowds, of panic and triumph, and so forth, called up by association with the words “Rome,” “was,” “occupied,” “by,” “Caesar.” Of these possible associates, as we have already seen, only such are selected as are compatible with the meaning of the judgment as a whole. That is to say, the associates called up by each word are profoundly modified by their connection with the associates of the other words which together go to build up the complex internal structure of the judgment. Are these elements further modified by the entrance of the judgment into a wider class? Does it make any difference to the associates selected, if the judgment is regarded as fictitious or as historical, as dramatic or as commonplace? There can be no doubt as to our answer. If the judgment is dramatic and tragic, we cer-

tainly tend to select such elements of our crowd-experiences, of our panic and triumph-experiences, *etc.*, as are connected for us with the dramatic and tragic. If, on the other hand, the judgment is regarded as commonplace, we select mainly such elements of our crowd-experiences, *etc.*, as are connected with every-day feelings. Thus we see that the external context into which the judgment enters, profoundly modifies the details of internal organisation, and in fact, that the construction which gives us the symbolic judgment varies as the external organisation varies, or, as we have seen in the previous judgment-types, the internal organisation is through and through dependent on the wider intellectual context.³

(D) In Transcendent Judgments.—Consider such a judgment as “God is a substance with infinite attributes.” In the sense in which we have understood it hitherto, *i. e.*, taken ideally, as realising all which it attempts to accomplish, such a judgment can not be said to fall into any larger, wider, more inclusive class. It is formed by taking various attributes, such as extension and thought, and expanding these to infinity, and as the resulting concept of God is explicitly all-inclusive—including not only all humanly possible experience, but also all possible experience in general, human, angelic, and Divine—it embraces already, in its internal context, *every* universe of meaning. There thus remains nothing outside, in reference to which it could be said to be “externally” organised. In other words, transcendent judgments, taken ideally, are co-extensive with the ultimate class, the class of thinkables.

It should, however, be clear from the nature of the case, that a judgment which really and in actual fact was all-inclusive, and embraced in the unity of a single act of thought not only all humanly possible experience, but also an infinity of experiences of which human beings cannot even frame a clear positive concept—*i. e.*, a judgment which really transcends human experience—cannot be made by a human being. In other words, it is only ideally that a transcendent judgment can be said to have no external context. In actual prac-

³ Cf. W. B. Pillsbury, *Fundamentals of Psychology*, 1917, p. 340: “The outlines of black and white that constitute the words start the association processes which lead to the ideas, and these associates are controlled by the *wider setting and wider knowledge* of the individual at the moment. . . . The revival of the earlier experiences is controlled by the laws of association and by *the context* in a degree that practically *amounts in many cases to new construction.*” (Italics mine.)

tice, the attempts to think metaphysically tend to be one-sided, largely formal, finite, and imperfect. They are, in fact, symbolic extensions of experience, labeled with the formal demand that they should be extended to infinity. But this demand remains purely formal, and expresses an ideal which we cannot realise in actual concrete thinking. In the case before us, for example, we think of God as possessing attributes which come within the realm of human experience. When we attempt to magnify each of these attributes beyond the scope of possible human experience, *i. e.*, to infinity, we tend to lose ourselves. For in infinity, in that which is presumed to lie beyond human experience, the distinctions which have meaning and value within our experience cease to apply. Infinite space, for example, cannot be measured out in inches or centimetres; infinite time is hopelessly incommensurable with our minutes and hours; and the infinite spirituality of the Divine cannot be expressed in terms of finite propositions taken from empirical psychology. The *symbolic* ideal of a better Self, that is, a Self better than our actual Self, but distinctly conceived as within the range of human possibility, has meaning and value for the direction of our lives. But the *transcendent* concept of an absolutely perfect Self, a Self which transcends infinitely the possibility of human realisation, is so remote that we cannot even form a clear positive conception of what it means; and its value for our lives diminishes in direct proportion to its remoteness.⁴

In dealing, then, with actual judgments of this group, we may treat them as a slightly more extensive kind of symbolic judgment; in which case, all that we have found true of the symbolic judgment will hold good here also. If these judgments are one-sided and imperfect, then it is possible for them to have an external context: and if they have an external context, then the general meaning of the judgment will alter according to the external context into which it enters. For example, if we think of God chiefly as a merciful judge of the weaknesses of humanity—*i. e.*, in reference to the class "merciful judges"—our thought acquires shades of meaning which differentiate it sharply from the thought of God in some other context, *e. g.*, as the object of mystical enjoyment, that experience of infinite unity in which we are alone with the

⁴ Cf. H. Sidgwick, *Methods of Ethics*, pp. 18–22.

Alone.⁵ In the same way, so far as we regard judgments of this group as essentially and in fact extended symbolic judgments, the external organisation will enter into the internal structure of the judgment, and will govern the selection of the elements out of which *S*, *P*, and the judgment as a whole are built up. In other words, transcendent judgments, as actually judged by human beings, are dependent, as to their internal organisation, down to the minutest details, upon the wider class within which they fall, and this class is ultimately the class of thinkables, the ideally complete organisation of all which can consistently be thought.

Conclusion—The Ultimate Intellectual Standard.—If we now put together what we have discovered about the meaning of external organisation for the various types of judgment, certain conclusions stand out with especial prominence. First and foremost, no judgment whatever stands by itself, but each is an integral portion of some wider universe of meaning, which constitutes its intellectual context. Perceptual judgments are portions of a concrete texture of thought which is by no means confined to the perceptual level. This intellectual context embraces, as we have seen, not only perceptual, sensory, judgments, but also summings up of these in classified form—*i. e.*, is continuous with experiential and symbolic judgments. Ultimately, as extending into the field of "thinkables," it is continuous with the intellectual context of transcendent judgments. Judgments of experience are similarly portions of a wider context which on the one side forms part of our popular, perceptual thought, and on the other is intellectually continuous with those summings up of experience out of which science is born. Ultimately, as being continuous with the texture of science, this intellectual context is continuous with the symbolic extension of experience which reaches out after infinity and is called transcendent. So too the intellectual context of symbolic judgments is on the one side experiential, and, on the other, transcendent, while transcendent judgments seem partly to be symbolic, partly to belong to an all-inclusive universe of meaning. Every judgment, then, is part of a wide system of meaning, which extends with unbroken intellectual continuity in the direction of what we have called transcendent thinking.

⁵ Cf. Plotinus, *Enneads*, Bk. VI, chapter ix, sec. 10 (in Bakewell, *Source Book in Ancient Philosophy*, p. 393).

Let us ask further, are the systems of meaning to which different judgments belong, themselves different? For example, does "The room is warm" belong to a different universe of meaning, a wholly different intellectual context, from that into which such judgments enter as "Rome was occupied by Caesar," or "God is a substance with infinite attributes?" Our answer must be, No. They belong, of course, to distinguishable systems of meaning, as sense-judgments can be distinguished from historical judgments, or as a judgment based on the warmth which we perceive with our senses can be distinguished from a judgment about God, whom we do not perceive with our senses. But these systems are only relatively distinguishable—they are not absolutely severed from one another. Ultimately all form parts of one and the same great system. In the case of the special sciences, for instance, we saw that a thread of mathematical and logical thinking runs through and so far connects and unifies most of our concrete thinking. If we apply this to the present case, we can see that all systems of meaning, however diverse in appearance, are at one at least in this, *viz.*, that they are all alike systems of *meaning*, that logical thought and intelligible significance enters into them all—in a word, that they one and all belong to the widest and ultimate totality of "intelligibles" or "thinkables." In this way we come to realise that the ultimate system, the circle of widest meaning to which our judgments can possibly belong, whether they are perceptual, experiential, symbolic, or transcendent, is the totality of thinkables, and that in an ideally complete intellectual organisation of our sensory consciousness, the various elements which go to form the *S* and *P* of our judgments—"the room," "freight-train-disturbances," "Rome," and "God,"—and indeed every judgment of whatever type, must be rational, comprehensible, and thinkable through and through, so as to be fit without further transformation, to take their place in a completely intellectualised experience, in which all elements whatever would be clearly interrelated, and be organic through and through with meaning.⁶

Our final conclusion, then, concerning the intellectual element in judgment is this:—The standards of identity and

⁶ Cf. Plato's ideal of Dialectic, as developed in *Republic*, Bk. VI, *ad fin.*, and F. H. Bradley, *Principles of Logic*, pp. 449-451.

diversity or difference are subordinate concepts within the wider conception of organisation, and the standard of internal organisation is subordinate to the conception of external organisation. The unity of whatever can be consistently thought as belonging to a single intellectual system is, then, the ultimate standard of the intellectual side of judgment. It remains to inquire into its validity.⁷

⁷ This ultimate standard is what Kant calls the Unity of Self-consciousness—i. e., of a single ultimate experience. Kant sometimes calls it the "transcendental unity of apperception." Cf. *Critique of Pure Reason*, tr. Meikeljohn, pp. 82-86. Cf. also Bosanquet, *Logic*, Vol. I, p. 144: "The course of judgment within the present whole of perception is determined by connections which refer beyond that accidental whole, to other more comprehensive totalities, and ultimately, in every case, to the system of the known world. The connections thus prescribed between part and part within some systematic whole are necessary *connections*."

FOR FURTHER READING

B. Bosanquet, *Logic*, Vol. I, pp. 72-92. W. B. Pillsbury, *The Fundamentals of Psychology*, pp. 335-344. Chr. Sigwart, *Logic*, Vol. II, pp. 508-528.

EXERCISES

1. What is the wider intellectual context of the following judgments, and how far does it enter into their internal organisation: The ice looks slippery. These skates feel sharp. This key is rusty?

2. What is the wider intellectual context of the following judgments, and how far does it enter into their internal organisations: In the spring a young man's fancy lightly turns to thoughts of love. Beans and cucumbers nearly always die of some blight. Life is not what it used to be?

3. What is the wider intellectual context of the following judgments, and how far does it enter into their internal organisation: Water is H₂O. Marius conquered the Cimbri and Teutones. Some day the air-plane will supersede the automobile?

4. What is the wider intellectual context of the following judgments, and how far does it enter into their internal organisation: Things in themselves are unknowable. I have taken *all* knowledge to be my province—I shall be omniscient. The Future Life will be the present life over again, but raised to the *n*th power?

CHAPTER IX

THE INTELLECTUAL ELEMENT AND VALIDITY.

The Question of Validity.—So far we have seen that the function of intellect in judging is to split up the material of sensory consciousness, to take it apart into its elements, and then re-shape it nearer to the desire for relevance, consistency, systematic unity. This is done by introducing the standards of identity-in-difference, difference-in-identity, and organisation, both internal and external, until at last the material of the sensory consciousness has been so worked over that it is able to take its place in a single organised system, in which every element is rational, relevant, coherent with every other element, and the whole structure down to its minutest details is consistent, thinkable through and through, and organic with a meaning which constitutes one great identity-in-difference, a single system, an ideal individual.

All this, however, is merely descriptive. It tells us how the intellect functions, what it does. But there is a further question, the question of validity. Granted that the intellect functions in this kind of way, is what it does *legitimate*? Can we justify the great changes brought about by intellectual organisation? By what *right* do we analyse and reconstruct? That our meddling intellect mis-shapes the beauteous forms of things, mutilates reality and imprisons it within a net-work of man-made frames, conceptual abstractions in which is neither life nor truth—in this view poet and philosopher are frequently at one, and never more so than at the present day. That the work of “discursive” thought is thus infected with falsity, is a conclusion not lightly to be set aside. The question must therefore be faced—How far is the work of intellect valid?

Here, for the sake of clearness, we must introduce a distinction between a more general, and a more special question. What our poets and philosophers mean, when they suspect the intellect of misleading us, is that truth resides

in intuition, sensation, sympathy, immediate awareness, whereas intellect gives us an awareness which is mediate, not the thing itself, but a conceptual model, a structure which we can substitute for the reality. We can understand this, because we have ourselves constructed it. It works as well as if it were the original, but—it is not the original. This objection to the work of intellect opens up a more general question than we are at present in a position to discuss. It compares and balances the respective value-claims of intellect and intuition, or intellect and sensation, and thus presupposes an answer to two more special questions, (1) concerning the validity of intuition or sensation, and (2) concerning the validity of intellect. So far, we have only answered the first of these more special questions. We have treated of the sensory element, in abstraction from the intellectual element. It remains, therefore, to attack the second of these special questions. We must leave on one side the question of sensation and sensory validity, and examine how far, if at all, the intellect, *qua* intellect, is valid, *i. e.*, the question of intellectual validity. After this question is answered, if we succeed in discovering the work of intellect to be intellectually valid, we can then proceed to the general question, and ask how far the working of a mind which is intellectually *sans peur et sans reproche* compares with sensory validity, as a factor in attaining reliable knowledge of reality.

Let us take an example. If the truth of a scientific hypothesis is called in question, we can, as we say, verify it by an appeal to sensory experience. This is true, and has been already considered. But there is a prior question, *viz.*, how far the hypothesis is intellectually satisfactory, how far the intellectual structure, the conceptual model, is really intelligible, whether it really hangs together and is rational and meaningful. It is the conditions of this meaningfulness which we must now examine. Let us take a concrete instance from the field of simultaneous equations. "If three hens lay on the average as many eggs as four ducks, and the number of eggs laid in a month by six hens and six ducks is a hundred and eighty, what is the weekly average of each hen and each duck?" The intellectual element in the solution consists in the introduction of sharply differentiated identities such as x and y , in such a form that the conditions of the problem

are expressed somewhat thus: " $3x=4y$, and $6x+6y=180\times 7$."

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The further manipulations of these conventionalised identities according to the rules of algebra or geometry inform us with mathematical certainty, that the weekly average of each hen is four eggs, and of each duck three eggs. Here, then, are judgments which are intellectually valid. On what does their intellectual validity depend?

It depends (1) on the introduction of identity-in-difference. If x did not have a core of identity in the various propositions, in spite of its varying fringe of relations, if, for instance, it meant in the second equation something entirely different from what it meant in the first, no result could be reached. It depends (2) on the introduction of difference-in-identity; for if x and y did not have some variations, some differences of meaning in their varying contexts, then, as we have seen, no movement of thought could take place; nothing would be judged. It depends (3) on the organisation of these identities and differences with reference to one another in a single act of thought; for instance, if the first and second equations were not somewhere brought together, no conclusion could be reached. Finally it depends (4) on the external organisation of these conventionalised elements within the wider system of algebra or geometry, with all which this implies. The result is reached by adhering strictly to the rules of algebra or geometry, *i. e.*, to the rules of mathematics, which is a kind of applied logic and belongs to the system of whatever can be thought. If we sum up what we have seen, we can state that intellectual validity in the case before us consists in observing strictly the rules upon which organisation of intellectualised elements within a single consistent system depends. So far as the analysis and subsequent reconstruction are governed strictly by reference to the ideal of this single system, the result is intellectually satisfactory. So far, however, as any of these rules are not observed—*e. g.*, if the elements are not identities, or are not to some extent different, or are not organised internally or externally—so far no result can be reached with which we can be intellectually satisfied.

Let us now proceed to ask how far, in our various typical

forms of judgment, the intellectual structure admits of intellectual validity.

In Judgments of Perception.—Take such a judgment as “This room is warm.” As we have already seen, sensation alone assures us primarily of a feeling of warmth. The judgment, however, that *the room* is warm, adds something further, an element of *interpretation*, which we have regarded as intellectual. This element consists, as we have seen, of an analysis of the sensory consciousness into certain elementary experiences which are then reconstructed, not merely in reference to present sensation, but also in the light of a wider system of knowledge, partly experiential and partly symbolic, partly practical and partly reaching out into physics and logic with all which these imply—*i. e.*, ultimately the elementary experiences are reconstructed in terms of the single system of organised knowledge, of what can consistently be thought. It is this reference to the wider structure which enables us to pass beyond the present sensation and state, not merely that *we* feel warm, but that *the room* is warm, and the furnace probably requires attention, or there may be danger of a fire, *etc.* The intellectual element, then, consists in realising, in making concrete, the lines of intellectual continuity which give our perceptual judgment its place in the system of thinkables.

On what does the validity of this procedure—if it *is* valid—depend? Let us consider the steps we have taken, in order. The first step consists, as we have seen, of analysing our sensory experience into elements which are conventionalised, taken out of their sensory contexts and made discontinuous, cut off and fixed by the mind so as to retain a certain identity of meaning, however various the intellectual contexts into which they may subsequently be thrown. These mental counters, then, which are thus utilised as bricks in building up the intellectual structure, are what we have called “identities-in-difference.” Those used in constructing the logical subject of discourse—the room—are predominantly spatial experiences, and those from which we construct the logical predicate—warm—are predominantly temperature-values. If these elements were not conventionalised, if one identical meaning did not underlie them and keep them substantially the same in different contexts, if they remained vague and fluctuating in meaning,—then farewell to consistency and

unity, *i. e.*, to intellectual validity. For what is inconsistent is invalid, and what does not form a unity is a multiplicity without coherence or consistency. As far, then, as these elementary experiences are strictly fixed by the mind and used as "identities-in-difference," *i. e.*, as retaining the self-same core of meaning, whatever the intellectual context into which they may enter,—so far we have a basis upon which a consistent and intellectually valid thought-structure may be erected. Conformity to the standard of identity-in-difference, then, is a condition *sine qua non* of intellectual validity.

A second step in the intellectual procedure is the introduction of the standard of difference-in-identity, *i. e.*, of different fringes of meaning according as our "identities" enter different contexts. We have already seen that apart from such difference, no movement of thought, and thus no judgment, could take place. The introduction of difference-in-identity—a fringe of difference which does not annul the underlying identity—is thus a second necessary condition *sine qua non* of intellectual validity.

The third step is organisation, internal and external. If the elementary experiences are not brought together and unified, *i. e.*, organised so as to give us the complex structures *S* and *P*, then there is no subject of discourse and no logical predicate, which means that nothing is judged about anything, *i. e.*, no judgment takes place. Further, if *S* and *P* are not brought together, if "the room" and "warm" are not held together in a single act of thought which unifies them without annulling their differences, no judgment takes place. For judging is essentially a function of unity, and we certainly unify the *S* and *P* concepts in judging that *S* is *P*, the room is warm. In other words, internal organisation is a *conditio sine qua non*, a necessary condition of judgment. Further, if the lines of organisation which have thus given us the internal structure of the judgment are not intellectually continuous with a wider intellectual structure—if the judgment is not continuous in meaning with temperature-judgments, practical judgments, and ultimately with the whole organisation of rational experience—then it is inconsistent and irrational, *i. e.*, intellectually unsatisfactory. Thus we see that it is only so far as our perceptual judgment is intellectually continuous with the system of thinkables—*i. e.*, only so

far as it is relevant, consistent, and rational through and through, that it can be regarded as intellectually satisfactory.

In Judgments of Experience.—Take such a judgment as “The freight-trains crossing the bridge are growing more troublesome every year.” As we have seen, the process by which the sensory consciousness is here raised to the intellectual level, consists in splitting up the spatial and temporal continuum and selecting elementary experiences of train-disturbances, taking them from their sensory context and transforming them into mental counters, identities-in-difference which are then utilised in the further construction which builds up *S* and *P* and gives us the organisation which is our judgment—a structure not merely internally organised, but in its main lines intellectually continuous with the wider structure of practical judgments, and ultimately of all which can rationally and consistently be thought in a single system of meaning. Our question is, on what does the validity of this intellectualising process—if it is valid—depend? For instance, take the first step. Would the procedure be intellectually satisfactory if our analysis did not result in giving us conventionalised elements, identities-in-difference? Let us see. If there is no identical reference-point governing the selection of elements out of which *S* and *P* are to be constructed, then the *S*-concept will be composed of all kinds of heterogeneous elements without real unity—a mixture of vague, fluctuating experiences not all strictly relevant to the concept “train disturbances.” Such unintelligent groupings would certainly not be clear enough or sufficiently to the point to be used as an intellectual subject of discourse, and, in fact, as we have seen in perceptual judgments, the strict introduction of identity-in-difference would seem to be a *sine qua non* of intellectually valid thinking. What is true of the elementary experiences which are grouped together in order to form *S* and *P* is, of course, true also of *S* and *P* themselves and indeed of the general meaning of the judgment. Each of these must have its own clearly apprehended meaning, an identical direction of thought underlying any change of intellectual context, if the resultant structure is to be intellectually satisfactory.

In the second place, the introduction of difference-in-identity, as we have already seen, is similarly a *sine qua non* of valid thinking, indeed of thinking at all. Without some difference

resulting from the varying fringe of relations contributed by the contexts into which *S* and *P*, for instance, enter, no movement of thought could take place, nothing would be judged. In the third place, organisation of the various elementary train-disturbance experiences so as to form *S* and *P*, and of *S* and *P* themselves within the single act of thought which is the judgment, is necessary; and finally, if the general lines of internal organisation are not strictly continuous with the general lines on which the whole system of thinkable, consistent, and rational judgments is constructed, then the result fails to cohere, and is thus intellectually unsatisfactory. In this way we realise that intellectual validity in the case of experiential judgments depends on strict conformity to the intellectual standards of identity, difference, and organisation. It is only so far as the analysis results in identities with fringes of difference, organised so as to form an integral portion of the vast system which includes all which can be consistently and rationally thought, that the judgment of experience can be regarded as intellectually satisfactory.

In Symbolic Judgments.—Symbolic judgments have already been dealt with, at least in principle, in the simultaneous equation case considered above. What makes this a case of the “symbolic” type of judgment is not so much the fact that algebraic symbols were employed in its solution, as that it reconstructs for us, by indirect methods, a summing up of experiences which goes beyond what we have actually experienced. The statement of the average number of eggs to be expected per hen, for instance, is more than a mere summing up of past experiences; it gives us a rule which holds *e. g.*, for future experiences, for experiences which have not been, and indeed may never be *ours*, in a word, the judgment moves in the field of *possible* human experience. It thus belongs to the sphere of what we have called “symbolic” judgments, and we may take what we saw there and apply it briefly to our typical instance of a symbolic judgment, “Rome was occupied by Caesar.” The steps by which, in this typical case, the sensory consciousness becomes raised to the intellectual level consist (1) in the analysis which results in mental counters composed of blue-sky experiences, panic and triumph experience, *etc.*, taken out of their original sensory contexts and fixed by the mind in the form of identities-in-difference, (2) in the introduction of difference, *i. e.*,

different shades of meaning according to variations of intellectual context—into the “identities”—and (3) the organisation, both internal and external of these differentiated identities, which results in giving us a judgment which does not stand by itself, but is part and parcel of whatever is rational and meaningful, and helps to constitute the single system of whatever can consistently be thought. It is unnecessary to inquire in detail concerning the intellectual validity of this judgment. Briefly and in principle, it is only so far as these intellectual standards of identity, difference, and organisation are followed, only so far as the resulting structure really does constitute an integral portion of the system of thinkables, only so far as it is intellectually continuous with all which is rational, consistent, and coherent, that it can be regarded as intellectually satisfactory.

In Transcendent Judgments.—Transcendent judgments are, as we have seen, a kind of extended symbolic judgment, so that the conclusions as to intellectual validity which we have found to hold good in the case of symbolic judgments can be transferred, without alteration in principle, to transcendent judgments also. And in fact, if we examine the thought of our professed metaphysicians, we find without much difficulty that it is on the standards already studied that they rely for the intellectual validity of their constructions. The attempt to construct the outlines of the Absolute Experience is almost always guided by considerations of systematic consistency, and the ideal of rational thinkability is openly acknowledged as the highest standard to which one can appeal. We shall therefore conclude without further examination, that in such judgments as “God is a substance with infinite attributes,” “Things-in-themselves are unknowable,” “The real is the rational,” and so forth, the intellectual validity of such constructions depends upon strict conformity to the standards of identity, difference, and organisation, and that it is only so far as such judgments are strictly consistent and cohere in a single, all-inclusive universe of rational meaning, that they are regarded as intellectually satisfactory.¹

Conclusion—Intellectual Validity.—If we now put together what we have discovered in our discussion hitherto, we realise, that while all judgments have a sensory, as well as an intel-

¹ Cf. e. g., F. H. Bradley, *Appearance and Reality*, latest edition, Appendix I; Bosanquet, *Logic* Vol. I, pp. 3-8.

lectual aspect, it is only so far as they conform to intellectual standards that they can claim intellectual validity. It is only so far as intellectually irrelevant elements—such as we find vaguely embodied in the continuous stream of sensory consciousness—are removed, and we have strict organisation of strictly intellectualised identities within a single system which includes in principle all that can consistently be thought—that the resulting structure can be regarded as intellectually satisfactory. A judgment, then, is intellectually valid, precisely in so far as it is fitted, without further qualification or transformation of meaning, to enter into a system of relations which includes every thought which is rationally consistent, and thus itself intellectually satisfactory, the kingdom of Truth.²

² Cf. Kant, *Critique of Pure Reason* (tr. Meiklejohn), pp. 26–27. The view taken in the text is approximately the same as Kant's when he says that only that which reason constructs according to rational principles is entirely intelligible and satisfactory.

FOR FURTHER READING

F. H. Bradley, *Principles of Logic*, Bk. I, chapter i. B. Erdmann, *Logik*, (2nd Edit.), pp. 194–197, 409–426.

EXERCISES

1. On what does the intellectual validity of the following judgments depend: You are looking sun-burned. This leaf is turning red. This new varnish feels sticky?

2. On what does the intellectual validity of the following judgments depend: We usually put on the storm windows towards the end of October. Our tomatoes ripen well indoors, if we allow the sun to get to them. The neighbor's baby nearly always cries at night?

3. On what does the intellectual validity of the following judgments depend: None but the brave deserve the fair. Demosthenes was a great orator, but a poor statesman. $5340 - 2189 = 3151$?

4. On what does the intellectual validity of the following judgments depend: All genuine knowledge is independent of experience. We shall be as God, knowing both good and evil. As it was in the beginning, is now, and ever shall be, world without end.

CHAPTER X

THE VALIDITY OF JUDGMENT

The Problem.—So far we have discovered by analysis of our thought two aspects of every judgment, (1) the sensory, and (2) the intellectual. We have further seen that each of these aspects has its own laws, and that it is only by strictly conforming to these laws that each aspect possesses validity within its own sphere. So far as our thought is sensory, it is valid provided that it is a continuous sensory expansion from the focus of sensory consciousness,—continuous, that is to say, in space and time. So far as it is intellectual, our thought is valid so far as it constitutes an integral portion of that organised totality which ideally contains all which can consistently be thought and can rationally cohere in a single system of meaning. Sensory continuity is one thing; intellectual consistency is another; and we have already seen that these are not perfectly proportioned to one another in our concrete thinking. In perceptual judgments, the sensory element is almost everything; but as we advance towards symbolic and transcendent judgments we notice more and more the gulf between what sensation can give and what the idealising intellect demands. The sensory element can be spread out, so to speak, so as to cover the field, not merely of actual, but also of possible human experience. But as we gradually approach the place where the symbolic begins to pass into the transcendent type of thinking, the sensory covering has become so thin that at last it is totally inadequate to satisfy the needs of intellect. Sense and intellect are thus, to some extent at least, heterogeneous, and sense covers a less wide field than intellect. What is their inter-relation, and which of them plays the major part in contributing, not to the specifically sensory, or specifically intellectual validity of judgment, but to the general validity of our thinking? This is our present problem.

Idealism.—Many thinkers, from Plato down, have assigned the palm to intellect. For such thinkers, "the sensory," or the content of the sensory consciousness, is not an object of

strict knowledge. It is too vague and fluid, and refuses to lend itself to the manipulations of accurate and systematic thought. It is only so far as we conform strictly to intellectual standards, and leave behind us the sensory element, that we attain to truth, or knowledge of real Being, a knowledge which extends infinitely beyond what our limited senses can hope to realise. For Descartes and Leibniz, for instance, in modern philosophy, sense is simply confused thinking, thinking confused by connection with our bodily sense-organs, and it is only so far as the mind separates itself from these disturbing influences and thinks by itself, that it can attain to clear and distinct apprehension of its object. So too in our own day it is not difficult for a writer like Bradley to show that the chief viewpoints of empirical science fail to conform strictly to intellectual standards, and thus present us with a kind of knowledge which deals only with phenomena, not with Reality—*i. e.*, not with the kind of object which would fulfill the ultimate aspirations of pure intellect. The great Idealists have thus almost all disparaged sensation and exalted intellect, to such an extent that, as notably in the systems of Plato and Hegel, intellect tends to occupy the entire field, and the field of knowledge is taken to coincide with the field of transcendent thought.

Sensualism.—A different group of thinkers, of whom perhaps Condillac is the chief representative, regard the sensory element as of predominant importance. For such thinkers, to *judge* is to perceive a relation between two ideas. The perception of such a relation is a matter of comparison, and comparison is a matter of attending to two sensations. Judgment thus means attending to two sensations. But attention itself is not an intellectual act. Attention means, having our capacity for feeling wholly taken up by the impressions made upon our sense-organs, and these impressions are modifications of our conscious selves, *i. e.*, of our sensibility. Thus we see that judgment is a matter of having our capacity for feeling wholly taken up by two impressions, or, more simply, having our sensibility modified in two ways, *i. e.*, having two sensations. Attention is thus not a specifically “intellectual” operation. It is simply a question of being conscious of sensations, and such thinkers tend to regard all the more elaborate structures of science and philosophy as valid only so far

as they can be reduced, without remainder, to simple sensations.¹

Solution of the Problem.—Both these views are one-sided. It cannot be too strongly insisted upon, that sensation without intellectual organisation is blind, and intellect without sensory content is empty. Everything which we regard as meaningful, rational, clear-cut, and intelligible is to some extent the result of intellectual operation. A "pure" sensation, *i. e.*, a sensation purified of every intellectual element, would be without organisation, without identity or fixity, a fleeting psychical entity which we could never quite grasp and apprehend. In fact, seizing, grasping, and fixing is essentially the work of intellect, an intellectual operation necessary to understand anything, introducing, as it does, order, system, rationality, meaning into what would otherwise be incoherent and chaotic. It picks out from the sensory consciousness everything which is relevant, and lets the rest go. It telescopes the sensation, so to speak, and makes the meaningful elements stand out in sharp relief. In other words, the intellectual concept is the sensation, but only so far as the sensation contained elements of meaning. It is the essence, the meaning, the concentrated extract, as it were, of the sensory consciousness. Just as a bottle of beef-extract is supposed to represent the concentrated food-value of the ox, so does the intellectual concept represent the concentrated meaning-value of the sensory consciousness. Scientific method is simply an efficient intellectual machine, into which one puts the vague and confused mass of feelings, sensations, *etc.*, in order to grind out clear-cut elements which can be used for constructing science.

On the other hand, without sensory content intellect can accomplish nothing. One cannot make bricks without straw; and the application of scientific method cannot, however methodic the scientist may be, extract from the data more than is there to be extracted. No amount of scientific manipulation of mathematical or physical material, for instance, can produce valuable decisions on ethical or religious questions,

¹ Etienne Bonnot de Condillac, *Traité des Sensations*, Paris and London, 1754. See Rand, *Modern Classical Philosophers*, pp. 347-375. This way of thinking is usually attributed to the British school of thought which commences with Locke. Cf. Locke's *Essay* (1690-1700), Bk. II. Cf. also J. S. Mill, *Examination of Sir William Hamilton's Philosophy*, chapter xi (Rand, pp. 690-702).

just as no amount of card-indexing our ethical or religious convictions can tell us anything about the validity of biological theories of evolution. We need, it is true, all the applied logic, all the scientific method, of which we are capable. But we must recognise, and never allow ourselves to forget, that logical acumen alone will never lead to valuable results. It is necessary also to have material with which to work, and this material comes from the sensory consciousness. Both intellect and sensation are necessary, if we are to attain to results of general validity.

Let us take an example. Any one who has had to solve many problems by means of simultaneous equations, for instance, knows well how easy it is to make some slight error in the preliminary analysis which results in the two x and y equations. The conventionalised expression often fails to represent strictly the concentrated extract of the popular expressions of general language. In such cases, the subsequent operations with x and y may take place with consummate skill, but if anything vital has been omitted or added, the final result, however valid intellectually, fails of attaining general validity. The answer, as we say, is *wrong*, though some mark may be given for the *working*. There has been a mistake in *fact*, and it is only by patient reference to the material, by repeated sense-experiences, that we can hope to rectify such mistakes.²

Application.—Let us apply this solution briefly to our typical instances. (1) The validity of "The room is warm" will depend wholly upon whether this judgment is a correct interpretation of the sensory consciousness, the feeling of warmth which I undoubtedly experience. My judgment consists in an organisation of this consciousness, the application of scientific method with all its standards, in order to extract from it the essence or meaning-value which it contains. The validity of the result depends, in the first place, upon whether the analysis and reconstruction is itself intellectually valid, *i. e.*, whether it is consistent and rational—this is a *sine qua non*—but also, in the second place, whether the result harmonises with the sensory starting-point. If I conclude that the temperature of the room is normal, while my sensations continue to assure me that I feel warm, then something is wrong. The hypothe-

² Cf. Erdmann, *Logik*, pp. 372-413, esp. p. 374.

sis of the *room's* being warm must be succeeded by a question as to whether I myself am in some pathological, feverish state, *etc.* That is to say, as in the simultaneous equation above, there may have been some mistake about the preliminary analysis, the pathological symptoms having been overlooked. In other words, in order to verify a perceptual judgment it is necessary (1) to be sure that the result is intellectually satisfactory, and (2) to be certain that it really does represent the meaning, the concentrated essence, of the sensory consciousness which it professes to interpret, and this can only be assured by patient reference to the sensory, as well as to the intellectual side of the experience.

(2) So too in the experiential example. In such a judgment as "The freight-trains crossing the bridge are growing more troublesome in recent years," error, or lack of validity, is almost always a matter of lack of thoroughness in the intellectual analysis. The disturbance is annoying, and without really making a clear-cut, exact, scientific comparison with previous experiences, we allow ourselves to jump—without thinking—to the conclusion that it is *more* troublesome than it used to be. Or it may be merely that *we* are personally growing more sensitive. It will be important, in dealing later with the railway company, to know precisely which of these is the case. The verification of our conclusion, therefore, will involve both the application of strict scientific method and patient reference to the sensory consciousness which extends back from the present to the past cases of similar disturbances. The judgment of experience will thus be valid so far as it is a correct interpretation of the experiences in question, *i. e.*, (1) correctly put together in accordance with intellectual standards, and (2) correctly applied to the particular experiences of disturbing freight-trains.

(3) The case of symbolic judgments has already been considered in principle in dealing with the simultaneous equation example above. It remains to apply our results briefly to the judgment of Rome being occupied by Caesar. The validity of our conclusion depends on the correctness with which we have interpreted the printed text of our history book, which in turn goes back to the interpretation of various classical manuscripts and archeological evidences. It is well known that momentous conclusions in history have at times been based upon very slender evidence. The ideal of reaching valid conclusions in

this field, then, demands not only the strictest use of clear thinking, but also the most careful study of the data before the student—*i. e.*, the visual sensations, or sensory consciousness of the reader. Only thus can we be sure that the interpretative structure is really based upon experiential, sensory foundations. Both intellectual and sensory validity are necessary to secure the general validity of the symbolic judgment.

(4) Transcendent judgments, in the nature of the case, are incapable of direct sensory verification. All that we can do is (1) to test our thinking concerning "things-in-themselves," or whatever our transcendent entities may be, by reference to intellectual standards, and then (2) to connect it with whatever sensory experience we may have which bears upon the question. These ultimate conclusions of speculative thought, these metaphysical edifices, are all supposed to represent the final truth of our experience, to give the concentrated essence or meaning-value of experience as a whole. But this includes our actual experiences here and now, so that the intellectual acumen of the philosopher represents only one side of the question. His results must really apply somewhere to our empirical experiences also, if they are to be regarded as of general validity. Not only intellectual, but sensory validity also is, then, necessary in the case of transcendent judgments.

Conclusion—Theory of Judgment.—Let us now try to put together the whole of our preceding discussions, let us practise what we have just been preaching, and endeavor to extract the kernel of meaning from the material which has surrounded it in varying ways from the first page to the present. Our conclusion is, that judgment is the intellectual organisation of sensory experience, the introduction of intellectual standards into the sensory consciousness so as to give us, in place of the even but vague sensory flow, a clear-cut intellectualised essence which is fit to take its place in the ultimate ideal of organisation, the system of knowables. This system is not only thinkable through and through, but must be connected with the sensory consciousness in such a way that our judgments can be verified, can be, not merely *thought*, but *known*. The conceptual, intellectualised essence must be the *essence of the sensory experience*, *i. e.*, must give us a meaning which is not a pure creation of intellectual manipulation, but is implicitly present from the very first, embedded in our experience even at the sensory level. Judgment, then, is both

sensory and intellectual; it is the intellectual organisation of sensory experience, and is valid precisely so far as it is what it professes to be. If the sensory side of the experience is acceptable to direct sensory apprehension, and the intellectual organisation is thoroughly consistent, and if, finally, the judgment is the intellectual organisation of the sensory experience in question, then the judgment is valid.

FOR FURTHER READING

B. Bosanquet, *Logic*, Vol. I, pp. 72-92. F. H. Bradley, *Principles of Logic*, Bk. I, chapter i. H. Lotze, *Logic*, pp. 140-148.

EXERCISES

1. On what does the validity of the following judgments depend: These gloves are very comfortable. This water is too hot to drink. This grain elevator is larger than that?

2. On what does the validity of the following judgments depend: We nearly always go to church on Sunday. As a general rule, the children seem—so far as I have seen—to prefer the slide to the swings. We seem to wake up to life again when the spring comes?

3. On what does the validity of the following judgments depend: Conscience is but a word which cowards use, devised at first to keep the strong in awe. Shakespeare borrowed nearly all his plots from other writers. Blue litmus paper turns red, when dipped into acid?

4. On what does the validity of the following judgments depend: The termination of this life is the beginning of a new existence. The aim of my life is mystical absorption in Divinity. The vision of absolute beauty raises us above the human level?

APPENDIX TO PART I

NEGATION

The Problem.—Traditional logic, from the time of Aristotle to the present day, has recognised a distinction of judgments as affirmative and negative. "*S is P*" is an affirmative judgment, and "*S is not P*" is a negative judgment. For traditional logic this distinction is of major importance. Modern logic, on the other hand, regards the distinction as of minor importance. Lotze treats judgment as the answer to a question "*Is S r P?*" (*i. e.*, Does *S* stand in a certain relation to *P*?), and observes that whether we answer Yes or No makes no difference to *r*, *i. e.*, to the logical character of the relation. Whether affirmed or denied, it remains the relation *S r P*, and a distinction where there is no difference of relation is of little importance for logic. Wundt and Erdmann somewhat similarly regard the distinction as of secondary importance, and we have accordingly not dealt with it in the text. But unless properly understood, negation gives rise to so many difficulties of interpretation, that it is necessary to deal with them in an appendix. The two chief difficulties are, (1) that negation is subjective, and (2) that negation is indefinite.

Is Negation Subjective?—Let us take a few instances. Things-in-themselves are not knowable. Virtue is not square or hexagonal. No men-who-are-not-brave deserve the fair. 47 plus 89 do not make 130. All is not gold that glistens. Stone walls do not a prison make, nor iron bars a cage. There is no such thing as "apperception." I am not in love. This color is not brown. . . . These examples cover the whole field of judgment, and with such instances in view we can ask, Does negation give us any information which could be called positive? Does it add to our knowledge of reality? Has it, in a word, objective value, or is it merely subjective?

Let us consider. "Things-in-themselves are not knowable." Does this tell us anything about "the knowable," or about "things-in-themselves?" Does it even assert indubitably that there *are* such entities as things-in-themselves or knowledge?

For modern logic, on the whole, the answer to each of these questions is No. "Virtue is not square" seems to note a positive failure on our part to connect ethical and mathematical values, perhaps comparable to the failure to get to know anything about things-in-themselves. To know that "no cowards deserve the fair" does not tell us that any one *does* deserve the fair. Perhaps merit does not enter into the case at all. To know that "47 plus 89 do not make 130" does not tell us what they *do* make. From such considerations, a tendency has arisen to regard negation as registering a failure of some ideal experiment of ours. We have constructed, in the unsubstantial region of the imagination, a relation $S \ r \ P$ and proceed to interrogate reality, to see whether it will accept our hypothesis. If we can verify it, if reality accepts our ideal suggestion, well and good; we have added to the sum of knowledge. But if reality refuses to accept our hypothesis, we must try again. The only positive conclusion is that we have failed. We cannot even be certain that reality is otherwise than we have supposed. It may be merely that we have failed to connect our supposal properly. Many a correct hypothesis in science has been abandoned for a time because in the then state of knowledge it could not be verified. In such cases it is not possible to say of such a relation $S \ r \ P$ either that it *is*, or that it *is not*. All that can be stated correctly is that *we do not know* whether or no. We do not know whether things-in-themselves are absolutely unknowable, whether ethics and mathematics are hopelessly disparate departments of experience, whether the fair can or can not be deserved by any combination of manly qualities. All that we *do* know is that *we have failed* hitherto to discover a satisfactory answer. In other words, the value of negation is subjective rather than objective. It throws us back upon ourselves, and ends in the Socratic recognition of ignorance.

Is this all that we can say? Is the search after knowledge, as the psychologists tell us, a matter of trial and error, hit or miss? Are we to say the whole value of the negative judgment consists in registering a miss, in realising that we are somehow in error and must try again? Even if so, we cannot stop with this statement. For to know that we are in error is to know something positive, to add to some extent to the sum of human knowledge. It may not tell us much about epistemology or ethics or mathematics, but it does tell us

something about ourselves. But since we are also elements within the real world, the Socratic conviction of our own ignorance will have *some* objective significance. To know that we have failed, that we cannot, at least at present, verify our hypothesis, involves some positive knowledge about the universe of discourse within which our ideal experiment is applied. Reality-as-we-conceive-it rejects our hypothesis. Good. We must, then, have some conception about reality, and our failure amounts to this, that we recognise an incompatibility between two of our conceptions about reality. One is accepted as—at least in part—verified. The other is rejected, at least as being inconsistent with the first. It is possible that the first is not entirely inconsistent with the second; it may be merely that *we* cannot, are not in a position to realise the deeper viewpoint which will ultimately reveal an underlying unity and consistency. We thus recognise an inconsistency within the realm of knowledge, and it is this recognition of inconsistency which convinces us of failure and throws us back upon ourselves. The negative judgment is thus at least the positive recognition of inconsistency, otherness or difference in the world of knowledge. We can, if we wish, define this as “subjective.” But if we can recognise differences within one sphere of knowledge, why not also in another, (say) in physics or mathematics?

Let us consider further. So far we have been proceeding upon a somewhat one-sided view. We have been assuming that the suggested relation *r* is affirmative in character. Let us now take the other alternative. Let us suppose the relation to be negative, a suggestion that perhaps *S* and *P* are objectively different. We form the ideal hypothesis that a square is not a circle, that virtue is not three-cornered, that cowardice, at any rate, does not merit feminine favor. We proceed to interrogate reality, and find that it accepts our idea. Well and good. The hypothesis is verified, and the sum of objective knowledge is increased. We have established a fact, the fact of some objective difference, and can no longer maintain our Socratic pose of ignorance. We know, and we know by means of a negative judgment. The negative judgment can thus serve to give us objective knowledge.

It is thus misleading to regard the affirmative judgment as monopolising objective knowledge, and to treat the negative judgment as exclusively subjective. The truth seems rather

to be that both forms alike possess not only a subjective, but also an objective reference. If then in affirmation we are in undeniable contact with reality, the same statement holds good of negation also. In both cases we are apprehending an objective relation, a relation which really obtains between elements of reality. The difference between affirmation and negation is not the difference between establishing and failing to establish contact with reality. Contact is established in both cases. The difference is only in the kind of relation apprehended. There are, on the one hand, relations of inclusion or identity, and on the other, relations of exclusion or difference; and both kinds of relation are equally objective. "An electric bulb is not a typewriter." This deals with a perfectly objective relation. An electric light is other than, different from, a typewriter. You cannot substitute the one for the other. Recognition of such differences and distinctions in the objective world is often even more essential to our safety than recognition of identities. "That ladder is not safe," "The ice is not strong enough," "The paint is not dry," "The train does not stop here, unless you ask the conductor," "X is not to be trusted in money matters." If we were unable to rise above the subjective stage of asking questions and failing to answer them, if we failed to establish contact with reality in such practical, every-day negations, who can doubt that our life, as Hobbes puts it, would be "nasty, brutish, short"?

Negation, then, has an objective reference. How are we to classify the "subjective failure" which is sometimes a fact of experience? Can we classify it as either affirmative or negative, or does it, perhaps, fall outside this distinction? Let us consider. In every judgment, whether affirmative or negative, so long as we succeed in establishing contact with reality, so far as reality accepts our ideal suggestion, we have a subjective reaction—the reaction which we may roughly designate as satisfaction in our success. Failure, then, is something different. It is not our reaction either when we establish an affirmative relation, or when we establish a negative relation. It arises only when we do not succeed in attaching our floating idea to reality, when this remains a floating idea and we still do not know whether reality accepts it or no. We have asked our question and cannot answer it. Our attitude is one of painful suspense, with a certain sense of failure. Our suggestion obstinately refuses to leave the unsubstantial region

of the imagination. Put simply, we do not succeed in making our judgment. We do not judge. The sense in which Sigwart and others take negation must be interpreted, then, as involving, not negative judgment, but absence of judgment. That is a subjective failure, and results in the Socratic confession of ignorance. But we must not confuse it with the establishment of a negative relation between *S* and *P* in the objective world. Either we judge, or we do not judge. If we do judge, we judge something positive; we assert a relation *r*, whether the relation is mainly one of identity or mainly one of difference. If we do not judge, that may represent a failure of ours, but we must not confuse ourselves by calling it negation or negative *judgment*—for it is not judgment at all, neither affirmation nor negation. “When we think,” asks Socrates, “do we think something, or nothing?” “Something,” is the answer. “And to think nothing means, not to think.”¹

Is Negation Indefinite?—Negation, then, is objective in its reference. But there remains a further difficulty. When a scientist states that his results are “*only* negative,” he is usually not satisfied. Negative conclusions do have an objective reference, but it is felt that they do not give us much information about the object to which they refer. “This is not Mr. Smith” does not do much towards establishing the identity of the Unknown. If we knew him to be Mr. Jones, that positive and affirmative knowledge would of itself be sufficient to rule out the possibilities of his being Mr. Smith, or Mr. Brown, or Mr. Robinson. As we sometimes say, there is only one thing which a thing *is*, but an infinity of things which it is *not*. Consequently, to establish a negative relation merely touches the fringe of this “infinity,” and does not always bring us much closer to what we desire to know. At best it does little more than narrow the field of enquiry, and its function is thus mainly preliminary to genuinely scientific work. It is for this reason that there seemed to be some truth in the theory of negation as failing to get into touch with objective reality. We know now that it does establish contact, but apparently only with the fringe or outside edge of the subject. This is expressed by saying that negation is indefinite—*i. e.*, indefinite in not stating the ground of the

¹ Plato, *Republic*, 477 E. ff., *Theaetetus*, etc.

negation. An affirmative judgment, it is thought, would be more definite, and would state the ground of the affirmation.

Consider the following instances: "This watch is not going," "He has not arrived," "Your letter did not reach me," "She was not there," "The medicine had no effect," "These cartridges did not explode," "Your orders can not be carried out," "He did not take the examination." In each of these cases the negation is objective in its reference, and is important. It establishes the fact that something is wrong. But it leaves it an open question as to *what* is wrong. It is necessary to search further for the ground. Some spring may be broken, one of the screws may be missing, the watch may require to be oiled, or I may merely have forgotten to wind it. He may have gone astray, his train may be late, he may have missed his train, he may have forgotten to start, he may have decided to stay away and break the appointment. The fact is established, but the explanation of the fact is still to seek.

Let us examine further. The watch is not going—yet the springs are not broken, the screws are not out of place, it has not run down and remained unwound—perhaps the reason is to be found in the fact that it has not been oiled for many years. So too the automobilist, in searching for the *causa mali* when his machine will not work, goes through such judgments as "No, there is nothing wrong with the carburetor, nothing is wrong with the oil-feed, nothing is wrong with the ignition," *etc.*, until finally he discovers "There is no gasoline." Every successive negation established narrows the field of enquiry, until at last—we have the reason, or at least enough of the reason for our immediate purposes. The watch does not go because it has not been oiled, the automobile refuses to move because there is no gasoline, the letter failed to reach me because it did not leave the writer's pocket, he did not take the examination because he was not prepared.

These instances, however, prove more than we were anticipating. The judgments which actually do express the ground (the watch has not been oiled, he was not prepared for his examination, *etc.*) are not affirmative but negative. A negative judgment *can*, then, after all express the ground. Or—to state our result in another form—negation is not confined to the "fringe" of a question. It *can* go straight to the heart of

the matter. "This coat will not do, because there is not a button in place," "This composition receives the mark *F*, because there is *no unity* and *no mass* in it," "He missed every shot, because he did not understand how to use the sighting apparatus." In other words, negative judgments can be perfectly definite and explicit in stating the ground. "No, I won't lend you a dollar. Why?—Because you do not pay back what you borrow." "You can't thread that needle—the eye is not large enough for the thread." It is difficult even to imagine more explicit statements of the ground.

It is not, then, impossible for a negative judgment to be perfectly definite. What are we to say about affirmative judgments? Is it impossible for them to be indefinite? Must they, one and all, state a ground clearly and unambiguously, or do they also vary from indefiniteness to definiteness? Is it only negative judgments which skirt the edge of a subject and gradually narrow it down? Is not the same true of affirmative judgments also?

Let us consider a few instances. "There is a department of study called philosophy. There is a part of philosophy called logic. A certain part of logic is called the theory of judgment. A certain part of the theory of judgment deals with intellectual standards. *Identity* is one of these intellectual standards." Here we have affirmative judgments which progressively narrow down a field of enquiry. It might, however, be questioned whether any of them could be called "indefinite." Let us proceed, therefore, to consider a different type: "Pyrrhus, I say, the Romans can subdue." "A man, whose last name began with B, called to see you," "He was traveling in Germany, or some such country, at the time," "Someone has been here," "There are times when I sort of wonder whether anything is worth while," "I feel something, but it seems rather hazy." It is unnecessary to multiply instances. The conclusion is simply forced upon us, that affirmative judgments also can be indefinite.

We should, perhaps, note further that many judgments can be expressed easily and naturally in either affirmative or negative form. The emphasis shifts as the form changes, but the general meaning seems to remain much the same. "Don't lend him money—he is not to be trusted in money matters." The general meaning is much the same if we are told "he is untrustworthy in money matters." Let us consider further

instances: "The patient is not yet strong enough—the patient is still too weak," "This coat is not long enough in the back—this coat is too short in the back," "He is not feeling well—he is feeling ill," "I shall not be here tomorrow—I shall be elsewhere tomorrow," "That is not Jones—that is someone other than Jones," "This color is not purple—this color is different from purple." From such examples it looks as though our thought, whether superficial or profound, whether indefinite or definite, can be expressed either affirmatively or negatively. Affirmation emphasises the identity-aspect, and negation the difference-aspect of our thought. As we have seen above, all thought has both aspects. It seems safest to conclude, then, that all our thinking has aspects which can be best expressed negatively, and other aspects which can be best expressed affirmatively, but that, as a matter of fact, all our thoughts can be expressed in either form, though one form will be more appropriate to certain situations, and the other form to other situations.

Our final conclusion is, that our thought, whether definite or indefinite, has an objective reference, and that negation is as objective and definite as affirmation, and affirmation is as subjective and indefinite as negation—the two being related to one another as are identity and difference in our judgments.

FOR FURTHER READING

F. H. Bradley, *Principles of Logic*, pp. 109–120. B. Erdmann, *Logik*, (2nd Edit.), pp. 496–520. J. G. Hibben, *Logic*, Part I, chapter viii. Chr. Sigwart, *Logic*, Vol. I, chapter v. W. Wundt, *Logik*, (3rd Edit.), pp. 200–211.

EXERCISES

1. Are the following judgments subjective rather than objective: Not a soldier discharged his farewell shot. This ribbon is not red. The train will not start for another ten minutes. God is not mocked. I never apologise. No combination of yellow and red will give blue. This conduct is not to be tolerated?

2. Are the following judgments more indefinite than their affirmative counterparts: Never say die. Not another word! Not a single match would light. No educated man would accept that statement. Julius Caesar never dreamed of the New World. My desk is not oak. This typewriter has not a blue ribbon. This electric light will not work. Nothing venture, nothing win. My father was not in the park at 10:15. 23547912 is not a prime factor?

3. Point out the affirmative elements in the following: No real lady would act thus. This material is not silk. We shall never get off this mud bank. I never dine before noon. 579 and 732 do not make 1234. Black is not a positive sensation. Not more than one man in fifty will vote for that program.

4. Point out the negative elements in the following: That is blue. I have accustomed myself to wearing white ties. Two plus two makes four. Pompeius was known as "the Great" in his own life-time. He was thinking of taking a walk. The fire was growing stronger. We shall pay you a visit tomorrow.

PART II.

THEORY OF INFERENCE

CHAPTER XI

THE GENERAL CHARACTERISTICS OF INFERENCE

What are the Marks of Inference?—What do we understand by inference or reasoning? Perhaps we can best discover by examining a number of typical instances. “If Winnipeg is north of Minneapolis, and Minneapolis is west of Chicago, then Winnipeg must be northwest of Chicago, and Chicago must be southeast of Winnipeg.” “If our guest tends to be gracious when he has celery for lunch, let us by all means lay in a large supply of celery.” “If a bird in the hand is worth two in the bush, then two in the bush are worth one in the hand, and it is still an open question which I may happen to prefer.” “If $x^2 + y - xy = 55$, and $x - y = 5$, it follows that $x = 10$ and $y = 5$.” “If Mrs. Smith is my wife’s mother-in-law, then—unless my mother has married again—my son’s name must be Smith.” “If I had \$5.00 when I started out, and if I only spent 10c for carfare and \$3.65 for dry goods, I certainly ought to have \$1.25 left.” “If this piece of blue litmus paper turns red, that is a sure sign that the fluid before us is acid.” “If oxygen and hydrogen are combined in the proportion of 1:2, the result is *water*.”

These cases are all fairly simple. For the sake of completeness let us add a few which are more complex. “If the Minnesota team beat Wisconsin, and Wisconsin beat Chicago, and Chicago beat Illinois, then Minnesota ought to be able to beat Illinois.” “If potatoes do well in Jones’ garden, and the soil in my garden is like his, and these potatoes are like those which did so well with him, and if I am as careful a gardener as Jones, then I ought to be able to raise a good crop myself.” “If the batteries are in working order, and the sparking plugs are all right, and the machine is well oiled, and the brake is off, and there is plenty of gasoline, then—if I can only get this self-starter to work—we ought to begin to move.” “If my testimonials are all they should be, and if I look at my best and am not nervous, and if there is no other candidate with better testimonials or a better appearance, then I ought to get the position.”

Dependence.—Let us examine these various instances to discover what they may have in common, in order that we may realise, at least in a preliminary way, what are the most striking characteristics of inference. One such characteristic stands out with especial prominence. In every case we notice an “if . . . then,” a “certainty,” or a “following upon” which we may call “dependence.” “If my testimonials . . . then I ought to be appointed.” That is to say, the probability of my getting the position depends upon these conditions, so far as I know. An inference thus seems to be a kind of judgment which asserts that a conclusion x depends upon, or follows upon, a condition or premise a , and the typical form of inference would thus appear to be “If a , then x ,” or, if we wish to do justice to the complexity of the cases, “If A is B , S is P .”¹

Dependence then, or the following of a conclusion upon a premise or condition, appears to be one constituent or characteristic of all inference. Can we say, it is *the* fundamental, distinguishing feature of inference—or must we look further? Let us consider. We have stated that inference seems to be a kind of judgment which expresses dependence. Can we convert this and say that a judgment which expresses dependence is an inference? For this is one of the tests of a good definition. Let us see. “My appointment depends upon certain conditions.” “The direction of Winnipeg from Chicago is connected with the relation of both places to Minneapolis.” “The surname of my son is definitely related to that of my father.” “Water is H_2O .” These formulations of our thought all express dependence, some more clearly, others less clearly. They are all, then, judgments expressing dependence. But are they quite what we should call inferences? Hardly. They look more like what we should call statements, and we should draw a distinction between a statement and an *inferred* statement, even when the simple statement expresses dependence. The expression of dependence, then, is not the chief constituent of inference, though it may be one of the characteristic features of reasoning. It follows, that, for the discovery of the fundamental features of reasoning, we must look further.

Analytic Expansion.—The objection to the foregoing definition was, that it attempted to reduce inference to a type of

¹This view is represented especially in the work of Sigwart. See his *Logic*, Vol. I, chapter iii.

judgment, whereas it seems to be something wider. If we compare such a judgment as "My appointment depends on certain conditions," with the inference "If this condition, and that condition, and the other condition, are all realised, then I ought to be appointed," we see at once that, while both express the same meaning—*i. e.*, have the same reference—the inference expresses it in a more expanded form. It represents a more thorough-going analysis, as a result of which all the steps from which the conclusion follows are set in array as explicit conditions or premises.² Let us take an example. " $AB=BA$." The form of statement is the same, whether it expresses an intuitive judgment—*i. e.*, is reached by "simple inspection,"—or whether it expresses the conclusion of a process of inference. But the man who has gone through all the steps³ has a much clearer and more reliable insight into the truth of the statement. He has not jumped hastily to the conclusion, but has a knowledge which is firmly based upon analysis, and such knowledge has been tested and examined, rather than left to first impressions. From this viewpoint the distinction between judgment and inference or reasoned knowledge is the difference between what we call "feminine intuition," *i. e.*, trusting to unanalysed impressions, and reasoned knowledge based upon methodical analysis.⁴ An inference is thus an *analytically expanded* judgment, and we may regard this characteristic as "analytic expansion."

But if we accept this and look no further, shall we be altogether satisfied? Our view makes of inference an analytic refinement upon judgment, in such a way that, instead of stating roughly and in general terms that " S is P ," we have analysed out the various factors involved, and know what it is in S and what it is in P which makes them stand to one another in this relation. That such analysis is of great value is beyond question. But, when all is said and done, are we not left just where we started? Have we succeeded in adding to the sum of our knowledge a single new idea? Our ideas are now far more clear and distinct. Our knowledge is highly polished and clear-cut, but—has it advanced a single step in a direction which could be called *new*? Have we *discovered*

² This view is represented especially in the work of Lotze. See his *Logic*, esp. §§ 97 ff.

³ For the actual steps, see *infra*, chapter xxix.

⁴ Cf. R. L. Nettleship, *Lectures on Plato's Republic*, opening lecture.

anything hitherto unknown? Let us see. We know that *S* is *P*, and we know, rather more than less, why we think so. We have made clearer to ourselves the elements which together constitute the complex *S* and the complex *P*. But surely we knew all this before; not so clearly, perhaps, and not with such certainty, but still in the main we did know it—so that at the end we remain with the same sum of knowledge with which we started.⁵ Is this perfectly satisfactory as a account of inference? Does it not remain a puzzle how we could ever discover whether *S* might be *Q* or *R* or *T*—or indeed how we ever managed in the first place to hit upon the idea that it was *P*? But briefly, if this were all, or even the most important part of inference, inference is the analysis and classification of knowledge which we have somehow managed to discover by some other means. But we have a suspicion that inference is a method of *discovery*, that it is one of the means of *extending* the field of knowledge and learning something which is *new*. Can we justify this suspicion?

Novelty.—Let us ask then, whether a third characteristic of inference is not that it leads to something *new*, or adds to sum of knowledge. Consider a few instances. The Abbé is chatting with the ladies. “Ah, ladies, my first experience in the confessional was terrible indeed. My first penitent was a murderer!” Soon after, their host entered the room. “Well, ladies, chatting with our good Abbé? Do you know, I was his very first penitent?” There can be no doubt that the ladies drew an inference which told them something new about their host—something which had not been told them either by the Abbé or by their host. The Abbé had spoken generally, without hinting at any names, and had merely emphasised the horror of his situation, as a young man, on being brought into relation with one who had committed so terrible a crime. The whole stress was on the shock to his delicate and

⁵ The student of Latin composition will be familiar with this from *e. g.*, “Bradley’s Arnold,” where he learns in one exercise that the present tense can be used to express not only present time, but time immediately past, time immediately future, and even time perfectly past (*i. e.*, can be used for the imperfect, future, and perfect tenses), and finds in subsequent chapters that the imperfect tense can be used for the present, the perfect, and the pluperfect, the perfect tense can be used for the present, the imperfect, and the pluperfect. When he comes to the end of the (quite elaborate) account of how these tenses can be used, he really knows little more than he did before. He is, if anything, made far less certain which form of expression would be appropriate in a given case, as he has to choose between approximately three forms, without really knowing which would be the best.

untrained nerves, at meeting with a real murderer. The secrets of the confessional were not revealed, though there was, perhaps, a slight indiscretion in speaking of such things at all. So too their host does not tell them that he is a criminal. He is merely emphasising the many years that he and the Abbé have known one another. In fact, it was at the very outset of the latter's ecclesiastical career that they had become acquainted. The reference to the confessional is slight and general—one of the usual incidents of their religion—and the whole stress is on the many years in which he has been intimate with their good Abbé. The inference, however, certainly leads to something new—in fact to a startling and shocking discovery about their host's early life.⁶

Let us take another instance. The new minister, in the course of one of his first sermons, happens to mention how young he was when he served in the Spanish-American war. "I was only eighteen at the time." Instantly every member of his congregation puts two and two together and discovers how old he is *now*. This is *new* information. It is something which they wished to know, and which he did not tell them. He was merely emphasising the extreme youthfulness of many of "the boys" at the time of that service. His parishioners, however, compare dates and come to a conclusion which is an interesting discovery and satisfies their curiosity on that head. Inferences of this general type are of very frequent occurrence in our every-day intercourse, and are certainly to be met with in science also. Many great discoveries have been made by a trained scientist's putting two and two together in this way, and we may accordingly regard novelty as at least an extremely important element in inference.⁷

Have we, in this characteristic, discovered the full nature of inference? Can we rest satisfied with a definition which tells us that inference is a form of reasoning which leads to the discovery of something new? Let us consider. Assume that all inference gives us knowledge which is new. But, we must still ask, is every process which gives us new knowledge to be regarded as inference? A sudden shoot of pain will produce knowledge which is new—it informs me that I have the toothache, that there is something wrong with my

⁶ Cf. Royce, *Sources of Religious Insight*, pp. 94–96. The instance is from Hibben's *Logic*.

⁷ Cf. J. S. Mill, *System of Logic*, Bk. III, chapter ii.

tooth, and that perhaps I had better see my dentist. The last two statements, which refer to the cause of the pain, and the way in which it could be removed, would probably be considered inferences. But the knowledge that I have a pain in my tooth, or the toothache, while certainly new, would not naturally or usually be called an inference. So too with all simple judgments of perception. "It is raining," "How warm it is getting," "I am thirsty," "You look quite pale," "This tree is turning brown,"—these judgments and a hundred others of similar type would usually be regarded as judgments rather than as inferences, as we can realise if we compare them with such statements as "The roofs are wet—it must be raining," "Look at the thermometer—how warm it must be getting," *etc.* And yet they certainly contain information which is new. In order to exhaust the nature of inference, then, we must look still further.

Constructive and Systematic.—Let us consider. An inference differs from a simple perception, though both alike may give us knowledge which is new. In what precisely does this difference consist? Perhaps mainly that in simple perception we just apprehend what is forced upon us, what affects us immediately in sensation, whereas in inference we go through some process which may, indeed, start from sensory data, but may lead far beyond what is present in sensation. This process consists in inferring or drawing or deducing a conclusion from premises, *i. e.*, from previous knowledge, which may or may not be of an immediately perceptual kind. In perception I just see. In inference, my thought completes itself by passing through a more or less elaborate process which gives me a result which has been deduced or reasoned out, which follows upon something else, which is true and certain, if that from which it follows is true and certain. Two or more previously acquired pieces of information are put together in such a way that they lead to a *tertium quid* which is new. That is to say, inference is constructive and systematic. In the two premises we have fragments of a potential system. The process of inference seems to consist in putting together these fragments so as to realise in our construction something more of the system of which they now form a part. The "something more" which our construction thus produces is the new information or conclusion. For example, from an arc of a circle we can construct

the rest of the circle, by first discovering the center, and then applying the postulate for circle-construction. That is to say, we manipulate the given material in accordance with the laws of the particular system, and thus acquire new information which is true within that system. So too in the inference "If Mrs. Smith is my wife's mother-in-law . . . , then my son's surname must be Smith," the conclusion can only be reached by constructing the system of relations of consanguinity accepted by present social conventions. So too in "If $a - b = (x+y) (x-y)$, and $a = x$," it follows with mathematical certainty—*i. e.*, it follows with certainty within the mathematical system—that $b = y$. This we can only discover by constructing the relevant portions of the system within which such algebraic relations are worked out. So too the calculation, in trigonometry, of the distance of a ship at sea is discovered by constructing the system of cosines and tangents which is appropriate to the special concrete situation. Inference is thus constructive and systematic,⁸ and this characteristic is obviously of such fundamental importance that we might, in a preliminary way, define inference as the discovery of new information by the construction of a system, or—as Mill graphically expresses it—by arguing (systematically) from the known to the unknown.⁹

Conclusion—The General Characteristic of Inference.—Our conclusion, then, is that inference is (1) dependent, *i. e.*, is a matter of "If . . . then," rather than of simple statement; (2) analytically expanded, *i. e.*, is reasoned knowledge as opposed to an unanalysed impression; (3) novel, *i. e.*, leads to a result which constitutes a genuine discovery of something new; (4) constructive and systematic, *i. e.*, reaches its conclusion by constructing the relevant portions of a system of knowledge which is appropriate to the concrete situation. Put briefly, inference seems to be a process of discovery by constructing an appropriate system.

But before we can advance from this preliminary sketch to a final view of inference, we must examine more in detail the four characteristics which we have discovered. This examination will be the task of the succeeding chapters.

⁸ Cf. especially Bradley, *Principles of Logic*, pp. 255 ff.

⁹ This is to be taken merely as a striking phrase. Cf. Schuppe, *Erkenntnisstheoretische Logik*, p. 260, Sigwart, *Logic*, pp. 360–362. How far, if at all, we can reason from the known to the unknown is considered below.

FOR FURTHER READING

B. Bosanquet, *Logic*, Vol. II, chapter i. F. H. Bradley, *Principles of Logic*, pp. 235-239, 396-411. B. Erdmann, *Logik*, (2nd Edit.), pp. 588-593. J. G. Hibben, *Logic*, Part I, chapter x.

EXERCISES

1. Point out the element of dependence in the following: You vote for *F* as captain, and I'll vote for your friend *B* as first lieutenant. Get a good lock for your car, unless you want to have it stolen. I didn't give him the order, because it seemed to me that he was trying to charge too much.

2. Point out the element of analytic expansion in the following: If she be not fair to me, what care I how fair she be? If that is my car, I must run. Anyone who can put two and two together must realise that sincere atheism is a self-contradiction.

3. Point out the element of novelty in the following: If I spend \$5.00 a month for rent, and \$10.00 a year for books, and \$30.00 a year for clothes, and other expenses in proportion, I ought to manage on about \$450.00. If this is what education leads to, it's back to the farm for me! If sunflowers, corn and mangel-wurzels grow as easily as that, it ought not to be much trouble to feed poultry.

4. Point out the element of systematic constructiveness in the following: If you can't find time to do this typewriting for me, then I shall have to take it to a public stenographer, and shall probably have to pay quite a sum for it. If my train leaves at 6:30 a. m., I shall have to get up at about 4:30, in order to be ready in time. If he won't help me with my work, I shall refuse to help him with his.

CHAPTER XII

THE DEPENDENT OR HYPOTHETICAL NATURE OF INFERENCE

Dependent Nature of Inference.—As we have seen, in inference we draw our conclusion from premises. If $x - y = 8$, and we know that $y = 2$, it follows that $x = 10$. That is to say, the truth of x 's being equal to 10 follows from, or depends upon, the truth of the premises. If the premises are correct, then the conclusion holds good. If oxygen and hydrogen are combined in the proportion of 1:2, water results. In other words, the phenomenon called water results from our experiment only if the requisite conditions are fulfilled, or is dependent upon the fulfilling of those conditions. It is not true absolutely, but only on condition of the correct proportion being observed. This characteristic of the conditionedness of inferred truths is expressed by calling them "hypothetical." Water is not regarded absolutely, as water, but as the resultant of certain complex chemical conditions. Music is not regarded simply as music, but as an effect produced by the intermingling of sound-waves in an order determined by a number of rules, *i. e.*, as resulting from the fulfilment of a whole complex of conditions. So too *Taenia saginata* is not treated simply as a certain kind of worm, but as a certain stage in a complex series of life-forms, each of which develops under certain definite conditions, and the whole series of forms can be brought to an end by interfering with any stage where the conditions admit of such interference.

Inference, then, is hypothetical, or the conclusion is dependent on the fulfilment of certain conditions. Can we analyse further this element of dependence, and come to realise in what it consists, or how it is constituted? Let us consider a few cases. "If I delay any longer, I shall miss the car." "If the corn is planted too early, the seeds will rot." "If children persist in sucking their fingers, they must be punished." What is it that I really *judge* in such cases? Do I judge that I shall miss the car, or that the seeds will rot?

Hardly, for perhaps I shall hurry, and thus not miss the car; perhaps the corn will be planted later, in which case the seeds will develop normally. I do not, then, judge that I shall miss the car. What is it, in that case, which I do judge? Do I judge that I shall delay longer, or that the corn will be planted too early? Again, we must say, this can hardly be the case; for it may be otherwise, and the "if," taken strictly, leaves it entirely unsettled whether or no. I do not, then, judge either that the seeds will rot or that they will not rot. In other words, I make no simple judgment at all.¹ What is it that I do? I go through a process of thought which is complex, and draw a conclusion from premises. My judgment is, the seeds will rot *if* planted too early, I shall miss the car *if* I do not hurry, children must be punished *if* they persist in wrong-doing. My conclusion is thus a conditioned conclusion, and what I judge is essentially the *connection* of premises and conclusion. I establish a law of connection—the connection of ground and consequent, or of cause and effect. The dependence of the hypothetical judgment is thus the dependence of consequent upon ground.

Kinds of Dependence: (A) From Cause, (B) From Absence of Cause.—The dependence is thus the dependence of an effect or consequent upon a condition. Let us proceed to ask, in what ways and to what extent are consequences dependent upon conditions? Let us take a few cases. "If I expose myself unduly, I shall catch cold." The consequence is here clearly dependent upon the functioning of certain general laws of health. Suppose the condition realised, suppose I do expose myself unduly. In that case, so far as my knowledge goes,² I shall certainly catch cold. So far, so good; this is reasoning from cause to effect. But let us now suppose, on the other hand, that the condition is not realised—suppose I wrap myself up carefully. How about the consequences? Do I, or do I not catch cold? As we commonly understand the laws of health, it would follow that, so far as cold-from-exposure is concerned, I do not catch cold. In other words, we can, in such cases,

¹ The view in the text is opposed to that of Sigwart, who regards both *if*-clause and *then*-clause as expressing judgments. But it seems inadvisable to treat them as judgments, since they are certainly not judged.

² The connection here is only "empirical"—i. e., a matter of imperfectly analysed and imperfectly understood experience. The "law" does not *always* hold good, but so far as I know, I expect it to work.

argue both positively and negatively. If the condition is realised, the consequent is realised, and if the condition is not realised, then the consequent is not realised either. We can argue from presence or absence of cause to presence or absence of effect. The rotting of seed-corn, for instance, depends upon the wetness present in the soil. If the soil is wet, the seeds rot; if the soil is not wet, the seeds do not rot. If I wait longer, I miss the car; if I do not wait, but hurry, I catch the car. If children persist in wrong-doing, they must be punished; if, however, they amend their ways, they must not be punished. If you work, you may some day amount to something; if you don't work, you will never amount to anything.

These are all instances in which the connection is a matter of empirical law. Experience shows that the connection holds, as we say, in the long run. But we do not have precise insight into the condition, as we do in the so-called exact sciences. Do we, then, find a different result in the exact sciences? Let us see. "If a triangle is equilateral, it is equiangular." If it is not equilateral—if, *e. g.*, it is scalene—is it, or is it not, equiangular? It is not equiangular, and the case is similar to what we discovered in the empirical cases. "If $(a+b)$ be multiplied by $(a-b)$, the result is a^2-b^2 ." If, however, $(a+b)$ be not multiplied by $(a-b)$ —if, *e. g.*, it be divided by it, or if it be multiplied by $(c-d)$ —the result is *not* a^2-b^2 . "If 32 be added to 57, the result is 89." If, however, 32 be not added to 57—if, *e. g.*, it be subtracted from 57, or be added to 41—the result is *not* 89. In other words, in the exact sciences, as well as in our more empirical thinking, if our thought succeeds in penetrating to a law which holds good, then we can say:—(1) if the condition is realised, the consequent is realised, and (2) If the condition is not realised, neither is the consequent. If A is B , S is P ; and if A is not B , S is not P .

(C) From Effect; (D) From Absence of Effect.—Let us consider further. A relation of dependence is two-edged. If A is in relation to B , B is also in relation to A . So far we have considered only what we can infer when the condition is, or is not, realised. Can we, however, start at the other end, and ask, given the consequent or effect, is it possible to draw any safe conclusion about the ground, or cause? Or again, granted that the consequent or effect has not been realised, can we infer, perhaps, that the ground or cause has not been realised? Let us consider. Suppose I have caught cold. This

is an effect. Can it be inferred—if it be an exposure-cold—that I have unduly exposed myself? As we commonly understand the laws of health, undoubtedly, Yes. Suppose, on the other hand, that I have not caught cold. Can it be inferred that I have not exposed myself? Not perhaps with the same degree of certainty, for the statement of the empirical law in question is not quite exact, and people do sometimes expose themselves without suffering the consequences. But on the whole, my immunity from colds is fair evidence that I have taken reasonable care of myself, and it would usually be argued, that if I have no cold, I have probably not been exposing myself unduly. Let us consider the next case. If the seeds have rotted, can we infer that the soil has been wet? Gardeners would say, Yes. If however, the seeds do not rot, but develop normally, can we infer that the soil was not wet? Or at least not unduly wet? This seems a little less certain—for the same reason as before, *viz.*, that the law is only empirically and imperfectly known—but on the whole it also would be answered in the affirmative. If the seeds show no traces of rot, it would be inferred that the soil had not been wet. So too in the other cases. If I miss the car, it can be inferred that I delayed too long. If I catch it, it can be inferred that I did not wait too long. If the children have been punished, it can normally be inferred that they have been doing something which they ought not to have been doing. If on the other hand, they have not been punished, that is at least presumptive evidence that they have committed no serious offences. Similarly if a man's success is pronounced, it would usually be inferred that he must have worked hard to earn it. If, on the other hand, he never amounts to anything, it would as a rule be inferred that he had not worked hard.

These cases being all empirical, the degree of certainty with which we can argue from consequent to ground, or from effect to cause, varies in the various cases. But on the whole we are certainly convinced that where the consequent is realised—where, that is, we have an effect—the condition or cause must have been realised also, than that where the effect is absent the suspected cause must also have been absent. Let us now review the cases taken as examples of exact science. If a triangle is equiangular, can we argue that it must be equilateral? Yes, certainly we can. If, however, a triangle is not equiangular—*e. g.*, suppose it obtuse-angled—is it to be

inferred that it cannot possibly be equilateral? Yes, again, quite certainly. Let us take the next case. If the result of multiplying $(a+b)$ by a second factor be a^2-b^2 , can we infer that the second factor must have been $(a-b)$? Yes, certainly. If, however, the result was something else—*e. g.*, $ax-y^2$ —can we infer that the second factor was *not* $(a-b)$? Most certainly we can. Our conviction depends upon our insight into the law in question—the divisor-dividend-quotient relation—and we can be in no possible doubt either in the positive, or in the negative case. So too, if we are acquainted with the laws of addition and subtraction, we know that if 32 plus *something* adds up to 89, the second element in question must be 57, and that if the result is not 89—but is, *e. g.*, 65—we can infer with mathematical certainty that 32 has not been added to 57. In other words, in the more exact sciences as well as in our more empirical thinking, if our thought has succeeded in penetrating to a genuine law of connection—of ground and consequent, or cause and effect—then, if the consequent or effect is realised, we can argue that the ground or cause must have been realised: and if, on the other hand, the consequent or effect is not realised, we can infer that the ground or cause cannot possibly have been realised.

Conclusion.—Expressed generally, our conclusion is, that if we are sure of the law which connects $A-B$ and $S-P$, so that we can say S 's being P depends on A 's being B , then we can argue from our law of connection in all the four ways discovered above, *viz.*, (1) If A is B , it follows that S must be P ; (2) if A is not B , it follows that S cannot be P (so far as our knowledge goes); (3) if S is P , it follows that A must be B ; (4) if S is not P , it follows that A cannot be B . That is to say, we can reason from ground to consequent, or from consequent to ground, from cause to effect, or from effect to cause, either positively or negatively, with reasonable certainty—the degree of certainty being precisely proportionate to the degree of our insight into the law of connection in question.

Further Consideration.—It is perhaps advisable to dwell further on the imperfect cases—in which we have not entirely succeeded in attaining a genuine law of connection—in order to see what can be inferred in such cases. Believers in apparitions, for instance, state as a hypothetical law that an abnormal event, such as a murder, tends to leave traces of itself in the locality—*e. g.*, in the form of vibrations which perma-

nently modify the structure of the walls or furniture—so that when a sensitive person is in the neighborhood, these traces will affect the imagination of such a person, in such a way that he will see a visual image or “ghost.” The cause is here the murder, and the effect is the apparition. But compare the two inferences:—(a) If a murder has been committed, then a sensitive person will see an apparition, and (b) If a sensitive person sees an apparition, then a corresponding crime must have been committed. Which of these two inferences would meet with the wider acceptance? The great majority of believers would feel more certain of (b) than of (a), and would justify their belief by reasoning that an apparition is an effect, and that, since every effect has a cause, an abnormal effect probably has an abnormal cause. In such cases the argument from effect to suspected cause has more weight than the argument from cause to effect. Let us now consider the negative side of the relation. (c) If no murder has been committed, the sensitive person will see no apparition; (d) If the sensitive person sees no apparition, then no murder has been committed. Which of these two inferences seems the more probable? To a majority of the “authorities” in this field, (c) would seem more reasonable than (d). That is to say, in such cases we could argue from the absence of the cause to the absence of its suspected effect, but could not so certainly infer from the absence of the effect to the absence of the suspected cause.

Let us take another instance of such imperfectly analysed thought. In the case of certain diseases, it is believed by physicians that if the patient is to recover, he must desire to get well. The suspected cause is here the “will” of the patient, and the effect hoped for is his recovery. Let us see what can be inferred. (a) If he earnestly desires to get well, he may be cured. (b) If he is cured, he must have wanted to recover. (c) If he does not wish to get well, he will not recover, but will probably die. (d) If he does not recover—or if he dies—that is a proof that he cannot have really desired to recover. Not one of these inferences is certain, but (a) would be thought slightly more probable than (b). and (c) would be thought slightly more probable than (d). That is to say, in such cases as this, the argument from cause to effect seems more probable than from effect to suspected cause, whether we are reasoning positively or negatively. It is, however, only

fair to add that no physician would risk his reputation by supporting any one of these inferences. The probability, in such cases, is usually realised *after* the event.

Conclusion.—Is there any one infallible rule for inferring in this field—the field of “popular,” half-analysed thought? Let us see. In the apparition case, it seemed reasonable to infer from the presence of the effect, or from the absence of the cause. In the mental healing case, it seemed allowable to argue from the presence, or from the absence, of the cause, but not from the presence of the effect. That is to say, both cases agree in permitting an inference from the absence of the suspected cause to the absence of the suspected effect. These instances thus agree with the seed-corn and car-catching cases, at least in the single particular of admitting an inference from the absence of the cause. Arguments from (1) presence of cause, (2) presence of effect, (3) absence of effect, appear to be admissible in certain cases, inadmissible in others. That is to say, the only inference which holds good invariably in the instances before us is the argument from absence of cause. Can we then state as our conclusion that in such half-analysed thought it is at least always permissible to argue from the absence of the cause to the absence of the suspected effect? Let us consider yet another instance. “If the locomotive ran over him, he must be injured, perhaps fatally.” What kinds of inference can here be drawn? (1) From absence of effect: if he is uninjured, it is scarcely credible that the locomotive can have run over him. (2) From presence of cause? Yes, with a fair degree of certainty. If the locomotive has run over him, we are pretty sure that he cannot possibly be uninjured. (3) From presence of effect? Let us see. If by “injured, perhaps fatally” we refer (as of course we do) to the kind of injuries received in being run over, the experts at the inquest would infer from effect to cause, and their inference would tend to be accepted. (4) Finally, can we argue from absence of cause to absence of effect? In other words, does this case agree with the others in permitting this kind of inference, or must we conclude that there is no one definite rule applicable to such reasonings? If the locomotive did not run over him, can we argue that he is not injured in that kind of way? Some logicians—*e. g.*, John Stuart Mill—would assert that an effect may be produced by a “plurality” of causes, and that from the absence of any one cause we cannot reason to

the absence of the effect. For the effect might have been brought about *quand même*, by any one of the other possible causes. For instance, he may have escaped the locomotive, but may have fallen beneath a street-car or an automobile. Is this objection to be taken seriously? Or is it not rather too superficial to merit attention at the present day? We do not infer from his not having been run over by the locomotive that he is, *e. g.*, alive and well at the present moment. That would be to go far beyond our information. We argue only that he has escaped injury of a particular kind—the kind caused by locomotives. He has escaped *locomotive-injuries*. About theoretically possible injuries from other sources not a word has been said. Our conclusion, then, is that however the admissibility of the other modes of inference may vary in empirical cases of cause-effect reasoning, the argument from absence of cause to absence of effect is reasonable, and further, that there is a tendency for us to feel, in all such cases, that if the connection in question is at all valid, there must be some degree of evidence for all four modes of inference.

Hypothetical Versus Categorical.—One further point remains to be discussed. Inference, as we have seen, is hypothetical, conditioned—*i. e.*, has an “if” in it. It might be thought that an unconditional form of statement—“categorical” as it is named—*i. e.*, a direct statement without even the suggestion of an “if” about it—would be more valuable, and thus should be the goal of inference. Compare, for example, the two forms of expression:—(1) “*If* that street-car coming round the corner is yours, you must hurry.” (2) “That street-car *is* yours; hurry!” There is no *if*-ness about the second, or categorical, form of expression. It is unhypothetical, does not admit of doubt or hesitation, but demands instant action. The hypothetical form does not tell us whether or no—it leaves the *if* in full force, *i. e.*, it leaves the question still open. The categorical form, on the contrary, leaves no room whatever for questioning or deliberation. It tells us outright that the fact *is* so. It thus seems to go further than the hypothetical form, and it might reasonably enough be asked, which form should be our ideal, and which attitude it is, on the whole, wiser and more logical to cultivate,—the hypothetical, or the categorical.

Let us consider a little further. The categorical form really seems to have more to offer than the hypothetical. For it does not end with an unanswered question, but is direct and

straight-forward, and, as we have seen, leads immediately and by the shortest path to action. It resembles the bedside manner of the practising physician, which cuts across half a hundred hesitations and disputed questions and boldly orders a definite line of treatment. It is, in fact, in general, the attitude of applied science and common sense, and, like them, betrays a certain impatience with the questionings, with the sceptical, impartial, judicial attitude of the theoretician, the "pure" scientist. Practical life needs quick decisions. Questions must be settled, one way or the other, and settled at once. Theory is all right in its place—but its place is the research laboratory or the research publication, and not in the office or the home. What the practical man needs is results, definite concrete rules which can be used, and not eternal questionings which lead nowhere.

On the other hand, the practical attitude seems somewhat dogmatic. To cut short deliberation and enquiry is, in the last resort, unprogressive, and leads to mental stagnation. The hypothetical attitude is concerned with discovery, with the establishment of laws, with the exact analysis of phenomena and study of their conditions. Not unduly under the influence of practical considerations, its interest is in enquiry, in testing, investigating, finding out what is to be learnt in the sure school of a science based upon experience. Not leaping to conclusions, but studying problems; not deciding too soon, but weighing arguments; not rapid, but very sure, and above all, progressive. It is the attitude of theory, of study and science, of logic and philosophy, as opposed to mere common sense and an interest confined to the immediate needs of practical living. Which attitude should be cultivated depends upon our character—on the kind of persons we are and the kind of persons we wish to be. For modern logic, the hypothetical interest in scientific method seems more valuable than the categorical, rule-of-thumb method of the half-trained practitioner, though it is not denied that, in its place, this too is valuable. Method versus results; deliberation versus action; progressiveness versus complacency. Which is the more valuable? Which attitude do we ourselves choose?

Concluding Summary.—Let us now put together what we have discovered in the present chapter. There are, generally speaking, two kinds of knowledge:—(1) scientific, in which we have attained a degree of insight into the working of some

law; (2) popular knowledge, the field of the practical man with his rule-of-thumb methods and his love of categorical statements. The first kind is more hypothetical, and is interested in discovering the particular *ifs* upon which certain conclusions depend. The second kind does sometimes throw its results into the *if* form, but is usually more categorical. In the hypothetical cases, if we have succeeded in discovering some law, we can argue or infer from the law in four typical ways. We can reason from the presence or absence of the cause or ground to the presence or absence of the effect, or consequent; or, *vice versa*, we can argue from the presence or absence of the consequent or effect to the presence or absence of the ground or cause. If our thought has only partially succeeded in its analysis, and we are not quite certain of our suggested law, we cannot argue with the same degree of certainty. The ideal is, here also, all four forms of inference, but in practice we have found only one which does not vary—*viz.*, the argument from the absence of the cause to the absence of the effect. This has seemed always reasonably admissible, but in proportion as our knowledge is greater and our thought more strict, even the popular forms of inference tend to allow all four types of reasoning.³

³ As a practical illustration of what we have discovered in the present chapter, we might state that if it is true that the workings of a law lend themselves to the four typical inferential forms, then it will also be true that wherever we can make all four inferences with reasonable certainty, that is a safe test of the validity of the law in question, and that where we are unable to draw all four inferences with equal certainty, that is a safe indication that we have not yet succeeded in discovering the law in the case in question. Cf. the rule *re* the convertibility of a definition, *infra*, chapter xxvii.

FOR FURTHER READING

F. H. Bradley, *Principles of Logic*, Bk. I, chapter ii. J. G. Hibben, *Logic*, Part II, chapter xii. Chr. Sigwart, *Logic*, Vol. I, pp. 326–338.

EXERCISES

1. Assuming the following inferences to be correct, what inferences can be drawn from ground to consequent, or from consequent to ground,* and which of these seem the more probable: If he writes to me again, I shall not read his letter. If the lawn-mower won't cut, I shall send and have it sharpened. If the car has no gasoline, of course it won't start. If this sand were only cleared away, it would be grand. If you want to be an engineer, you will have to study and go to college?

2. Assuming the following inferences to be correct, what inferences can be drawn from ground to consequent, or from consequent to ground:* If you will not help me put the car into the garage, I shall not take you out riding with me again. If this history book says so, it must be true. If the engine is really off the rails, it will be necessary to get a derrick for it. If you love me, you will love my dog. If you are a good shot, you should score 100% at that range?

3. Assuming the following inferences to be correct, what inferences can be drawn from ground to consequent, or from consequent to ground:* If $x = 5$ in the equation $2x - 3xy + 4y = 10$, then $y = 1$. If the plane figure in question is such that it is rectilinear, and a closed figure, and such that any of its exterior angles is equal to the sum of the interior opposite angles, the figure in question must be a triangle. If the quotient is 61, and the dividend is 305, the divisor must have been 5.

* *i. e.*, both from the presence of ground (or consequent), or from the absence of ground (or consequent).

CHAPTER XIII

THE ANALYTICAL CHARACTER OF INFERENCE

Nature of Analysis.—So far we have considered the difference between the categorical and hypothetical viewpoints, and have seen that inference or reasoning is hypothetical rather than categorical. Let us now proceed to compare statements and inferences from a new viewpoint, in order to bring out what we took to be the second main characteristic of inference, *viz.*, its analytical nature.

The practical man never analyses, if he can help it.¹ He just acts, and acts in accordance with habit and routine. For most of the purposes of life—which are instinctive—little analytical reasoning is required. Most of our ends are easily attained, and a conventional attitude of mind as well as a conventional appearance—what we call the professional manner—carries us safely through most of the social contingencies which present themselves. It is, in fact, only when things cease to run smoothly, when something goes wrong, that analytical thought is called into play. A man who finds the door refuse to open behaves, at first, precisely like a trapped animal, such as a dog, cat, or monkey. He pulls, pushes, kicks, and knocks, and makes a fuss generally. It is only when this method fails that his attention is directed to investigating the cause, and to taking measures to have the conditions remedied—*e. g.*, by telephoning to the janitor or locksmith, before descending by way of the fire-escape or window. The occasion for inference is thus the breaking down of our customary methods of procedure before novel circumstances. These necessitate the adoption of new methods especially adapted to the new occasion—require a new analysis of the situation, a substitution of analytical investigation for the practical rule-of-thumb method of “common” sense.

So much for the *occasion* of inference. The way in which we actually reach our solution is still, in the main, the method

¹ Cf. W. B. Pillsbury, *Fundamentals of Psychology*, chapter xi (on Reasoning).

of trial and error. Various associations are aroused, tested mentally, and rejected, until at last there comes into our minds one which fits the case and leads to action. Explicit analysis takes place only when we direct our attention to the chief elements in the problem, and to the chief steps of a solution which will be just to them all.

In this way, then, we analyse practically a practical situation. Let us proceed to consider a case which is less immediately practical. I am reading Kant's "Critique of Pure Reason," and am doing my best to understand the "psychological" part of the "transcendental analytic." I find that a great deal of the constructive work of knowledge is performed by the "imagination," but when I try to make clear to myself just what imagination does, especially as compared with sense on the one hand and understanding on the other, I find myself baffled. I try all the ways out which suggest themselves. Has imagination, in Kant's sense, anything to do with mental imagery?—That doesn't seem to fit in with its "transcendental" functions. Is it, perhaps, like the creative imagination of the poet?—That also does not seem very helpful. I consult the explanatory literature which is at my disposal—but that seems to play fast and loose. At one moment imagination seems the same as sensory perception, at another it exercises much the same functions as understanding—with the peculiar qualification that it is "blind." I give up this way of trying to learn. Unmethodical guesses, or looking up, at haphazard, literature which was obviously written without any feeling for my particular problem, gets me nowhere. I decide to find out for myself, if possible. There is only one way to find out, and that is, by exhaustive analysis of the situation itself. I make a collection of all the passages in which the term is used. I then collect leading statements about sensory experience, and about the work of understanding. I proceed to classify the data with which I have thus furnished myself, putting together all statements which seem to bear on some one point—*e. g.*, (1) the "blindness" of imagination, (2) its "synthetic" function, (3) the relation of reproductive to productive imagination, (4) "schemata" as distinguished from images, *etc.*—until I have exhausted all the distinguishable points which appear to be treated in Kant's writings. I then, very gradually, find what he seems to have had in mind, and my problem is solved.

Here again, although the material is very different, the general method of dealing with the situation seems much the same as before. I do not analyse until the rough and ready method of trial and error breaks down. Here also my analysis seems to be simply an attempt to appreciate, one by one, the various distinguishable features of the situation, in the hope that, if I take them one at a time, some ray of illumination may dawn upon me, and I shall be helped out of my difficulty. It differs, however, from the old attempt by being methodical and exhaustive, rather than haphazard and at random. It tries *all* the ways, rather than those which chance to present themselves to me, and thus leads to some sort of satisfaction. For if I have tried *all* the ways, I know that I have done my best, and that my non-success is not a mere matter of chance—which a little perseverance might remedy.

Special Features of Analysis.—Let us focus our attention a little more closely upon certain features of our analytical method. In the first place, we do not bring in hypotheses *ex machina*. To find our solution we split up into its elements the situation which is before us, and act accordingly. If I wish to learn to play music, it will never do just to read books on “the three B’s,” Bach, Beethoven, and Brahms, but I must patiently and methodically practise scales, chords, and passages, until I can solve the technical problems presented in the particular sonata or fugue which I wish to perform. There are also musical problems to be solved, which also will require specialised studies appropriate to the occasion. We have to remain very close to our data. Thus no one would dream of inferring that Socrates must die on the general ground that “all men are mortal,” but for the specific reasons that the Athenians have condemned him to death, that he is in the condemned cell, that the fatal morning has arrived, that he has, in fact, drunk the hemlock and it has begun to take effect. So too no botanist would infer that plants grow upwards because they love the sun, or even because they have a positive heliotropism and negative geotropism, but would dissect the various stem-cells, and would show the precise way in which certain starch-granules rest normally upon the sensitive protoplasm at the bottom of those cells, and thus furnish a delicate mechanism for appreciating the influence of gravitation, analogous to the statocysts of primitive animals. So too the X-chromo-

some and the mitotic subdivision of cells were discovered, not by general reasonings, or by haphazard guesses, but by patient and methodical experimentation with the various factors shown, by histological analysis, to be present in the concrete situation. It is thus the situation before us which is the direct and sole subject of our analysis.

A second question concerns the "elements" to which we analyse. These vary in the various concrete situation. If we are studying language, it is in units which are grammatical or phonetic, that our analysis terminates. If it is plane geometry that we are studying, our analysis terminates in lines and points. If it is history, then we end up with the various types of data which can be regarded as furnishing "testimony," whether documents, archaeological remains, or what not. We do not find tones in histological analysis, or centrosomes in musical analysis. That is to say, our units or elements differ qualitatively according to the qualitative differences of our various universes of discourse. But in spite of these differences—differences so great that we can seldom hope to argue from what takes place in one field to what may be expected to happen in another field—there are certain important respects in which our elements agree. Analysis as such always involves interference with the concrete situation. It is split up. It ceases to exist in its natural form. What is irrelevant for our purposes is discarded, and only what is strictly to the point is taken into the focus of attention. This involves a certain artificiality, and our elements are all abstract, intellectualised entities, bloodless concepts. The trail of the intellect is over them all.

This gives to the products of analysis a certain unity which we should never anticipate from what we have seen of the qualitative differences of the various universes of discourse—a unity derived, of course, from the intellectual nature of analysis itself. The products of analysis are, for instance, almost always numerable, and almost always parts of wholes; and they always have a (varying) number of other characteristics in common. From this unity of form, it sometimes comes about that the instruments forged by intelligence to deal with one type of situation can, as a matter of fact, be utilised, with but slight changes, to deal with another type of situation. Our analytical methods thus become schematised.

Just as the artist has a body of working schemes² for representing a man, a tree, or a house, just as, in social intercourse, we acquire generalised ways of approaching other people with what we call "tact,"—so the trained scientist has at his command a body of generalised ways of dealing with his kind of problems—a method of analysis which can be applied easily and with but slight modifications to fit all sorts of special cases. A logician like Bosanquet³ or Royce can apply the schemes of biological analysis to logic. A metaphysician like Spinoza can apply quantitative methods to the concept of *Deus sive Natura*—and there seems to be no way of deciding how far this can be done. "Transgressing into another kind," as Aristotle named it, has become a fine art, and, as a matter of actual practice, we all recognise that "scientific method" is much the same, whatever the special material to which it is applied.

A fourth feature of our analytical method is, that it does not, and can not, go beyond its data. Schematised and general as it is, it can never extract, in the way of elements, more than is present to be extracted. It splits up the confused and concrete situation, omits what is irrelevant, and takes up what is relevant. These elements stand out far more clearly after the analysis than before, as in Aristotle's analysis of friendship,⁴ or Windelband's analysis of Plato.⁵ But they were there all through, embedded in the concrete situation, and only awaiting our analysis for their discovery. This is so obvious, that it would be unnecessary to emphasise it, if it were not so incessantly sinned against. To reconstruct Locke's thought in the light of Condillac⁶ or of Kant⁷

² Cf. F. C. Ayer, *The Psychology of Drawing*, esp. pp. 8–9, 74–75, 100, 159. A "scheme" is a method for representing in an image, as e. g., the method of representing pleasure or grief in the human countenance by curving the eyes and mouth upwards or downwards, respectively.

³ Bosanquet's book is "Logic or the Morphology of Knowledge."

⁴ See Aristotle's *Nicomachean Ethics*. Bks. VIII–IX, with the introduction of Sir Alexander Grant to Bk. VIII.

⁵ Windelband, in his small volume on *Plato*, by treating in separate chapters Plato as a Teacher, as a Philosopher, as a Theologian, as a Prophet, and as a "Social Thinker," has given an impression of amazing lucidity—such as without that analysis would have been impossible.

⁶ Cf. e. g. Victor Cousin, *La Philosophie de Locke*.

⁷ Cf. e. g. the treatises comparing Locke and Leibniz written by G. Hartenstein and by G. v. Benoit—to mention only two out of a goodly company.

is just as illogical as to reconstruct Kant in the light of Hegel,⁸ or to read monistic idealism⁹ or Freudian psychology¹⁰ into Shakespeare. The analysis of Locke's thought should lead to elements which are Lockian; the analysis of Shakespeare to elements which are Shakespearean, just as the analysis of space leads to elements which are spatial, or of time to elements which are temporal. Otherwise we are embarking upon the hopeless quest of explaining everything in terms of something else—*i. e.*, in terms of something which is not it—which is the *reductio ad absurdum* of such explanation. Our analysis must, then, restrict itself to what is *given*, and must not attempt to read into its data something which is extraneous and strictly irrelevant.

Analysis and Intuition.—So far we have seen that the analytical method, in splitting up the given situation, (1) keeps as close as may be to its data, (2) is schematic or slightly more general than the specific occasion seems to require, (3) leaves us with elements which are somewhat artificial, and yet (4) were there, embedded in the concrete situation, only awaiting methodical analysis to become evident. It remains to compare it with a method which employs no explicit analysis, but reaches its conclusions by simple inspection—the intuitive method.

Compare for example, (1) "His character must be good, for he has consistently acted in this, that, and the other kind of way," and (2) "His character is good—I couldn't say why, but I just feel sure of him." The analytical inference is less certain than the intuition, less confident of itself. It sums up the evidence, and rests the responsibility for the conclusion upon the certainty of that evidence. The intuitive judgment says nothing about the evidence—there is no appeal to logical reasoning, for the person in question feels sure—he "just knows." So too we might compare, (1) "This way must be shorter than that, for if you count up the number of blocks, you will find that there are two more blocks that way," and (2) "This way is shorter than that—It just seems so somehow." The intuitive person seems less hesitant, seems to feel

⁸ Cf. *e. g.*, the accounts of Kant given by Kuno Fischer and by Edward Caird.

⁹ Cf. A. C. Bradley, *Shakespearean Tragedy*.

¹⁰ Cf. Ernest Jones, "The Oedipus-complex as an explanation of Hamlet's mystery," in *American Journal of Psychology*, Vol. XXI, 1910, pp. 72-113, and *Psychoanalysis*.

less in need of the longer way round to his conclusion. In some cases the longer way—that of analytic inference—seems less applicable. Compare, *e. g.*, (1) “This picture must be finer than that, for (a) you can see what this is intended for, (b) it is drawn according to the rules of perspective, and (c) the colors are bright and clear,” with (2) “This picture is finer than that—I couldn’t give any reason, and I don’t need to—it is a matter of the aesthetic intuition. Either you have it or you have not. If you have it, you don’t need to argue about it—you just know.”

Both intuition and analysis arrive at conclusions—but only analysis states the evidence on which the conclusion seems to depend. Intuitive thought is far more common than analysis. For most purposes it is sufficiently correct, and for some purposes—*e. g.*, in judging works of art—it seems more correct. But we may reasonably ask, which of the two attitudes of mind is it wiser to cultivate? On the one hand we have the widespread belief in intuition. “Give your decisions,” said the experienced judge to the new appointee, “they will probably be right. But keep your reasons to yourself—they are sure to be wrong.” So too our modern psychologists¹¹ tend to regard all motivation as welling up out of the depths of our subconscious nature, while all the specious reasons which we give for our conduct tend to be discounted as mere “rationalisations,” *i. e.*, as disguises by which we hide our motives from ourselves and others by letting them appear only as clothed in language appropriate to the system of ideals recognised by the conventions of social usage. On the other hand we have the equally widespread belief in reflection, analysis, deliberation. To which of these two opposed beliefs ought we to yield the more loyal allegiance?

By intuition we mean, in general, that attitude of mind by which we put ourselves in the other fellow’s place, and try to feel as he would be feeling. It is a matter of feeling rather than of reasoning, and seems to be immediate, structureless, simple—just knowing. It is unreflective and unmethodical, and feels a certain distrust of elaborate arguments, as of instruments which somehow come between us and what we are studying—which distort and falsify our view, warping our judgment until we don’t really know just what we do believe.

¹¹ Cf. *e. g.*, W. James, *Varieties of Religious Experience*, and Bernard Hart, *Psychology of Insanity*.

We feel the value of intuition especially in ethics, in aesthetics, and above all in religion. It is especially in these fields that we trust our intuitions most completely, and feel most distrust of reasoning. "Metaphysics," says Bradley—himself a metaphysician—"is finding bad reasons for what we believe by instinct."¹²

On the other hand, in mathematics, in chemistry, in biology, and in the sciences generally, we should never dream of trusting to intuition. In these fields analysis, patient, methodical experimentation, with all the powers of reasoning as well as of observation which we possess, is universally admitted to be the only feasible method. So too in much of our every-day life we should never think of trusting intuition. Do I have an "intuition" that this is my street-car, that the rain will be only a shower, that the business deal which I have just completed is going to turn out a magnificent success? Yes, I do have intuitions in this field, but experience has taught me to distrust them, and to replace them by analysis and experiment. Scientific method, in business as well as in purely scientific study, reliance upon analysis and reasonings verified by methodical appeals to experience, is the only safe guide here.

What are we, then, to conclude? Are we to believe that in practical life and in science, analysis should be our rule, but that in all which concerns the inner life,—art, goodness, and religion,—intuition—unanalysed feeling—is a trustworthy guide? There is, of course, no doubt that this is precisely the solution which most men accept—the system of watertight mental compartments—one for the office, another for the church; one for the opera-house, another for home life. In business relations, in scientific research, they analyse and infer. Nothing is taken upon faith. In the realm of ideals and "values," faith, trust, intuition is their guide. We must ask, however, not whether this attitude is commonly accepted, but whether it is wise, consistent, logical, whether it is not rather the source of our unprogressiveness in the things of the spirit, in the essentials of civilisation, while in externals we seem to have advanced by leaps and bounds.

Let us consider. Intuition is not radically distinct from analytical reasoning. Both often reach the same conclusion, and often take a similar path. The only difference is, that for

¹² Cf. F. H. Bradley, *Appearance and Reality*, preface. Cf. also B. Russell, *Problems of Philosophy*, pp. 39–40.

intuition the reasons are not made explicit, are not weighed and tested, but just followed. All the emphasis is placed upon the conclusion—the way by which that conclusion has been reached remains out of sight. Our choice, then, is between (1) accepting conclusions with unbounded faith, but without careful weighing of the evidence and setting forth the grounds which might influence a reasonable man, and (2) only drawing inferences after the fullest consideration and analysis of all the facts before us. So stated, there can be no longer any hesitation. Reflective analysis as a method for the conduct of life is immeasurably superior to unreflecting intuition—wherever and whenever such analysis can be fruitfully applied. The reason why it has not been accepted as the sole reasonable method is because the inner life cannot easily be subjected to scientific analysis, as well as the pressure of practical needs, which so often confine our attention to the external things of life. But there can be no doubt that, as our psychological insight gradually develops, we shall be able to extend the method of analytical inference over more and more of the inner life, and that the progressiveness which is so marked a feature of the fields to which that method has hitherto been applied, will continue to invade the new territory also. Wherever applicable, then, analysis, deliberation, weighing all the evidence, bit by bit, is a safer guide than intuition, and the reasoning, deliberative, analysing habit of mind is the one to cultivate.

Conclusion.—Our conclusion is, then, that inference is analytical, *i. e.*, does not treat the material before it as a simple unanalysable whole, to be reacted to by an intuition, but as a complex situation which must be split up into its elements. Such inference remains always close to the concreteness of the situation, and examines the elements, one by one. As produced by an analysis which has torn them from their living context, such elements are somewhat artificial. But they are, after all, not created by our analysis, but discovered by it, and are clear and helpful in enabling us to understand the concrete situation before us, when we attempt to put it together for ourselves. Wherever this method can be applied, analysis is superior to intuition, and the analytical habit of mind is the one which should be cultivated.

FOR FURTHER READING

F. H. Bradley, *Principles of Logic*, pp. 414-415.

EXERCISES

Compare the working of the intuitive method with the method of analytic expansion in the following situations: (a) In seeking a lodging in a strange town. (b) In dealing with a case of sickness. (c) In estimating the value of "popular" music as opposed to the music of Bach, Beethoven, and Brahms. (d) In accepting the tenets of a particular religion. (e) In investing one's savings in stocks. (f) In estimating the ethical value of charity.

CHAPTER XIV

NOVELTY IN INFERENCE.

The Problem.— $2 + 2 = 4$. $2 + 3 = 5$. This procedure is typical of all inference. We put together two or more premises and the resultant conclusion is an item of information derived from neither premise alone, but from both taken together. Let us take a more complex example. $2 \times 4 - \frac{3}{8} = 7\frac{1}{2}$. The number of possible premises—*i. e.*, the number of distinct items of information which can be combined to yield a single conclusion—is theoretically unlimited. Any addition sum is an inference which furnishes us with information derived from *all* its premises, and expressing, from a certain viewpoint, the whole truth concerning those premises, however numerous they may be. So too with statistical information. The arithmetical mean, in conjunction with the probable error, furnishes us with a kind of telescoped information as to the behavior of a large group, however many its members. A simple curve can tell us at a glance how students at Cornell or Harvard tend to be marked by their instructors.¹ A different, but equally simple, curve can inform us of the rate at which practise makes perfect.² Both curves, however, though simple, express the result of innumerable distinct premises, and are, in fact, valuable in direct proportion to the number of cases which they sum up. So too a chemical formula or botanical law may express briefly and clearly the result of years of patient observation and experimentation; a poem or picture may sum up the experiences of a lifetime; and a philosophical speculation may express the soul of humanity.

The problem before us in the present chapter is to ask how far such information is *novel*. That it is attained by methodical analysis of the premises, we have already seen. It remains to ask whether what we obtain by summing up,

¹ See Finkelstein, *The Marking System in Theory and Practise*, 1913.

² For some recent work in this field, see J. Peterson, *Experiments in Ball-Tossing; the Significance of Learning Curves*. Jour. Exper. Psych., Vol. II, pp. 178-224.

or by putting together, our premises is something which we already possessed, something which was ours all the time, or whether it is in any strict sense *new*. As Mill expresses it, can we "argue from the known to the unknown"? That is to say, can any logical manipulation of what we know, by any possibility lead us beyond what we know—extend the bounds of knowledge, and raise the veil which yet conceals the unknown from our eager vision?

Sensory Novelty.—Let us consider for a moment. There are two sources of knowledge, (1) sense-perception, and (2) intelligence. So far as sensory experience is concerned, there can be no question that the information with which we are furnished through these channels is essentially novel. Color, sound, taste, touch, smell—an experience deprived of these would be poor indeed, and direct experience of these qualities is furnished us by the senses alone. No logical manipulation of sensory data—however intelligent that manipulation³—can give us a new *sensation*. That is beyond the possibility of logic. That veil, at any rate, cannot be raised by the intellectual function of inference. Are we to conclude from this that the function of intelligent inference is, after all, to accept data derived from another source, and to proceed to classify? Are we to believe that sense-perception alone is the source of novelty in our experience? Are we to accept the view that intelligence can combine, analyse, and shift the positions of sensory data relatively to one another, but that to add to our positive knowledge is beyond the power of intellect? In other words, is the desiderated extension of the bounds of knowledge purely a matter of sense-perception, and is inference restricted to the organisation, to the re-arrangement of contents which cannot be altered, and above all cannot be increased by any logical manipulation whatever?

For instance, I do not know whether there will be any mail for me today. Will inference help me? I turn over the probabilities in my mind. I do what I can with the knowledge which is at my command. I analyse it, turn it over and over, alter its arrangement by looking at it from different angles of approach. But do what I will, any conclusion to which I can come remains only probable. "There *may be* a letter from X or Y." The only way in which my lack of

³ Cf. Hume's suggestion to the contrary, *Enquiry into Human Understanding*, Section II, last paragraph but one.

knowledge can be converted into actual knowledge, is here by sense-perception. The postman's ring, followed by actual receipt of the letter, is in such cases the only satisfactory evidence. Shall I receive a certain appointment? Again I make use of all the knowledge at my disposal. I turn over and over what I know about myself and about the other candidates. I bring to bear all that I have heard of the disposition of the man with whom rests the final decision. Still, there is a gap. I do not know enough to feel certain. The only satisfactory evidence is, here again, by sense-perception—the receipt of an official notice of my appointment. Again—is the liquid before me an acid or an alkali? My knowledge tells me that *if* it is the one, it will turn blue litmus paper red, and *if* it is the other, it will turn red litmus paper blue. But I do not know which of these *ifs* will be realised. My knowledge does not reach far enough, and no amount of inference will stretch it so as to bridge over the gap. The only reasonable thing to do is to dip in a piece of blue (or red) litmus paper, and see, by actual sense-perception, what takes place. Then, and then only, shall I know. These and a thousand similar instances serve to illustrate the value of observations and experimental appeal to sensory experience, over arm-chair theorising.

Intellectual Novelty.—It looks, then, as though observation, the appeal to sense-perception, is essential in discovering information which is new. And yet, before coming to a final decision, let us examine a different group of cases. I have a thousand dollars. I wish to invest profitably, and yet safely. There are two Government loans, both redeemable in three years, but the one pays 5% interest and sells at par, while the other pays 4% interest, and is selling at 92. Both are equally safe. The only question is, which investment is the more profitable? I do not know. If I could find out with certainty, I should be acquiring information of the utmost importance to me. Would it also be *new*? It would at least be welcome news—novel in the sense that I do not know it now. If I could find out, there would be an addition to what I know. It might even turn out to be novel in the sense of unexpected. I may, in fact, be on the point of making an unwise investment. Perhaps, then, we can safely regard such information as new. And yet, no one doubts that this information can be acquired, and acquired by processes of

inference familiar to any mathematician. And further, a study of the analytical method examined in the preceding chapter should reveal the fact that such inference is restricted to the re-arrangement of contents which themselves remain unaltered. It looks, then, as if it must be possible, at least in some cases, by logical manipulation of given contents, to obtain knowledge which is reasonably regarded as novel.

Let us take another case. I wish to know what some writer means by a technical expression peculiar to him—*e. g.*, what Locke understands by "simple mode" or Plato by the term "idea." I have in each case a vague notion, derived from grasping the author's meaning, as best I can, as I read through his works. But my notion tends to change with each new instance of the term in question, and, in short, the evidence is so conflicting, that the only safe conclusion is, that I do not know what is intended. I cannot put it together and make one thing out of it. If I could really discover what the author means, I should acquire information which I certainly do not possess at present. I should add to my knowledge. The result would be new. It might also be novel, in the sense of unexpected—perhaps even as contradictory to the view which, in my present state of ignorance, seems least unreasonable. How can I find out? Mere sense-perception will not tell me. The instances are far too numerous, and the viewpoints far too complex, for that. I collect all the instances—which may be regarded as given contents, not to be altered in the course of the investigation. Then, by classifying these data in such a way that all which bear upon this point or upon that are put together, and every case which is irrelevant to the point at issue is excluded, I find that I can gradually settle one disputed point after another, until in the end I am able to formulate a hypothesis which is just to all the facts, and in short gives me the information which I was seeking. I have thus discovered something new, and have discovered it by processes of inference which were confined to re-arrangement of contents which remained unaltered throughout.⁴

Let us take yet a third case. I am playing a game of chess, and am reasonably familiar with the moves and conventional gambits. At one stage of the game, no amount of inference could tell me, or anyone else, who is going to win. We must

⁴ Cf. Lodge, *The Meaning and Function of Simple Modes in the Philosophy of John Locke*, 1918.

fight it out and see. But at a later stage, my opponent suddenly says, "Mate in four moves." I know enough about the game to see that I am seriously endangered, but I can not see that I must necessarily lose, whatever I do. I move. *One*, counts my opponent, as he also moves. I move again. *Two*, he counts. I look again. Yes, now I also can see that a check-mate in two more moves is inevitable. From my knowledge of the moves and from the position of the pieces on the board, I see that there are only a few moves which are possible for me, and that which ever of these I take, I must assuredly fall a victim in two more moves, provided that my opponent continues his attack. In this case my inference gives me knowledge.

So far, then, we have seen that information which is new can be derived in two ways. There are some cases in which sense-perception alone can suffice, and no amount of theorising, or re-organising what little knowledge we have, can be substituted for it. There are other cases in which sensory experience alone seems to be useless, and where the re-arranging of what we know leads to information which is both important and new. It should be clear from an earlier chapter⁵ that both sensory and intellectual elements enter into the acquisition of any and every sort of knowledge. So that the question which now faces us is: How is it, that in some cases inference helps us to knowledge, while in others no amount of "theorising" leads anywhere?

The Field of Relations.—To this question there are two answers. In the first place, admit that sense-perception alone is the source of certain kinds of knowledge—*viz.*, knowledge of sense-qualities such as red, warm, hard, painful, *etc.*,—still, there are other kinds of knowledge of which intellectual perception alone seems to be the source. A knowledge of straight lines and of circles—*i. e.*, demonstrative knowledge based upon figures which are *ideally perfect*—seems to go beyond what sense-perception gives us. These geometrical figures are constructed in conformity with intellectual demands, rather than somehow taken from sensory experience; for in fact, it is very doubtful if ideally straight lines, for instance, have ever been met with in sensory experience. So too arithmetic, the knowledge based upon the relations of the elements of

the number-series 1, 2, 3, . . . seems to transcend what we meet with in sense-experience. These units, each one of which is, from the viewpoint of quantity, ideally equal to every other, which extend in a progressive series from zero to infinity, in the plus or minus direction—are also intellectual constructions in accord with intellectual demands. Here also sense-perception plays a role which is at least subordinate. So too with the a , b , c , the x^2 and y^2 of algebra, with the *cosine alpha* and *tan theta* of trigonometry—in a word, with mathematical relations generally. These all furnish us with information which is of great importance for life and for science, and which is also undoubtedly new. And—what is, perhaps, surprising—the mathematical sciences demand, and receive, very little aid from sensory experience. The knowledge of mathematical relations, then, seems to arise less from sense-perception than from intellectual construction and intellectual perception.

So also with other branches of knowledge. "Conclusions drawn from premises which are true, are themselves true," "Entities related to the same entity are related to one another," "Every event has a cause," "If things-in-themselves are unknowable, then it is impossible to know that they are unknowable," *etc.*⁶ To put it shortly, knowledge of *relations* appears to arise from intellect rather than from sense, whether such relations are mathematical, physical, logical, or what not. Knowledge of qualities is furnished us by sense-perception; knowledge of relations by intellectual perception.

While knowledge of relations is thus not derived from sense-perception, it must not be supposed that it has no application to the sensory field. A man who was born blind, but has studied physical and psychological optics, including, for instance, the theory of color-vision, can draw inferences which are perfectly correct within the field of color-vision, although as a matter of direct, personal experience, the blind man can never verify his own deductions by the appeal to sensation. His thought is, of course, moving in the realm of laws, *i. e.*, of intellectually apprehended relations, and the accuracy with which this can be done, even in the complete absence of the corresponding sensory experiences, can be recognised when

⁶ For further instances, and a discussion of such "apriori knowledge," Cf. Bertrand Russell, *Problems of Philosophy*, esp. chapters vii-viii.

we study the writings, *e. g.*, of Helen Keller, which abound in sense-imagery, although this can have only a symbolic significance for her. So also a comparative psychologist can write intelligibly about peculiar sense-experiences of certain animals, where their sense-organs are very different from ours, and their sensations can be apprehended only symbolically by us. So too many short-sighted persons make up for their sensory deficiencies by using their powers of inference to a greater degree, and to some extent can, as they say, see with their intelligence rather than with their eyes. All knowledge of this type is, however, really given us by inference from laws and relations, and not by direct sensory experience.

Latent Knowledge.—The second answer to our question is also concerned with relations, but is not so indifferent to the presence of sensory experience. Let us consider a few cases. If I know that acid turns blue litmus paper red, I can apply my knowledge in a particular case, and thus discover whether the liquid before me is or is not acid. The appeal is here to sense-perception—but to a sense-perception organised so as to supply an answer to an intellectually prepared question. I arrange or organise my data in a particular way. This is a matter of intellectual construction. I then observe the result which follows upon my construction. I have not added anything to the data, the contents before me, but have merely altered their order or arrangement, *i. e.*, their relations to one another. By interfering with their relations to one another, *e. g.*, by dipping the litmus paper in the liquid, I set them working upon one another in such a way that they themselves produce a result which I observe.

Let us take another instance. I wish to discover the mental age of a child suspected of being backward. I apply the Binet tests up to six years, seven years, eight years. In the tests for the eighth year, the child breaks down. He is also unable to perform the tests for the ninth and tenth years. I conclude, on this evidence, that he has a mental age of seven years. The security of this conclusion depends partly upon the experiences which have been summed up in the Binet scale, partly on the carefulness with which I have examined the child. The summing up of experiences which has determined the order and interrelation of the tests has been a matter of intellectual construction rather than of sensory perception. The behavior of the little child resembles the behavior of the

litmus paper in the acid test—it is something which is observed, and then interpreted in the light of the law.

In such cases I discover information which is new, but is not confined to the realm of relations. I discover it by manipulating data, by changing their relation to one another, including some and excluding others, introducing a certain order, approaching nature in the attitude, not of a mere pupil, but of a judge, who determines what questions shall be asked and compels the witnesses to reply to those questions. Do I, however, always get an answer—*i. e.*, does this method always furnish me with information? We must admit that it is not uniformly successful. Just as no judge can legitimately extract from a witness more than that witness knows, so no scientific method of manipulating data can extract from those data more than is there to be extracted. The evidence must in some sense be there, awaiting only the proper method for its discovery.

Arguing to the Unknown?—So far we have seen that novelty in the case of inference is either a matter of restriction to a particular field—the field of relations—or, if inference leads to discoveries within the field of sense-experience also, this is brought about by some re-organisation of our experience which renders explicit sensory elements which were somehow there, but in a latent form—*i. e.*, such re-organisation clears the way for observation. We should, perhaps, note that in the case of relations also, these come to be apprehended as a result of some reconstruction of our experience. Such reconstruction, then, is present in both cases. The only difference between the two cases is, that in dealing with relations we are dealing with “form”-elements of our experience, while in sense-perception we have to do with “content”-elements. Neither form nor content is found alone, but the form is always the form of the content, and the content is always found in a particular form. Thus, in reasoning with reference to a triangle drawn on the board, a mathematician deals only with the “triangularity” features of the figure, with the geometrical relations of lines and angles ideally considered. But in addition to the triangular form, there is always a sensory content also—*e. g.*, the chalkiness of the lines, their position on this particular black-board, the time of day, *etc.* The mathematician restricts himself to the field of mathematical relations, and neglects the sensory features as such.

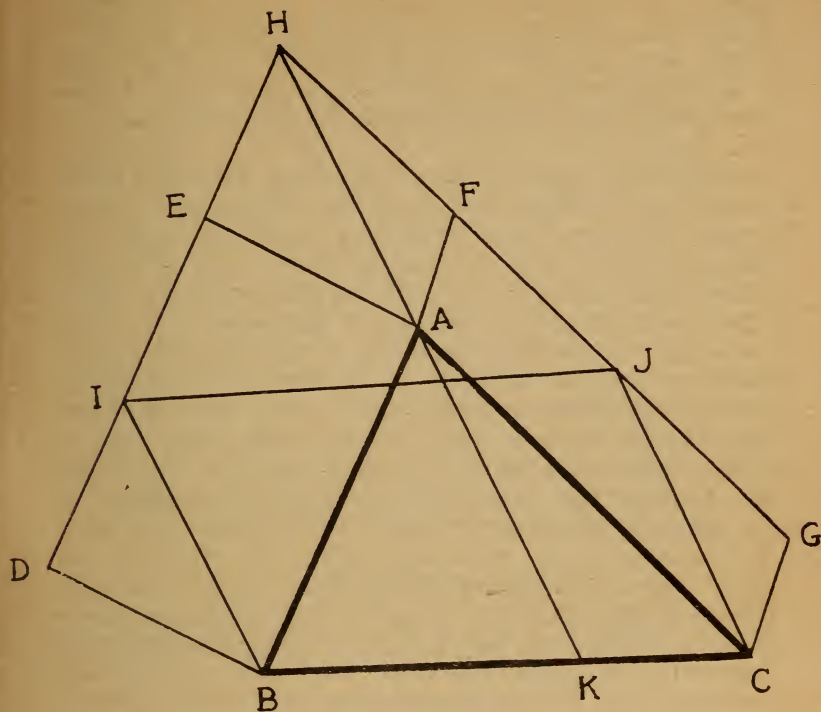
But in many cases in geometry, simple sensory inspection and direct sensory comparison of unanalysed figures will suggest to us the idea that two given triangles are probably equal, even when we are deficient in respect of that insight into relations on which the mathematical proof properly depends. It is, then, in the apprehension of elements hitherto unapprehended, whether such elements are of relational or sensory nature, that inference, so far as we can at present see, consists, and we are now in a position to answer our main question in this chapter, *viz.* How far our conclusion gives us information which is strictly new, or—as it is expressed by Mill—whether we can argue from the known to the unknown.

We have, perhaps, by this time a suspicion that knowledge attained by careful inference must in some sense have been there all along, awaiting the proper construction to become visible to us. And in the case of discoveries within the field of sensory experience, we have already seen that this must be the case. We cannot construct a new sensation, but can only clear away obstacles, or take a new viewpoint, and thus come to discover what was there to be discovered. The case of relations, however, has not been adequately considered. Can we, in such cases, argue to the “unknown”? Let us consider a typical instance. Suppose I discover that it is possible to construct, on any side of any triangle, a parallelogram equal in area to parallelograms of any size whatever constructed upon the other two sides of the triangle. This was something which I used not to know, something which I should, perhaps have thought difficult, if not impossible, before I went through the proof.⁷ In going through the proof, then, which leads to a result so remarkable and unexpected, do I at any place argue from the known to the unknown? Let us see.

Let ABC be any triangle, and let BDEA be any parallelogram on the side AB, and AFGC any parallelogram on the side AC. It is required to construct upon the third side BC a parallelogram equal in area to the sum of the parallelograms BE, FC.

Construction.—Produce DE and GF to meet in H. Join HA and produce it to meet BC at K. Parallel to KH, draw BI and

⁷ This “problem” was solved by Porphyry (5th Cent. A. D.) For the “proof” which follows, I am indebted to the late Professor Cook Wilson, of Oxford.



CJ, to meet DE and GF in I and J respectively. Join IJ. Then shall BIJC be the parallelogram required.

Proof.—(1) To prove that BJ is a parallelogram. Because BH is a parallelogram, $BI = AH$; and because AJ is a parallelogram, $AH = CJ$. Therefore, since each $= AH$, $BI = CJ$. And they are by construction parallel. Therefore, (by joining IC or BJ) it can be shown that the opposite sides IJ, BC are equal, and that the opposite angles of the figure BJ are equal. That is to say, the figure BIJC has been proved to be a parallelogram.

(2) To prove that $BJ =$ the sum of the parallelograms BE, FC. Because BE, BH are parallelograms on the same base BA and between the same parallels BA, DH, therefore they are equal in area. Similarly, IK and BH are equal in area. Therefore, since each $= BH$, $BE = IK$. Similarly it can be

shown that $CF = JK$, for each $= CH$. Therefore the whole parallelogram $BJ =$ the sum of the parallelograms BE, FC .

That is to say, upon BC , the third side of the triangle ABC , a parallelogram has been constructed which is equal in area to the sum of the parallelograms on the other two sides.

Q. E. F:

In the above figure we have constructed something which we did not previously know could be constructed. That is to say, we have really discovered something which was unknown by us. Have we, however, at any point in the procedure, taken the "inductive leap," and gone beyond our knowledge by arguing to the "unknown"? Let us see. Our whole reasoning is based upon the known truths concerning parallelograms, as applied to the figure before us. The construction which gives us the particular figure allows us to apply our knowledge in a way in which it was, perhaps, never applied before, and thus to obtain an insight which is novel. But our procedure could not strictly be described as arguing to the unknown, as reasoning from something which we do know to something which we do not know. Our argument consists throughout in making clear to ourselves facts which are already implicit in what we know, facts which are present in our knowledge, but remain latent, awaiting the appropriate construction to become clear to us. We remain on *terra firma* the whole time, and take no "inductive leap" to the unknown.

Could we perhaps say—to shift our ground—that arguing from a particular experiment with this single representative of the class "triangle" to all future triangle-experiences, or to all possible triangle-experiences, is arguing from something which we now know (this particular case) to something which we do not know (the "possible" cases, which we have not yet experienced)? Certainly not. The argument is not concerned with the "here" and "now," but is based upon insight into the nature of the triangle and the parallelogram as such. It is insight into the non-temporal realm of mathematical relations, and the question of time, whether past, present, or future, is strictly irrelevant. If I really grasp the proof. I have, in grasping it, apprehended the interrelations of such figures for *any* triangle, and we may safely conclude that, so far as insight into relations is concerned, we never argue from the known to the unknown, but always make explicit something which is implicit, *i. e.*, something which is already

contained in what we know, and is merely awaiting the proper method of construction to make it explicit and clear to us.

Conclusion.—Reasoning from the known to the unknown, then, is impossible. Our inference can never take a leap and somehow transcend its data, the premises with which it starts. This must be abandoned as a chimera, as an attempt to get something out of nothing. Inference is always a matter of analysis, of disentangling something which is in some sense present—present, *i. e.*, in the sense of awaiting discovery, awaiting the application of some method which will disentangle it and make it plain. This method of analysis, the method of disentangling in question, is what we understand by inference, and the so-called “novelty” of inference consists, not in somehow adventuring into realms unknown, but in so analysing and re-organising our experience as to let us see our way where, without such aid, we were at a loss. Analysis has been already considered. The other feature of inference, the construction which makes discovery possible, remains to be investigated in the succeeding chapter.

FOR FURTHER READING

B. Bosanquet, *Logic*, Vol. II, pp. 8–9. F. H. Bradley, *Principles of Logic*, pp. 419–429.

EXERCISES

Consider, in the following cases, how far the “discovery” is really novel: (1) By looking up a date in a reliable history book, I discover the date, which I did not know before. (2) By mixing approximately equal amounts of blue and yellow on the color-wheel, I discover that they make, not (as I expected) green, but gray. (3) By practising carefully, passage by passage, I find I can gradually learn to play Liszt’s “La Campanella”—which I had always thought impossible for me. (4) By drawing a curve, I find that the center of a ladder which slides down the side of a house, describes the arc of a convex circle—whereas I had previously supposed it would have been a concave curve. (5) By eating porridge, I discover that it tastes good—whereas from its optical appearance I had always supposed it would not taste good. (6) By the use of a little trigonometry, I discover how far distant a certain ship is from the ship on which I am.

CHAPTER XV

THE SYSTEMATIC CONSTRUCTIVENESS OF INFERENCE

Examples.—Add one to one. It makes two. Add one to two. It makes three. If we have grasped the principle, we can now add any number to any other number, or indeed can multiply any number by any other number. We can also easily learn to subtract and divide, with any numbers, or indeed with any quantities $a, b, c \dots n$, whatever. The whole of arithmetic and algebra can be constructed from these simple operations.

Take a ruler and place it upon a sheet of paper. With a pencil against the edge of the ruler, draw a line. If we have grasped the principle, we have learnt to draw any number of straight lines. Take a pair of compasses and describe an arc of a circle. If we understand the principle of the instrument, we can describe circles of any radius. Incidentally we understand that, by keeping the compasses open with the same radius, we can mark off equal portions along either straight lines or circumferences of circles. With these three simple premises, let us proceed to construct something difficult—*e. g.*, a regular polygon such as a pentagon.

To construct a regular pentagon: Draw any straight line AB, and on it mark off five equal divisions, AC, CD, DE, EF, EG. With center A and any radius, describe an arc HIJ, cutting AB in I. With center G and the same radius (AI), describe an arc HKJ, meeting HIJ in H and J. Draw a straight line joining H and J, and cutting AG in L. The line AG is thus divided into two equal portions AL, LG, and we now know how to bisect any straight line. With center L and radius LA (or LG), describe a circle. On the circumference of this circle, with compasses open to the extent AC (or CD, DE, *etc.*), mark off equal portions AM, MN, NO, OP. Join in straight lines AM, MN, NO, OP, PA. The resulting figure is the regular pentagon required. If we have grasped the principle of this construction, we now know how to construct a regular heptagon, nonagon, or n -sided polygon, and

indeed, can in principle construct the whole of plane and solid geometry, and even the meta-geometries.

Let us leave mathematics, and consider life. Reflect upon the egoism of childhood, and its negation by the altruism of adolescence. Note how both alike are negated by the mature man, who, by living through both and finding both unsatisfactory, gradually rises to a level of ethical development which is above such one-sided interpretations of life. If we grasp the principle of this, we have the key to understanding ethical evolution generally, in the individual and in the race, and indeed are able to interpret much of life and history which is otherwise confusing.¹

Let us take yet another case of inference. What can we infer from the following passage?

"The man *ex veritate* in the first place understands (with an understanding which is at the same time a living in the fullness of feeling) that the goods of this world are not true goods, and would not be unmixed with evil even if we could have them all. Pleasure, health, power, consideration, even the sweetest affection so far as it is fixed upon a creature who will one day vanish like ourselves—all is vanity. Even the inward peace, which is the reward of conduct praiseworthy from a human standpoint, is illusory and fundamentally sad. A life spent in the pursuit of such experiences would not be worth living.

"In the second place he understands (and feels) that personality, his own as well as that of others, has an intrinsic value. Our doing, our suffering, our aspiring after something better, must be justified. They are not vain appearances, but reality. Whether they take place or do not take place, cannot be all one—cannot be indeterminate and inconclusive. Our sight is darkened, our desires are disordered and impure, because all (or nearly all) of us miss the road. But the true road exists."²

We can, by processes of inference, construct at least three diverse theories of life. (1) From the first paragraph, by negating the negations of the *ex veritate* man, we can obtain a positive theory which asserts that the goods of this world are the only true goods, and that a life spent in the pursuit of pleasure, wealth, power, love, and a good conscience, is funda-

¹ The above principle comes from Hegel.

² From B. Varisco, *The Great Problems*, 1914.

mentally joyous and the only life worth living—in short, the theory of materialistic optimism. (2) From the second paragraph, by proceeding in a similar way, we can construct a view of life which asserts that personality is valueless, that the pursuit of one end rather than another, and in a word, aspiration, effort, and hope, are without significance. Forward or back—it's all one. Upward or down—it's just the same. Life is meaningless illusion, and there is no "true road." This view we might call materialistic pessimism. We can further (1a) by negating the pessimistic statements at the materialistic level, obtain additional statements of an optimistic kind, such as that aspiration and effort, so far as productive of pleasure, power, *etc.*, are not illusory, but "reality," and that a personality developed in the pursuit of such aims has a very genuine value. So too (2a), by negating the optimistic statements, we can obtain additions to our construction of the pessimistic view—such as that pleasure, health, love, *etc.*, are thoroughly transitory and illusory, and that a life spent in pursuing such aims—indeed *any* life—is not worth living.

On the other hand, (3), by negating the optimist and pessimist view both, from a standpoint which is not materialistic but spiritual, we can construct a positive statement of the *ex veritate* view. For the *ex veritate* man, it is only so far as lived at the materialistic level, that life is not worth while. But if our eyes are fixed upon higher aims, if we live our life in an idealistic spirit, then pleasure, health, love, conscience, *etc.*, take on a new significance. They are no longer evanescent and transitory, but—when taken up into the higher life of the spirit—become representative of the eternal values. Personality ceases to be the mere organisation of animal desires at an animal level, and becomes the reflex of Divinity, and in the service of the true values comes to realise the value which is properly its own. Such a life is life at its deepest and best, and is fundamentally joyous, and well worth the living.

The Problem: (A) How Does Inference Construct?—The above examples are typical of the constructiveness of inference. It is well known that occasions slight in themselves, sometimes entail a vast number of important consequences; that from the twinkle of a man's eye, or the tone of his voice, a popular novelist can deduce "whole volumes" of

information about his character, that from two or three facts a historian can reconstruct a whole epoch, that from two or three bones a palaeontologist can reconstruct the whole animal. This power which we have, to construct the whole from its part by means of inference, seems very remarkable, and in order to understand it we must endeavor to answer these two questions: (a) How does inference construct—what are the principles, laws, conditions of such construction? and (b) How far is inferential construction valid?

The first requisite of construction is to have the materials in such form that they can be put together. That is to say, construction pre-supposes analysis. The material must have been separated out into elements which can be used in a new construction. For geometrical constructions the straight line, and some form of conic section, such as the circle, are indispensable pre-requisites. For the solution of many a problem in physics the a , b , c , x^2 and y^2 of algebra are conditions without which nothing could be accomplished. For psycho-analysis, prepared and standardised association-experiments constitute the units. For the diagnosis of mental ability, standardised intelligence-tests furnish the material. And speaking generally, some kind of unit or element, the product of a preliminary analysis, is the very first requisite for inferential construction.

Elements alone, however, are not sufficient. From straight lines and circles we might construct triangles, squares, pentagons, polygons regular and irregular, *etc.*, *ad infinitum*. From the units furnished by the grammatical analysis of a language, we might construct a dissertation in prose or verse in the language in question, or we might construct a grammar of the language. From algebraic units, we might build up solutions of purely theoretical problems, or we might solve practical questions in almost any of the applied sciences. From the units revealed by psycho-analysis, we might proceed to cure a hysterical person by reconstructing his dissociated personality,³ or we might use the material as evidence in support or refutation of some cherished psychological hypothesis. From elements alone, anything or nothing might be constructed.

A second requisite of construction, then, is the presence of some aim or purpose, some ideal to be realised. If we

³ Cf. Morton Prince, *The Dissociation of a Personality*.

seat ourselves at the organ, and allow our fingers to wander aimlessly over the keys, we are not likely to strike a "grand amen," or indeed to produce anything which could be dignified by the name of music, any more than the machine described by Gulliver after his visit to Laputa was likely to create literature of a permanently valuable character.⁴ It may be true that our aims become realised by a process of trial and error which is largely governed by chance,⁵ and our constructions so far resemble the Laputan machine. But even so, the presence of a definite aim is necessary, at least in order that we may know whether our trial was an error or a success. If we attend courses at a university with the aim of taking a degree, we are far more likely to attain a degree than if our attendance is without aim or definite purpose. If we endeavor to solve a definite problem in simultaneous quadratics, we are far more likely to succeed than if we are just amusing ourselves with putting together x^2 and y^2 , a and b , etc. A second requisite, then, of constructive inference, is the presence of a definite aim or purpose.

Even this is not quite sufficient. It is not enough to have (1) a number of elements and (2) an aim of some sort, or even of the same sort as the elements. It is not any and every one who can put together the *dissecta membra* and make them into poetry. For that, it takes a poet, one who understands the spirit of poetry. So too, in order to build a cathedral, something more than good intentions *plus* the materials is essential, namely, a knowledge of architecture at least. So too, in order to play Beethoven's sonatas, something more is necessary than a knowledge of the position of the notes on the instrument, *plus* an ambition to play the works of the Master. Long practise and the gradual building up of a technique is essential, as is also an understanding of musical problems and of the sonata-form. So too, in order to re-construct a historical epoch, something more is necessary than the desire to do so, *plus* certain archeological and literary remains—*viz.*, the historian's familiarity with the principles of historical construction, both in general, and in connection with that special period.

What is this *tertium quid* which seems to be necessary? The technique of the poet or musician, the understanding of the architect or historian—what have these in common?

⁴ Gulliver's Travels, *Voyage to Laputa*, chapter v.

⁵ Cf. W. B. Pillsbury, *Fundamentals of Psychology*, pp. 501 ff.

At least this: familiarity with the means of connecting up materials and ideal, a grasp of the schemata and principles of construction applicable in these particular fields, an understanding of the structure of poetical or historical composition, an apprehension of the nature of the system in question. A botanist and a poet observe, let us say, the same aim—*e. g.*, a field of poppies. They have, let us say, the same aim—*e. g.*, to describe the nature of those poppies. But the scientist describes their nature with reference to the systematic study of plants. For him they are “phanerogams” with certain medicinal properties. For the poet, on the other hand, they are a vehicle for expressing his deepest emotions and aspirations. They are symbolic of the laughter and tears, the grief and joys of life. The third requisite of construction, then, is a grasp of some system—scientific, artistic, or what not—in which we can put together the elements which we have analysed out, and can build them nearer to the heart’s desire, shape them into some semblance of our ideal. Given an understanding of the principles of systematic construction in our particular field, *plus* elements appropriate for use in such construction, *plus* an aim which is realisable with such elements and by such methods, we need nothing further. We can proceed to construct our inferences.

(B) Validity of Inferential Construction.—Not every system constructed by inference in all we could wish. All inferential construction is hypothetical, and there are many possible systems, many rival hypotheses. We are even advised, as a point in scientific method, to keep in view, throughout our research, a number of alternative possible explanations, so as to avoid narrowness. But valuable as is the “method of multiple working hypotheses” from a practical viewpoint, we cannot suppose that, in strict theory, all the hypotheses suggested are equally true. Some of them are mutually exclusive—in which case, one at least must be false. How are we to choose between rival hypotheses? How can we discover which of the conceivable systematic constructions is the best?

Do plants whose leaves are particularly sensitive to light have an organ analogous to the primitive eyes found in some parts of the animal system?—Or must we seek some other hypothesis? Is the three-color hypothesis, or the four-color theory to be accepted as an explanation of the facts of human color-vision? Is Locke a critical idealist, or is he an empiri-

cist with leanings towards realism? Are Plato's "Ideas" to be regarded as Super-things, or merely as scientific methods? In every field of research this problem is pressing. Is there any method or criterion which will enable us to choose with certainty between rival explanatory constructions?

There is no royal road to such knowledge, no short cut across the devious, trial-and-error methods of experimental inquiry. Progress in knowledge can alone decide, progress in verifying or in refuting some suggested hypothesis by strict and careful reference to the data. What hypothesis is the best, can, in the end, only be discovered, not by some system introduced *ab extra*, but from a study of the system which is already present in the data, at least in germ. But to find out what system actually is present in the data is often a matter of prolonged experimentation with results which obstinately remain doubtful. And it is only very gradually and tentatively that a rival hypothesis can be eliminated, and a more correct one established.

What is the system, or what are the systems, according to which animal cells multiply? Could we think this out? We might imagine that the cell would grow by taking in nutrition, until the tension of its increasing bulk became too strong for the containing membranes, and it broke, aided by the pressure of external forces—such as the beating of waves in the case of water-animals—into two or more cells, each small enough to maintain itself in relative equilibrium over against the forces of the environment. Beyond this, perhaps, one could hardly go *a priori*. The functions, however, of nucleus and nucleolus, of centrosome and chromosome, and in a word of all the finer and more intricate elements which play their part in the division of cells, could never have been discovered in this way. Observation and patient experimenting alone could have discovered the system which is the *system of the data*.

It is necessary, then, to obtain insight into the appropriate system—the particular system to which the data, in some sense, already belong—if we wish to construct a system which will be valid. Given a curve to be completed, it makes a serious difference whether we try to complete it by constructing the form of a circle, of an ellipse, or of a parabola. The evidence, however, which is to guide our choice, is to be discovered only by a careful analysis of the data, *e. g.*, of the

curve in question. So too the principles which are to guide our reconstruction of the thought of Plato or of Locke, must be sought, not in a study, *e. g.*, of Neokantian philosophy, but in a careful study of Plato himself, or of Locke himself. In the last analysis, it is there, if anywhere, that the Platonic system or the Lockian system will be discovered. Any other studies, however, ingenious and however erudite, can lead only to unverified hypotheses.

Conclusion.—Our general conclusion is that from elements which are either given, or discovered by analysis, in a concrete situation, inference constructs a system which either puts together the elements from a new viewpoint, or carries further a system which is already present in a fragmentary or partial form in the elements themselves. Choice between the different possible systems, or in general the validity of inferential construction, can be determined only by strict analysis of the concrete situation, leading to the discovery of a system which is already, to some extent at least, present or implicit in the data, and then using this system, and this system only, as a principle of construction.

FOR FURTHER READING

B. Bosanquet, *Logic*, Vol. II, pp. 36–42. F. H. Bradley, *Principles of Logic*, pp. 412–414, 419–429.

EXERCISES

1. Construct from the following passage a theory of good and evil. When you have constructed the theory, point out (1) how you have proceeded, and (2) how far your result may reasonably be regarded as valid:

"We are asking whether goods and evils and obligations exist in physical facts *per se*. Surely there is no *status* for good and evil to exist in, in a purely insentient world. How can one physical fact, considered simply as a physical fact, be 'better' than another? Bet-terness is not a physical relation. In its mere material capacity, a thing can no more be good or bad than it can be pleasant or painful. Good for what? Good for the production of another physical fact, do you say? But what in a purely physical universe demands the production of that other fact? Physical facts simply *are* or are *not*; and neither when present or absent, can they be supposed to make demands. If they do, they can only do so by having desires; and then they have ceased to be purely physical facts, and have become facts of conscious sensibility." (James, *Will to Believe, etc.*, p. 190).

2. Construct from the following passage a theory of the *habitual* work of thought. Then point out (1) how you have proceeded, and (2) how far your result may reasonably be regarded as valid:

"Our intelligence can place itself within the mobile reality, and

adopt its ceaselessly changing direction—can grasp it by means of that *intellectual sympathy* which we call intuition. This is difficult. The mind has to do violence to itself, has to reverse the direction of the operation by which it habitually thinks, has perpetually to revise, or rather to recast, all its categories. But in this way it will attain to fluid concepts, capable of following reality in all its sinuosities and of adopting the very movement of the inward life of things. To philosophise is thus to invert the habitual direction of the work of thought." (Bergson, *Introduction to Metaphysics*, trans. Hulme, pp. 69–70).

3. Construct from the following passage, (1) a theory of the way of philosophy, (2) a theory of the way of life, and then point out (a) the nature of your procedure in each case, (b) the reasons for which your result may be regarded as valid:

"This I take to be the way of philosophy, of any philosophy which seeks to be consistent. It is not the way of life or of common knowledge, and to commit oneself to such a principle may be said to depend upon choice. The way of life starts from, and in the end it rests on, dependence upon feeling, upon that which in the end cannot be stated intelligibly. And the way of any understanding of the world short of philosophy still rests on this basis. Such understanding may despise feeling, and may claim to have risen into a higher region, but in the end it will be inconsistent and be found to stand on that which, taken as truth, does not satisfy. Outside of philosophy there is no consistent course but to accept the unintelligible, and to use in its service whatever ideas seem, however inconsistently, to work best. The man who stands on particular feeling must remain outside of philosophy. If you are willing to be inconsistent, you can never be refuted, and that is why philosophy can be said to depend upon choice." (Bradley, *Essays on Truth and Reality*, p. 235).

CHAPTER XVI

THEORY OF INFERENCE (1)

The Problem.—So far we have seen that inference has the four main characteristics of dependence, analytical expansion, novelty, and systematic constructiveness. The relation of these characteristics to one another appears to be somewhat as follows: First we analyse down to elements which exhibit a certain inter-dependence or law. Then we proceed, in accordance with this law of inter-dependence, to construct a system which either makes explicit the inter-relation of the elements discovered by the analysis, or carries further and completes the system of which these elements already constitute a fragment—as a given curve may be completed in the form of a circle or parabola. In this way, whether, by carrying the system further, we construct something new, or whether, by our mere analysis, we discover elements present but hitherto unnoticed, inference leads to discovery, to novelty, and inference may accordingly be defined as the discovery of knowledge by constructing a system based upon analysis of a given concrete situation.

The problem of the present and succeeding chapters is to construct a “theory” of inference comparable to our theory of judgment, to re-state our present findings in terms of the sensory and intellectual factors in our thought, and in short to supply an answer to two questions—(1) the question of fact, the question of the part actually played in inference by sensory and intellectual factors, and (2) the question of validity, the question of the reliability and satisfactoriness of inference. In the present chapter we shall deal with the first of these questions.

The Sensory Elements, (A) In Dependence.—Let us first of all consider the more empirical instances. “If the ground is wet, the corn will rot.” “If I do not hurry, I shall miss my car.” Is any part played here by sensory, as opposed to intellectual, factors? Let us see. The instances appear to be analogous to what we called “judgments of experience” in

an earlier chapter. Such cases are all generalisations from experience, and from an experience which has been largely sensory. The wetness of the ground, and so far as that goes, the earthiness, the ground-ness of the ground, are matters of sensory experience. So too with the appearance of the corn. The discoloration, softness, *etc.*, produced by the rot are all largely matters of seeing with the eye, testing with the touch of the finger, *etc.* The question of causation is usually regarded as intellectual—we do not *see* what it is that causes the rot, but *infer* or *reason*. Still, if we were right in attributing a certain continuity to our sensory experience, the gradual change from the firm yellow corn-seed to a mushy discolored lump of pulp might be regarded as sensory, provided that we abstract from all attempts to account for or explain the changes in question. That is to say, the mere apprehension of the mushiness at each stage of the degeneration of the seed can be regarded as a matter of sense-experience. But, it may be asked, does sensory apprehension of this kind really touch the question of “dependence” at all? Our answer must be, No, or at any rate not directly. The *dependence* of rot upon damp cannot be claimed as a matter of direct sensory experience. The nearest we can come to this is in the *continuous* apprehension of degeneration in the damp medium. This may be regarded as a sensory correlate of causal explanation, but in general, it is in apprehending the facts to be explained, in taking in the wetness of the soil and the mushiness of the seeds, though without putting two and two together and drawing a conclusion, that sensory apprehension plays a part here. So too sensation assures me of my hurrying or not hurrying, of the car standing still and of my reaching it, or of the car moving and of my remaining upon the spot. But the connection of these facts, the *dependence* of my reaching the car upon my hurrying, in a word, apprehension of the *relation between* these two elements, seems to be a matter of intellectual, rather than of sensory experience. Thus we see that, in both cases, the role of sensation seems to be to furnish us with a concrete situation, with facts, material, or data, but not in any sense with explanation, with apprehension of relations of dependence, whether logical or causal. The “hypothetical” element, as such, is non-sensory. There is no “If . . . then” about sensation. All that

sense-experience can do in such cases, then, is to supply us with the materials for inference, with a concrete situation.

What are we to say about the less empirical cases, *e. g.*, about mathematical dependencies? The equilaterality of a triangle is logically connected with its equiangularity, so that there is a relation of interdependence. Does sense-perception play any part here, or is this, perhaps, a case of "pure" reasoning, of reasoning, that is to say, unmixed with anything sensory? So too, in such a case as $x^2 + y^2 = 40$, if we already know that $y = 6$, we know with mathematical certainty that $x = 2$. That is to say, the conclusion $x = 2$ is dependent upon the two premises. Does sense-perception play any assignable part here? Or are we to accept the theory that mathematics is the creation of pure thought? There is no doubt that, whatever role sense-perception may play in such cases, it will be, at best, a very subordinate one. In the case of geometry, we may point to the fact that, while all our geometrical reasoning is general, and is concerned with *any* triangle and not with *this* particular triangle, still we do not seem able to proceed without a figure of some sort. Such a figure has definitely sensuous characteristics. If drawn on the board, it is white and chalky; if drawn on paper, it is black and inky; if drawn in the imagination, it is vague and wavering in outline. In any case, as apprehended by sense or in sensuous imagination, it is concrete, connected with a definite background, and contains a thousand other characteristics which are equally irrelevant to the strictly intellectual grasp of the dependence in question. So too in the case of algebra or arithmetic, the x^2 and y^2 , the 1, 2, 3, *etc.*, are represented on the paper or in the sensuous imagination by visible symbols without which we do not appear able to proceed, and here also sensation supplies our thought with a sensory context which is largely irrelevant to the mathematical law in question. From a consideration of such cases we come to realise that, while inadequate as a representative of mathematical laws, sensation does play a part in steadying the attention, in giving the intellect a *point d'appui* for its various operations, in short, in providing us with a concrete situation.

In summary, then, we can state that, if the more empirical cases resemble "judgments of experience," the less empirical cases resemble what we have called "symbolic judgments," and in both cases we see that sensation plays a part without

which no inference would be possible. For it supplies us with data, with a concrete situation on the basis of which we can proceed to reason. This function of sensation is not hypothetical, but categorical. All explanation or hypothesis, all rising above the datum to a grasp of principle, of a law of dependence, belongs to factors other than sensory. But the situation above which we rise and whose law we come to grasp is actually present, and we are assured of it by sensation. Of this there can be no question. It is categorical.

(B) **In Analytical Expansion.**—When faced with a given situation, inference does not, like intuition, regard it as a totality, as something to be accepted or rejected *in toto*, but proceeds to take it apart, to break it up into smaller factors, and generally to reduce it to its constituent elements, so that what is relevant can be picked out from what is irrelevant, and elements which have passed unnoted in the unanalysed datum may one by one be brought under the mental microscope of our attention, and nothing may escape. In this process, what part, if any, is taken by sense-experience? That it furnishes us with the situation from which we start, we have already seen. Does it, however, do more than this, and enter in some way into the analytical process itself?

In a way, yes. If, during the process of analysing, we come upon some element which has passed unnoted in the totality, we apprehend it in a way which is at least in part sensory. In fact, each and every element discovered by our analysis, whether previously noted or not, is now, at any rate, apprehended in a manner which we can consider as sensory. Sensation, then, accompanies every step of our analysis, and accompanies it *pari passu*. But should we, on this account, regard sensation as itself exercising the function of analysis?

Further consideration will convince us that the presence of sensory apprehension, in the final as well as in the earlier stages of analysis, is simply the presence of the datum, of the concrete situation itself. Analysis cannot go beyond its material, but must remain in closest contact with what it is analysing, and must guide its every step forward by reference to what has been given to it. Hence the presence of sensory elements at each step of the analysis. Sensation enters into our analysing in no sense other than that in which the given situation itself enters in—which is, as a guiding and controlling influence, at once the material and the

ne plus ultra of the process. The sensory factor, then, is strictly not analytical, but rather intuitive, accepting what is given as a whole, and controlling the process of analysis by providing something upon which factors other than sensory may exercise their functions—a complex concrete situation.

(C) In Novelty.—Inference is not content with analysing a given situation. It stoops to analysis in order to make discoveries, to progress, to conquer for knowledge something which is *new*. In this respect it resembles the symbolic judgment, which, from data drawn largely from actual experience, makes judgments which are valid beyond the field of actual experience and hold good for possible experience. In such cases, cases in which inference leads to the discovery of something new, what part, if any, is played by sensation?

There are, as we saw above, two classes of cases: (1) where the novelty, the new information in question, is more of the sensory type, and (2) where the new discovery is more confined to a special field, the field of relations, which we decided to belong more properly to the intellect. In the first case—*e. g.*, where, by dipping red and blue litmus paper into a liquid and by *seeing* the blue paper turn red, we infer that we have before us an acid and not an alkali, the novelty is of the sensory type. As such, it is strictly analogous to the case in which analysis reveals the presence of sense-perceivable characteristics hitherto unnoticed, and the novelty consists in our now noticing it. That is to say, the function of sense-perception in such cases is merely to give us the concrete situation in its full concreteness. The *inference*, however, by which we conclude that the liquid is *therefore* an acid, goes beyond sense-perception and belongs rather to the field of relations—in this case, of causal relations.

In the second case, in which the new discovery belongs more especially to the field of relations, as where we make discoveries in geometry, or where a person born blind studies physical and psychological optics, sensation plays a very subordinate role. But in some form it seems always to be present. The figure appears to be necessary in geometrical discoveries, and the blind man interprets what he studies by substituting for the missing visual sensations tactile or kin-aesthetic sensations, so far as this is possible—much as *we* try to understand the functioning of the statocyst in Medusa, or of the faceted eye in insects, symbolically, as translated

into sensory terms which we can connect up with our actual experience.

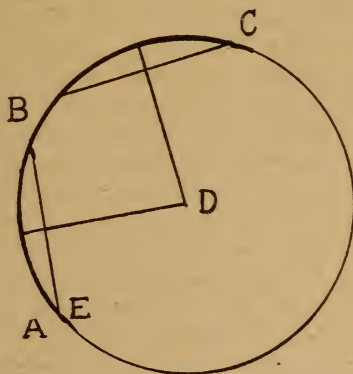
In both cases the function of the sensory element appears to be to steady attention by concentrating it upon a particular field.¹ This special field, within which we proceed to discover relations, is the concrete situation from which we start. In the case of novelty, then, as well as in the preceding cases, while the novelty as such is discovered by factors other than sensory, the function of sensation appears to be, to furnish us with the concrete situation from which we start, and by manipulating which we somehow advance to new discoveries. In itself, however, sensation is not progressive or novelty-seeking, but accepts passively and conservatively the given situation.

(D) **In Systematic Constructiveness.**—Discoveries are made either by our reorganising the material in such a manner that we come to see something which had escaped our notice, whether a sensory element or a relation, or else by carrying further some system which is already present in the data. Given a curve, a few simple geometrical experiments at construction will inform us whether it is part of a circle, an ellipse, or a parabola. Given an examination paper written by a student in logic, it is possible to form a fair estimate of that student's mental ability so far as logic and kindred studies are concerned. Given a few experiments upon memory, we can, by constructing a graph from our data, come upon striking and suggestive ideas for remembering what we are studying. In such cases, what part is played by factors which could be called sensory?

There appear to be two ways in which such factors come into play. On the one hand, just as in the preceding cases, sensation furnishes us with the concrete situation from which we start. The given curve, the examination paper, the results of the memory experiments—in apprehending all of these, a part is played by sensation, and so far, the function of sensory factors resembles precisely their function in the cases we have considered. But on the other hand, it seems to do something more. Let us examine an instance. Let ABC be any given curve. We proceed to test it, to experiment in order to see if it is a circle. We draw two chords, AB, BC, and bisect

¹ Cf. G. F. Stout, *Analytic Psychology*, 1896. Bk. II, chapter II.

each chord at right angles. The two bisecting lines meet in D. With D as center, and with a radius extending from D to B or C, we describe a circle BCE. If the point E coincides with the point A, so that the curve ABC coincides with a portion of the figure BCE, we conclude that the given curve is an arc of a circle. But if E and A do not coincide, we conclude that the given curve must constitute a part of some other figure. Either it is a portion of an ellipse, or of a parabola, or perhaps it is altogether irregular. We proceed to test each of these hypotheses in a similar way, until we come upon one which seems to fulfil all the conditions. At each step of the construction, sense-perception is present. We *see* the chords AB, BC. We *see* each step of the construction by which they are bisected. We *see* the bisecting lines meet in D. We measure out the radius with the aid of sense-perception. We see the curve extended so as to form a circle, or so as not to coincide with our circle, as the case may be. At every point of our construction, an appeal to sensory factors is inevitable.



In the process of analysis considered above, we noted that sense-perception accompanied every step of the process. Are the cases of analysis and construction perhaps similar? In the case of analysis we saw that the sense-perception in question was in fact another name for the concrete situation. Analysis of the datum revealed elements which were there, but had not been noticed. In the present case, however, there is a difference. No analysis as such could extend or pro-

duce a curve, and while in some sense we might say that extension or legitimate production of a curve is "given" along with the curve—for we do not create our extension arbitrarily, but construct in accordance with the law of the system which is given along with the given portion of that system, *viz.*, the curve—still the extension of the curve is not in any sense present for sense-perception, awaiting only the direction of our attention to it. It cannot be seen before we have constructed it. For sense-perception, it is not there. The role of sense-perception in construction is thus slightly different from what we found it to be in the case of analysis. And yet, in spite of this difference, we feel that in both cases its function is to apprehend what is given, and not to add or extend, or in any way reconstruct or alter the given situation.

What, then, are we to say its role *is*? Does sensory perception recognise, perhaps, in some way that the analysis or the construction has been carried through correctly? Is it a kind of verification of our hypothetical and methodical analysis or synthesis? Let us consider. All students of mathematics are familiar with cases in which, owing to some slight error in drawing the figure, perhaps accidental and unnoticed, a proof will end with the conclusion that a certain angle is both equal to, and greater than, a right angle, or that two straight lines are both parallel and not parallel, *i. e.*, are parallel, and yet meet when produced. In all such cases sensuous perception plays precisely the same role as when the figure is drawn correctly. By its aid we follow each step of the process, and notice that, *e. g.*, *A* does or does not coincide with *E*. But whether it does or does not so coincide, is not usually taken to be a matter of sensuous perception as such. No mathematician would dream of resting any portion of his proof upon an empirical and varying sensory observation. The appeal is always, not to sensation *qua* sensation, but to what is called "mathematical intuition," by which we realise, *e. g.*, that two straight lines *as such* cannot enclose a space, that two plus two *as such* add up to four, *etc.* There is no *as such-ness* about perception so far as it is merely sensuous.² Verification, then, is the function of

² Mathematics is not empirical in the same sense as *e. g.*, my belief that the sun is shining. It is not a matter of direct sensory perception. The difference is the difference between *empirische* and *reine Anschauung*, or between the *constatation empirique* and the *constatation logique* of a logician like M. Goblot. (Cf. E. Goblot, *Traité de Logique*, 1918, sect. 40).

factors other than sensory, and the function of sensation appears to be rather as an accompaniment, a *sine qua non*, of the process of construction. We could not do without a figure of some sort, and the function of sensation appears to be to steady the attention and concentrate it upon the features of the concrete situation which are to be utilised in guiding and controlling the non-sensuous process of systematic construction. In itself, however, sensation is not constructive, but simply accepts without question the concrete situation which is given to it, whether this is given as a starting-point, or as a result of construction.

The Intellectual Factors: (A) In Dependence.—"If *A*, then *B*; if not *A*, then not *B*; if *B*, then *A*; and if not *B*, then not *A*." The cases thus symbolised represent what the intellect demands as the four phases of a genuine dependence. Taken together, they constitute a system, a mutually supporting group of intellectual demands or standards with which we approach the empirical data in our attempts at inference, at obtaining insight into some law. As such, *i. e.*, as constituting a systematic group or organisation, they are subject to the standards which govern such groupings, that is to say, the standards of identity, difference, and organisation, internal and external. On its intellectual side, our thought directs its attention exclusively to those features of the concrete situation which are relevant to this point of view, and the result of the application of such standards is always to leave us with a mere skeleton of the concrete situation with which we started out, an abstract system of relations which satisfies the intellectual demand so far as it *can* be satisfied without going illegitimately beyond the given material. The details of this have been already sufficiently considered in our study of the application of intellectual standards in the case, for instance, of the symbolic judgment. The only significant difference is that here, instead of standards of organisation in general, and of consistency within a system in general, we have a very definite system with four main aspects, the system of logical dependence.

(B) In Analytical Expansion.—Analysis is a peculiarly intellectual function, and especially involves the standards of identity and difference. The concrete situation is envisaged as a complex, and this complex is taken apart and resolved into its constituent elements, one after another. For

example, I have applied for a position, and am using what powers of inference I possess, in order to discover if I can, what my chances are. The concrete situation from which I start is, in this case, all the information of whatever kind I have which in any way bears upon the subject. It is, however, not clear to me, but is hopelessly entangled in a mass of prejudices, hopes, and fears, which effectually interfere with my intellectual vision. By making use of the standard of identity, I run over this mass of material with a single idea in mind, and select every element which in any way bears upon, *e. g.*, my own merits, taking these one by one. By means of the standard of difference, I reject everything which does not bear upon each particular point, as associations bring up fresh material, and by using both standards in this way in clearing up one point after another, I at last have the whole of the irrelevant material separated off, and the whole of the relevant material carefully sorted out, so that I can proceed to weigh the evidence scientifically, with as full knowledge as possible and without undue bias from emotion or from the confusing mass of the material.

Let us examine another case, of a different kind. Take the following passage:³

"This good, then, which every soul pursues, as the end of all its actions, divining its existence, but perplexed and unable to apprehend satisfactorily its nature, or to enjoy that steady confidence in relation to it, which it does enjoy in relation to other things, and therefore doomed to forfeit any advantage which it might have derived from those same things;—are we to maintain that, on a subject of such overwhelming importance, the blindness we have described is a desirable feature in the character of those best members of the state in whose hands everything is to be placed?"

This passage constitutes a single sentence. But it is so difficult that without analytical inference we can hardly hope to understand it. Let us analyse it, using the standards of identity and difference so as to take one thing at a time. Suppose we take first our own predicament—the predicament in which "every soul" finds itself. We are pursuing some good as the goal of our every action. We intuitively believe in the existence of this goal towards which we are working,

³ Plato's *Republic*, Bk. VI, pp. 505 E to 506 A. (Trans. Davies and Vaughan).

but are unable to apprehend its nature. We are perplexed, and do not know what the goal actually is. In other cases, (*e. g.*, money, pleasure, learning, peace of mind), where we know what these are and understand their nature, we are not perplexed, but enjoy a steady confidence in regard to them. But in the case of this goal towards which we believe ourselves to be working—not exactly knowing what it is, we are without confidence, and do not know where we stand. Further, not knowing what the goal of life is, we do not know how we should arrange our life in respect to these other things (money, *etc.*)—we do not know how to select wisely, and therefore cannot make the most of them. Is ignorance of the goal of life, from which it derives its whole meaning—is ignorance in a matter so fundamental, desirable? If it is not desirable in our own case, is it not still less so in the case of the rulers of the State? If our great men also are blind, will not the blind be leading the blind?⁴

From this instance we see that taking a single view-point and omitting everything which is irrelevant introduces considerable clearness into our mental grasp of what was there to be apprehended. In the same way, by proceeding to make “the good” the object of our investigation, or the distinction between “divining the existence of” the end of life and “apprehending the nature of” other things such as money, pleasure, *etc.*, and similarly taking up one point after another, we could make the meaning of the passage as a whole much clearer. In general, we can say that the function of the intellectual factors in analysis is, while not leaving the starting-point or in any way going beyond the information which is given to us, so to introduce the standards of identity and difference, as to lead up to a carefully arranged articulation and classification of all which is relevant, and a strict removal of everything which is irrelevant. This sifting out of every element to which we attend leads to such an organisation of our thought that, in place of confusion and mental chaos, we have order and clearness. It may be at times that the given situation does not contain enough information to satisfy our every question, reorganise it as we may. But after such analysis has been completed, we at least know where we

⁴ So as not to take up too much space by making *two* analyses, there is included in the above the result of a previous analysis which discovered that the good is the goal of life.

stand. No item of information has escaped us, and if an answer is there to be discovered, methodical analysis is practically certain to bring it to light.

(C) In Novelty.—Inference is not fully satisfied with the clearness which results from analysis. It is not merely to make our thought clear, to purge it of confusion, to see our way more plainly, that we infer. We infer so as to discover an answer to some question which we cannot possibly answer otherwise, to make our way out of some difficulty which is otherwise hopeless, to obtain some advantage which otherwise we could not obtain. Inference discovers something which is *new*. In such discovery, what part is played by factors which could be regarded as intellectual?

Let us consider. In the above passage from Plato, for instance, we did not know, until after our analysis, what was meant by the clause "and therefore doomed to blindness" Probably we did not feel sure about what was meant by the "blindness" of the statesmen. Our analysis thus not only made our thought clear, but made us see our way where previously we had been entirely in the dark. In this way elements concealed by their context can be discovered by analysis, and so far "novelty" receives the same treatment as "analysis." It is by applying the standards of identity and difference with methodical regularity that we make discoveries of this character, that we light upon something which was already there, awaiting the application of precisely such methods.

There are, however, other cases which are not so simple. For instance, most problems worked out by simultaneous equations give us information which is new, but which can hardly be said to have been present, merely overlaid by an irrelevant and confusing context, and thus concealed from view. I miss my train, and two hours later take an express which follows it at a slightly more rapid rate—*e. g.*, ten miles an hour faster. When and where shall I catch up with my proper train and make my transfer? There is no doubt that an answer to this question in some sense is "given" along with the conditions, and that it can, as we say, be "worked out" and duly discovered. But there is also no doubt that something more is needed than merely taking to pieces the data and scrutinising them one by one. By analysis I learn that the first train moves at forty miles an hour; that my

train moves at fifty miles an hour; that the first train is two hours ahead; *etc., etc.* But just when and where the express will catch up with it—*that* I cannot by this method discover. It has to be worked out. In such cases a certain extension of the intellectual framework of the concrete situation takes place, and it is in relation to this extension that the answer is worked out and the discovery actually made. But the general method does not differ very seriously. I use x and y in working out my problem—*i. e.*, mental counters, sharply differentiated identities—and it is by the substitution, for the concrete situation, of such results of the application of the standards of identity and difference, that I am able to solve my problem. In the case of novelty, then, as in the preceding cases, the function of intellect is to apply the standards of identity, difference, and organisation so as to substitute an intellectualised model which is confined to essentials for the concrete reality which contains so much which is irrelevant and does not admit of that intellectual manipulation which leads to discoveries.

(D) In Systematic Constructiveness.—Inferential construction goes beyond mere analysis, but still without changing fundamentally the character of its data. In such construction we produce a straight line, for instance, to twice or thrice its given length. It is still, however, a straight line. We complete a curve, *e. g.*, by adding the missing portions of the ellipse of which it constitutes a fragment. The curve is still the same curve. On the third side of a given triangle we construct a parallelogram equal to the sum of any two parallelograms given on the other two sides. We have not, however, by so doing, altered the nature of the given triangle or the given parallelograms. So too we put together all we know of the character of Mr. X, and infer to his probable course of action in a certain contingency. We have not, however, altered the nature of our information by so extending it.

What intellectual factors are involved in such construction? In the first place, we use the standard of identity. In extending a straight line, we must continue to produce it in one and the same direction. In completing the ellipse, we must hold fast to the method of constructing the ellipse of which the given curve forms a part. In calculating Mr. X's probable action, we must follow the identical line of thought

established by our previous knowledge. In the second place, we use the standard of difference. We refuse to swerve from the identical direction of the given line. We refuse to wander from the path established by the given portion of the ellipse. We decline to allow hopes or prejudices to influence our calculation of Mr. *X*'s probable course of action. In the third place, we make use of the standards of organisation, internal and external, in constructing our system, whether that system be simple, as in the case of a straight line or ellipse, or complex, as in the case of character-analysis and character-construction. That is to say, in constructing systems we employ the same intellectual factors which we used in analysis, but on a slightly more extended scale—the standards of identity, difference, and organisation—and employ them in such a way that we start with a concrete situation and end up with a mental model, an abstract plan or system which omits many of the factors present in the given situation, and now includes elements which go beyond the situation with which we started, not however by altering its character, but by extending it.

Summary—The Sensory and Intellectual Factors in Inference.—In inference, then, the function of sensory factors is categorical, intuitive, conservative, receptive. Sense receives without question what is given to it, accepts and preserves it without alteration, and in a word presents us with the material for intellectual operations, a concrete situation which forms a starting-point for inference. The function of intellect is hypothetical, analytical, novelty-seeking, constructive, progressive. It receives nothing without question, works over what is given to it, omits here and extends there, re-arranges, sifts, and is not satisfied until it has substituted for the given material an intellectualised structure from the inter-relation of whose parts it obtains an insight which is new, and which leads us far beyond the concrete situation which was originally given to us. In so doing, however, it attempts to confine itself to alterations and additions which are legitimate. How this is possible, we must now proceed to inquire.

FOR FURTHER READING

F. H. Bradley, *Principles of Logic*, Bk. III, Part I, chapter vii.
J. E. Creighton, *An Introductory Logic*, chapter xxiv. J. Royce,

Sources of Religious Insight, chapter iii. W. Wundt, *Logik*, (3rd Edit.), Vol. I, pp. 296-301.

EXERCISES

1. Construct, in systematic form, a statement of the part played by personality in the world, according to the following passage. Then point out (1) the sensory elements, (2) the intellectual elements, used in your procedure:

"I do not see why the very existence of an invisible world may not in part depend on the personal response which any one of us may make to the religious appeal. I do not know what the sweat and blood and tragedy of this life mean, if they mean anything short of this. If this life be not a real fight, in which something is eternally gained for the universe by success, it is no better than a game of private theatricals from which one may withdraw at will. But it feels like a real fight, as if there were something really wild in the universe which we, with all our idealities and faithfulnesses, are needed to redeem.—For such a half-wild, half saved universe, our nature is adapted." (James, *Will to Believe*, etc., p. 61).

2. Construct from the following passage, (a) a theory of Truth, (b) a theory of Reality. Then point out (1) the sensory, (2) the intellectual elements used in your procedure:

"Reality for me is one individual experience. It is a higher reality above our immediate experience, and above all ideality and relations. It is above thought and will and aesthetic perception. But, though transcending these modes of experience, it includes them all fully. Such a whole is Reality, and, as against this whole, truth is merely ideal. It is indeed never a mere idea, for certainly there are no mere ideas. It is Reality appearing and expressing itself in that one-sided way which we call ideal. Hence truth is identical with Reality in the sense that, in order to perfect itself, it would have to become Reality. On the other side, truth, while it is truth, differs from Reality, and, if it ceased to be different, would cease to be true. But how in detail all this is possible, cannot be understood. (Bradley, *Essays on Truth and Reality*, pp. 343-344).

CHAPTER XVII

THEORY OF INFERENCE (II)

VALIDITY OF INFERENCE

The Problem.—So far we have seen what part is, in fact, played in inference by factors which are intellectual. Sense supplies us with the concrete situation from which we start, while intellect operates by taking apart this situation and presenting us, in its place, with a mental model which contains only factors which we can understand—a working model which we can take to pieces and put together again because we apprehend its principle of construction. Our present problem is to consider how far this procedure is valid, how far inference is reliable, how far a mental model which we can take to pieces and put together again represents accurately and truthfully the reality with which we wish to make contact, or to keep in contact.

In view of our earlier discussion,¹ it is unnecessary to inquire now in what sensory validity and intellectual validity respectively consist. In such cases as we have in mind, validity is purely a matter of direct sensory apprehension, or of direct intellectual apprehension of the results of applying strict intellectual standards. If we avoid misleading associations, and subject the sensory elements to direct apprehension, this is, as we have seen, ultimate, the last court of appeal, and is so far satisfactory. If, again, we have constructed our mental model after strict analysis into intellectualised elements—sharply differentiated identities which can be used as mental counters—and have constructed it in accordance with the principles of identity, difference, and organisation, then on the intellectual side it is so far satisfactory. The only question which remains is the question of *general* validity as opposed to a validity which is specifically sensory or specifically intellectual. How far is our

¹ See chapters viii and ix.

intellectual structure satisfactory from a viewpoint which is not merely intellectual, but includes sensory aspects also? How far is our sensuous apprehension valid from a viewpoint which includes also the aspect of intellectual demand?

Dependence.—As we have seen, the function of sensation is to give us our starting-point. Granted that every association which might mislead has failed to do so, granted that there is no flaw in the direct apprehension with which we see with our eyes the corn-seeds once so yellow become discolored, and with our finger feel their firmness change to a soft pulpiness—is this continuous sensory apprehension all we need to assure us of the presence of a law of cause and effect? Can we state, on the basis of an apprehension which is merely sensuous, however valid it may be, that a continuous sensory sequence is governed by a law? Can we state that the visible discoloration and palpable rottenness is caused by the visible and palpable wetness of the soil?

There can be no doubt that such sensuous observation provides us with the evidence by reference to which we verify our conclusion. But of itself, an apprehension which is merely sensuous can furnish us with nothing which is not itself sensory. A law is not something which one can see with the eye or handle with the fingers. A hypothesis is not directly apprehensible by the organs of taste or smell, and consequently, however direct and accurate sensuous apprehension may be, it is always unable to give us direct assurance, valid or invalid, of dependence, whether causal or logical. Sensation supplies us with the material for reasoning, with a given situation from which we proceed to infer and draw conclusions. But this basis or starting-point being given, the inferring or reasoning itself must be accomplished by elements other than sensory, and it is accordingly in respect of these further elements that we must ask how far inference is valid.

From the sensory starting-point described above, and by applying the standards of identity, difference, and organisation, I come to the conclusion that if the ground is continuously damp, the seeds become rotten; that if the ground is not continuously wet, the seeds develop normally; that if the seeds become rotten, the ground must have been unreasonably damp; and finally, that if the seeds develop normally, we may be reasonably sure that the ground was not unduly

moist. That is to say, I infer that there is a causal connection between excessive moisture in the environment and degeneration of the seeds. On what does the validity of this conclusion depend?

Assuming the mental pattern of cause and effect, why do we select this particular cause for this particular effect? That is, why do we select from our sensuous experience just this one element—excessive dampness of the soil—as the factor which counts? Not merely because it is in accordance with what we know of the effect of moisture in general, but because such a supposition fits in with, and is supported by, the sensory evidence in the case in question. It is satisfactory only so far as experience bears it out. If we could plant corn with impunity in the early days of spring, when the ground is especially moist, we should not accept the hypothesis in question. We should say, it sounded well in theory but did not work in practice—in other words, that it was not verified in sensory experience. But in the long run, experience does verify it, and accordingly farmers wait till the damp season is over before they plant their corn.

If, then, we may take this case as typical of relations of dependence, we can say that the validity of inference, in this field, depends upon our introducing, as a principle of organisation, a mental pattern which is satisfactory, not merely to the intellect with its general desire to understand, but also to sensory experience, which presents us with this particular situation as a problem to be solved. So far as the mental model is not only intelligible but also the pattern in accordance with which the concrete situation actually seems to behave—so far as we get insight, not merely into artificial, intellectual constructions, but also into the workings of the concrete situation itself—our inference is valid.

Analytical Expansion.—Here also sensation furnishes us with the concrete situation from which we start, and, of itself, sensation is peculiarly unable to analyse. It accepts intuitively, as a totality, whatever is given to it. Here also, then, the question of validity of inference will be concerned mainly in examining the claims of our intellectual operations to general validity. Let us take an example:

“We do not see the actual things themselves; in most cases we confine ourselves to reading the labels affixed to them. This tendency, the result of need, has become even more pro-

nounced under the influence of speech; for words—with the exception of proper nouns—all denote genera. The word, which only takes note of the most ordinary function and commonplace aspect of the thing, intervenes between it and ourselves, and would conceal its form from our eyes, were that form not already masked beneath the necessities that brought the word into existence. Not only external objects, but even our own mental states, are screened from us in their inmost, their personal aspect, in the original life they possess. When we feel love or hatred, when we are gay or sad, is it really the feeling itself that reaches our consciousness with those innumerable shades of meaning and deep resounding echoes that make it altogether our own? . . . Mostly, we perceive nothing but the outward display of our mental state. We catch only the impersonal aspect of our feelings, that aspect which speech has set down once for all because it is almost the same, in the same conditions, for all men. Thus, even in our own individual, individuality escapes our ken.”²

Let us regard this passage as a given situation and proceed to analyse it. We select a single viewpoint, and direct our thought into a single channel, taking in from the given situation every element which is of significance for this viewpoint, and omitting or rejecting everything which is irrelevant. Let us take for our first aim, to make a list of everything which is stated or clearly implied about “Things.”

Things are other than we see them—are more than the labels which are attached to them—have, in addition to the superficial aspects, aspects which are profound, unusual, unfamiliar—aspects of a deeper nature which is beyond naming, and is profoundly individual—which, to be apprehended as it is, must be regarded from some viewpoint which goes beyond merely practical interests, *etc.*

So too with our innermost *feelings*. Like things, these have an intrinsic essence, a life and individuality of their own, infinitely more profound than the superficial aspects which do duty for every-day purposes. If we wish to grasp them in their true individuality, we must use some method different from discursive thought, which seizes only upon aspects which are general, and can be expressed by names.

Then again we can take our own *knowledge*. We are thor-

² After Bergson, *Essay on Laughter*.

oughly practical, in science no less than in ordinary life. We grasp at the easiest, most superficial aspects of things and of our selves, being satisfied with what will work for practical purpose. We are driven to do this by the pressure of practical necessities, and as an aid to this kind of life have invented language—general expressions which come between us and ultimate realities, until we have come to live in a world of mental fictions, general concepts which we can use for scientific purposes, but which conceal from us the vital essence of our selves no less than of things.

So too, we might pay especial attention to *language*, or to *individuality*, etc., or to the interconnection of these concepts, and so make one element in the passage after another the subject of a detailed study, until every single element had been considered in turn and nothing had been omitted. In this way the whole passage would have been analysed and expanded.

Regarding this instance as typical of the work of analysis, let us proceed to ask how far such procedure is legitimate. The end-point is very different from the starting-point. We have taken each element out of its given context, have placed together in a new context elements taken from different parts of the passage, in such a way that they seem to throw upon one another a light which is new. At any rate, the patterns according to which the passage has been analysed and the results classified together in the above groups, were introduced by us in analysing, and it may well be questioned how far such interference with contexts is legitimate.³

That such procedure adds to our insight, is certain. The new patterns of arrangement, the taking one point at a time, the putting together every item of meaning which refers to one and the same point, is undoubtedly helpful in introducing clearness and intelligibility into the given passage. The only question which rises in our minds is, whether our procedure is not, perhaps, after all subjective, a mental luxury which makes our-convenience-in-understanding the first thing and is unjust to the passage—or whether our analysis has an objective significance. There is no doubt that such tampering with contexts is sometimes illegitimate. Under the influence of political or personal feeling, men analyse each other's utter-

³ Cf. Bradley, *Principles of Logic*, pp. 499–500.

ances in a way which makes sufficiently clear *to themselves* how base are the intentions of the other party. But it is well known how biased and unreliable such analyses are. Analysis, then, may be biased or possibly unbiased, and may be reliable, or possibly unreliable. What is to be our criterion? How are we to know when an analysis is reliable, and when it is not?

The answer is plain. Analysis is to be trusted, when it is the analysis of the given situation, when the substitution of one context for another makes clearer a meaning which is already there, but is not, perhaps, so clearly expressed. Taking one point after another is justified, so far as such a method makes intelligible the sense of the given passage. If it throws light upon the datum, and makes clear the intellectual context of the concrete situation itself, any such procedure is thoroughly justified. If however the introduction of new angles of approach obscures or falsifies—as prejudice, for example, obscures and falsifies—it is illegitimate. The verification, then, of an analysis involves patient and constant reference to the given material—reference at each forward step, as well as at the final conclusion, and at the final conclusion as well as at each forward step. Only thus can we be certain that the introduction of the intellectual standards of identity, difference, and organisation is satisfactory to our sensory, as well as to our intellectual, apprehension.

This seems, perhaps, so simple as to be hardly worth stating. And yet, everywhere one looks, one finds examples of what to avoid. Without the most patient and continuous reference to the given situation, the most ingenious and persistent attempts at analytical expansion serve only to divert from the path and to lead to constructions which are built upon no reliable foundation. The student who is familiar with modern attempts at Shakespearean interpretation is well aware that many of the attempts, literary as well as psychological, evince the utmost subtlety of analysis, the most rigid and patient following out of one path at a time. And yet, sometimes half-way through, and almost always at the end, the reader of these novel explanations, while forced to admire the analytical dexterity and logical or psychological acumen of the critic, and the extreme clearness and fascinating persuasiveness of his mental patterns, is forced also to

ask himself—can this possibly be Shakespeare? Only too often the answer is in the negative.

The validity of analytical expansion, then, is determined in the last resort, not merely by applying standards which satisfy the intellectual demand for clearness and consistency of outlook, but also by reference to the concrete situation from which we started. Only so far as the meaning which our methods succeed in extracting from the material is the meaning of the situation itself, is our analysis verifiable and accepted as legitimate.

Novelty.—The persistent application of methodical inference usually results in an insight which reveals something novel, discovers something new, or at least previously unnoticed. By putting two and two together, we discover something which otherwise, perhaps, would have escaped our most careful scrutiny—such as the age of a new minister, some vexed problem of authorship, or the solution of some historical problem, for example, the question as to the pass by which Hannibal crossed the Alps. By the careful and methodical analysis of a complex situation, we discover something which would otherwise have been missed by that superficial acquaintance which does duty for every-day purposes. By the simple extension of something which we already know, we may make discoveries of enormous importance for our whole future development. To the student who can at last read a hundred lines of the *Aeneid* in an hour, it occurs, with something of a mental shock, that he can read two hundred lines in two hours, or, further, that since there are only between seven and eight hundred lines in the average book, he can read a whole book in a single day, or even—illuminating and inspiring thought!—the whole *Aeneid* in less than a fortnight. To the student who has been accustomed to spending the greater part of a term over a single book, this extension of what he already knows comes with all the force of novelty, and is the source of a new insight which may alter his whole future plans of study, and indeed may affect radically his choice of a profession.

How far is such novelty trustworthy? It is by analysis, by extension, or by putting two and two together, that we make discoveries of this kind, and it is so far as these methods—of analysis, extension, and construction of systems—are valid, that their results can be accepted without misgiving. The

case of analysis we have already discussed, and at the present stage we can say definitely that so far as our intellectual operations upon the given material result in unearthing something which was already there, present in the concrete situation but unnoticed, so far such inferences are sufficiently reliable. The case of extension and construction of systems will be left for the next section, but at present we can say that so far as such extensions are legitimately involved in the concrete situation—*i. e.*, so far as they follow reasonably upon what is given—they are to be trusted, and, in general, our conclusion is that inferences which result in new discoveries are valid so far as they bring to light something which was already present, or is logically implied by what is present, in the given situation from which we start.

Constructiveness.—Inference is constructive. If a somewhat lengthy package is delivered to our neighbor from a delivery van with the sign "Sporting Goods," and we see our neighbor digging in his garden for worms that evening, any reasonable person will reconstruct for himself the scene which is to be enacted the following morning, and the part which will be played by the contents of the package—even though he is without special knowledge of his neighbor. If a psychoanalytical examination reveals undue hesitation in reacting to the stimulus-words "Ring," "Woman," "Marriage," "Voyage," *etc.*, we should draw our own conclusions as to the state of mind of the subject, and could reconstruct certain of his chief interests and portions of his recent history with fair accuracy, even though we had never seen him before. So too from a few scattered notices taken from the works of ancient critics, a modern scholar will reconstruct the outlines of a lost play by Euripides or Menander, much in the same way as from a few bones an anatomist will reconstruct a mammoth or eohippus, or an anthropologist will reason from the disposition of the remains of a pre-historic man that he had a religion and believed in the immortality of the soul.⁴

What are we to say as to the validity of such inferential constructions? They are all, of course, hypothetical, and none of them are absolutely certain. But when we thus go beyond the immediately present facts and enlarge upon our data, regarding them as fragments of some greater hypothetical

⁴ Cf. R. R. Marett, *Anthropology*, p. 206.

system, is our procedure so reliable as to be beyond reasonable suspicion? Let us consider what we do in making such constructions. We analyse the concrete situation which is given to us, take it apart into its elements, intensify the identity and difference aspects of these elements until they can be used as mental counters, contextless or almost contextless entities which can be put together in accordance with almost any intellectual pattern, until we end up with a mental model which we can take apart and put together again with fair insight into the principle in accordance with which we have constructed it. Instead of the concrete situation with which we started, we have an almost contextless artefact, intelligible indeed, but connected, at best, only remotely with the facts of sensory experience with which we started. How far is this inferential procedure to be accepted as reliable?

That such systems are at times far from trustworthy, is beyond doubt. From the paranoiac or the adolescent who sees everyone in a conspiracy against him, to the optimist who sees everything without exception working wholly for his best in this best of all possible worlds, the products of intellectual construction do not escape a certain suspicion, and even in the sphere of scientific research, many a brilliant theory has been discarded, as going too far beyond the evidence. N-rays whose "faint luminosity" is invisible to everyone except their discoverer,⁵ Phlogiston, the Geo-centric view of astronomy—how many doubtful hypotheses have been constructed in the tragi-comedy unfolded in the history of culture? What is it, then, which makes us trust one construction, and regard another system as wholly unreliable?

Let us consider. In the case of rival hypotheses—different and opposed theories to account for the same group of phenomena—on what basis do we make our choice? For example, do plants grow upwards because they love the sun, or because they have a special organ analogous to the statocyst in the animal kingdom? That is to say, is the explanation of their growth to be in terms of a simple, unmediated heliotropism, or in terms of a more complex organ for appreciating the influence of the earth's gravity? Or, if perhaps both factors are present, which plays the greater part? There is only one way to find out with reasonable certainty. We must devise

⁵ Cf. Gobiot, *Traité de Logique*, pp. 49-50.

experiments which will bring only one factor—*e. g.*, gravitation—into play at a time, and see how our test-specimens actually behave. We must also dissect a reasonable number of plants, and find out whether there is present anatomical evidence of the complex organs in question. That is to say, we can choose between rival hypotheses, or for that matter verify a single hypothesis, only by careful study of the concrete situation itself, in order to discover whether the suggested system is present in, or implied in, the data. If the suggested explanation is intellectually satisfactory—*i. e.*, if we can take the mental model to pieces and put it together again with a reasonable degree of insight into the law of its construction—the only question as to its validity is, whether it is or is not a system present in, or implied in, the data. If it proves to be the system of the concrete situation, then we can regard our reconstruction as an explanation of the data, and if it is in accordance with all that is known in that particular field, we tend to regard the explanation as to be accepted. Validity, then, in the construction of systems, depends wholly upon whether the intellectual context which we thus construct proves to be the intellectual context of the data themselves, or at least a reasonable extension of such context.

Conclusion—The Theory of Inference.—In this way we come to realise that the theory of inference and the theory of judgment are one and the same. The movement of our thought, whether in judgment or in inference, if it is to be regarded as valid, must satisfy certain conditions. It starts with a datum, a concrete situation with which we are in touch largely by means of sense-perception. This datum it analyses and remodels until it is shaped more in accord with intellectual demands—*i. e.*, until the intelligible elements in the given situation which are relevant to our special viewpoint are placed in their proper intellectual setting, so that their interrelation can be sufficiently understood. The validity of such thought-activities depends wholly on whether the operations of sense-perception and of intellectual reconstruction are performed accurately, not merely in themselves, as taken singly and separately, but as taken together, with especial reference to each other. Each single step of the process, from the first sensory apprehension to the last remodelling in the intellect, should be verifiable by explicit reference to the concrete

datum, and each element of sensuous apprehension should be purified from carelessness and misleading associations by a critical attitude which ensures as close conformity to intellectual standards as the nature of the case admits.

FOR FURTHER READING

F. H. Bradley, *Principles of Logic*, Bk. III, Part II, chapters iii-iv. Chr. Sigwart, *Logic*, Vol. II, pp. 5-23, 464-480. W. Wundt, *Logik*, (3rd Edit.), Vol. I, pp. 302-308.

EXERCISES

On what does the validity of the following inferences depend: (1) A sailor, coasting along the shore of an unknown land, one morning notices that he is passing scenery which he passed four days before—and infers that the unknown land must be an island. (2) Writers of almanacs tell us beforehand when eclipses are to take place. This has been inferred. (3) We leave home in the morning, and infer that when we return in the evening our home will still be there.

4. We place a letter on our private mail box, and after a few hours, notice that it is no longer there. We infer that the postman has taken it. (5) We look out of the window in the morning, and see the ground white with frost. We infer that the tomato plants have been killed. (6) We read in the paper that Mr. X has been elected president, and infer that this statement is true. (7) We look at the clock and find that it indicates an absurdly impossible time. We infer that we must have forgotten to wind it. (8) We read in the paper "Wanted, a man to drive a team with religious views," and infer that a comma must have dropped out. (9) We listen to a piece of music, and infer that it must have been composed by Beethoven. (10) We infer that, if we continue to work hard along the lines which interest us, we shall certainly be successful in the end?

PART III

SCIENTIFIC METHOD

CHAPTER XVIII

GENERAL CHARACTERISTICS OF SCIENTIFIC METHOD

Unscientific and Scientific Method.—*A* and *B* have each a business in the same town. *A* has three clerks, and each clerk does what he can. During the busy part of the day, every one keeps busy attending to customers, but during the slack times, when there is less to do, less is done. Each clerk knows as much about the business as the others, and any one of them is ready to do anything, from buying in goods to answering the telephone. *B* has three clerks, but their work is more specialised. *B* does the buying in himself, and each of his clerks has his definite department. His store seems to be kept more neatly, and while none of his clerks seem to have as wide knowledge about the business as *A*'s clerks, each one knows a great deal more in his own particular department. When business is slack, his clerks are tabulating and classifying the goods in their own departments, and during the busier part of the day it takes them less time to do business than at *A*'s store, because each knows exactly what he has in his own department, and can put his hand on it at once. In two years' time, *A* is going out of business, while *B* is enlarging his premises.

C and *D* are students in the same college. *C* attends classes and reads his assignments in a general sort of way, but does not get much out of his course. He takes things as they come along. If he happens to understand a subject easily, he passes. If there is any difficulty, he—takes another subject. *D* puts in no more time on his work than *C*, but he goes about it in a different way. He takes notes of what seems important in class, and, in preparing his assignments, makes brief analyses, noting down the important points and running them over before class. He soon learns what sort of thing to look out for, and has little difficulty in mastering each subject he takes up. Finally, he elects courses which belong together and give him what he wants to get from his college work. At the end of his course, he has developed a great deal, while

C leaves college without a degree and with a marked distaste for study.

These instances illustrate the difference between drifting along in a general sort of way, and applying scientific method. It is the difference between blundering along with a trial-and-error method, and finding one's way with a trial-and-success method. There is a right way and a wrong way of doing anything. The right way is the efficient way which leads to success. The wrong way is the inefficient way which leads to failure. What is called "scientific method" has arisen from a study of the methods which "work," that is to say, which lead to success. Such methods have been studied in commerce, in art, in the technical, applied sciences, and above all in the laboratories in which genuine discoveries have been made. Efficient methods of salesmanship differ from efficient methods of using tools, and efficient methods of using tools differ again from the efficient methods by which the original thinker solves his problems. Each branch of work has its specialised application of methods found valuable for its special purposes. But it has been found, in spite of these differences of detail, that there are a number of general respects in which all such methods, whatever the branch of work, agree. It is the systematic study of these general characteristics of scientific method which will be dealt with in the remainder of this book.

What are the most general characteristics of scientific method, the characteristics which all successful workers in every field agree in regarding as important? If we ask, what the *method* is, rather than how it should be *applied*, we find that it has two functions which are most generally considered of importance. (1) Certain methods are concerned with investigation, discovery, finding out something new. (2) Certain others are concerned with exposition, explaining to others, organising what we already know. Investigation, then, and Exposition are the two functions of scientific method which are most generally regarded as important. Application comes only after the demands of science in the way of investigation and in the way of exposition have been satisfied.

Methods of Investigation (A) Analysis.—What are the chief methods employed in investigation? The very first is analysis. Before we can solve any problem which is at all complex, it is necessary to take stock, as it were, both of what we know

and of what we do not know but have to find out, with explicit reference to the case before us. One of the very first things which a recruit learns to do with his rifle is, to take it to pieces. One of the first things which a research scientist has to do, is to analyse his problem and tabulate his data. One of the first things which a physician has to do is, to make a list of the symptoms of the patient to whom he is called. Before any steps of a more advanced kind can be taken towards solving a problem, it is necessary to realise exactly what the problem is, what are its conditions, or—as the scientist puts it—what are the *data*. This first step which is preliminary to all further work upon a problem, whatever, the field and whatever its specialised name—as diagnosis, taking stock, tabulating data, *etc.*,—is, in its essential nature, analysis.

(B) Abstraction.—A second general method of investigation, analytical in its nature and following closely upon the preliminary analysis just mentioned, is the method of abstraction. Given the analysis of a concrete situation into its general elements, it is usually necessary to pick out certain of these elements, and set on one side, as irrelevant or at least negligible, certain other elements, all of which were equally present in the concrete situation with which our analysis dealt. This process of picking out from the data those elements which are important for some special purpose, and neglecting the others, is a method of isolation. By its means, we isolate a special group of elements for special investigation, and are thus enabled to concentrate our attention upon the problem bit by bit, instead of having to deal with it as a whole. “Divide and conquer” is true not only in military science, but in all scientific study; and this process by which we isolate certain elements so that we can solve the problem piece-meal, is abstraction. As an example of this method at work, we can consider the way in which geometry, which deals with the nature of space, does not attempt to study space as a whole, but splits up the problem, and begins with a study of two-dimensional space, especially with a study of the properties of straight lines, triangles, circles, parallelograms, *etc.* So too Aristotle, in writing upon the subject of Friendship, does not deal with the whole question in a general way, but divides it up into friendships of pleasure, friendships of business, *etc.*, and discusses each of these in abstraction from

the others. By this method he is enabled to introduce a degree of clearness and distinctness into his discussion which is the admiration even of present-day thinkers.¹

(C) **Determination.**—A third general method, following closely upon the isolating method of abstraction, is the method of determination. Given a narrow group of isolated elements, *a, b, c, d, . . .* it is possible to proceed further by a careful comparison of *a* with *b, c, d, . . .* resulting in what are called new “determinations” of *a*. When friendship has been analysed into the special groups of friendships based upon community of purpose in some higher sense, as in moral, educational, or scientific collaboration, it is possible to “determine” each of these groups further by comparing instances viewed in varying circumstances. They can be compared in respect of durability, in respect of ethical value, in respect of their value for science or art, in respect of economic, social, or religious value, *etc.*, and each comparison tends to bring out new determinations, until in the end we know far more about both elements and groups—and indeed about friendship in general too—than we did before this method was applied. This method, then, which appears to be less of an analytical, and more of a synthetical nature, is known as determination.² It leads to definition and classification, though these are usually regarded as coming under Exposition rather than Investigation.

(D) **Synthesis.**—A fourth general method of investigation is synthesis. The word *synthesis* means putting together, and it may be the precise reverse of analysis, as when the recruit who has taken his rifle to pieces is taught to put it together again by exactly reversing the order of procedure by which he took it apart. But an exact reversal of the analytical procedure is, in actual fact, somewhat rare. It is far more usual to take the elements which analysis and abstraction have set before us, and to put these together in some order which is new. We may omit certain of the elements originally present, on the ground that they are irrelevant and unnecessary, and much of the value of synthesis

¹ See Aristotle's *Nicomachean Ethics*, Bks. viii-ix, and the introductory remarks of Sir Alexander Grant, in his edition, immediately before the beginning of Bk. VIII.

² The meaning and value of Determination are especially treated of by John Locke, in the second edition of the *Essay*. See Fraser's edition of the *Essay*, pp. 22-24

as a scientific method consists in its use in experimentally discovering just what *may* be omitted without prejudicing the result. White light is analysed into a mixture of various wave-lengths of ether, but we can synthetically produce the appearance of white—*e. g.*, on the color-wheel—by mixing, not *all* the rays of the spectrum, but as few as three or even two of the spectral colors revealed by analysis.

Further, by using the point, straight line, and surface revealed by analysis, synthetic geometry can construct any number of mathematical forms which are of the utmost importance for progress, not merely in geometry itself, but in all the sciences which admit of the application of geometrical methods. That is to say, synthesis is by no means limited to the concrete situation from which analysis started, but can go beyond it in its new constructions, creating forms like the eikosihedron, which is not found in nature, and such ideas as a fourth, fifth, or *n*th dimension of space, which is not observed by us.³ Synthesis, then, is valuable in at least three distinct ways: (1) It may be used as a check upon analysis, as when we verify analytical results by reconstructing the concrete situation. (2) It may be used in the discovery of simpler, more economical or more efficient methods of producing a desired result, as synthetic chemistry constructs substitutes for natural products which are hard to obtain. (3) It may be used, as in mathematical constructions generally, to give us information which reaches far beyond our original data, as we see especially in the use of graphs. This third use, however, is not by any means confined to mathematical constructions, but is true generally, as every applied science bears witness.

(D) Synthetical Abstraction and Determination.—A sub-form of abstraction is largely of a synthetical nature. There is, in addition to the abstraction which isolates elements, an abstraction which isolates laws or generalisations from experience. It is by the aid of this generalising abstraction that we neglect the particular and accidental element in individual experiences, and select for especial attention the general or

³ Such symbolic extensions of ideas originally given in sense-experience, are what Locke calls "Modes." Simple Modes are expansions of a single idea, as 2, 3, 4, 5. are expansions of the arithmetical unit, and Complex or Mixed Modes are formed by uniting different ideas, or their expansions, as "running" is a composite, consisting of (1) complex movements and (2) the sense of power. Cf. *Essay*, Bk. II, chapters xviii, xxii.

more universal element. This may look as though we were isolating the *law*-aspect of our experiences, and consequently, as though the method of generalising abstraction were really analytical. In practise, however, it always represents a summing up of many experiences, and is thus synthetical. A composite photograph, for example, is formed by putting together many negatives in order to obtain a single positive. The final positive brings out all the elements which the different negatives had in common, and omits the elements which appear in only one or two negatives. In this way we obtain a type-form, and it is obtained by abstraction.⁴ But, as resulting from many negatives and summing them up, such generalised products of abstraction are synthetical, much as a graph—such as the practise-curve, or the memory-curve—sums up the results of many experiences and is usually regarded as synthetical. That “determination” is largely synthetical, we have already stated. These methods of generalising abstraction and determination lead naturally to classification, but this is generally treated under the head of Exposition.

(E) *Induction*.—A fifth method of investigation, which depends upon both analysis and synthesis in all their forms, is induction. By some logicians induction is regarded as the investigatory method *par excellence*. But there can be little doubt, from a modern viewpoint, that it is a special form which presupposes both analysis and synthesis. A just criticism frequently leveled at Mill’s “inductive methods” objects that when phenomena are already analysed into *A, B, C, etc.*,—as in his exposition is always taken for granted—the real work of investigation is almost completed before the specifically inductive methods are brought into play; and in general, there can be no doubt that analysis and synthesis are more general methods, methods of wider application, and that induction proper is somewhat more narrow and specific, and presupposes both analysis and synthesis.⁵ The typically inductive method starts from a concrete situation which has already been analysed and further prepared by isolating

⁴ Cf. Fr. Galton, *Inquiries into Human Faculty*, Appendix II, pp. 349–354.

⁵ Cf. e. g., R. W. Sellars, *Essentials of Logic*, p. 216. Cf. also F. H. Bradley, *Principles of Logic*, p. 331, “The discriminative analysis . . . is the real agent which . . . contains the ‘induction.’” Also pp. 332–334, 335–336.

abstraction and by determination, and assumes some hypothesis or suggested law which is intended to account for the concrete situation, and then proceeds to test this hypothesis by the method of trial and verification. The inductive method thus resembles the second kind of abstraction—generalising or synthetical abstraction—except that induction sometimes goes rather beyond the data, treating them as a fragment of some wider system than what is actually observed in the immediate concrete situation, whereas abstraction, as such, never goes beyond its data. Induction is thus a process which uses, as necessary elements in its construction and verification of hypotheses, the two more general methods of analysis and synthesis.

(F) **Deduction.**—A sixth and final method of investigation is deduction. By some writers deduction is regarded as suitable only for purposes of Exposition, the function of Investigation being peculiarly the office of induction. But a little consideration will show that this can hardly be the case. By “deduction” is typically understood the arguing from a general principle to its consequences, and in almost all subjects capable of being investigated this method is of enormous importance. In fact, the typical inductive method is known as the “*deductive* method of induction,” because, assuming a hypothesis to account for our data, we proceed to deduce what would follow if this hypothesis were true, and then compare our deduced consequences with what we find empirically to be the case. A student who can reason deductively—*i. e.*, can draw conclusions from premises and see into what consequences the adoption of a principle will lead him—can usually see his way into a problem better than a student who has not mastered this method. The importance of such methodical insight for solving problems is obvious, and we shall accordingly regard deduction as a method of investigation. Like induction, it is not independent of analysis and synthesis—in fact certain modern logicians tend to distinguish an analytical form from a synthetical form of deduction, as methods appropriate to different classes of problems.

Summary.—Analysis, then and synthesis, abstraction and determination, induction and deduction, are the most generally recognised methods of scientific investigation. The most wide-reaching and the most universally present, are analysis and synthesis, but in solving any complex problem, in life or in

science, it is usual to employ every one of these methods. Definition and the concept, classification and the organisation of systems, are also pressed into the service of investigation. But these subjects are usually treated under the head of Exposition, to which we next turn.

Forms of Systematic Exposition.—Exposition cannot be sharply distinguished from Investigation. In scientific practise, there is no form of Exposition which is not frequently used in investigating new problems, and there is no method of Investigation which cannot be used in systematising one's conclusions or in explaining them to others. In fact, some of the best expositions are deliberate repetitions of the methods by which scientific problems are investigated. The laboratory method of studying known scientific laws is of this kind, and so is the attempt to understand ethical problems by the "case-method,"⁶ or the study of a poem like *Kubla Khan*, or a book like the *Critique of Pure Reason* by following the *Werdegang*, the processes through which the author passed in coming to write it.

But in spite of this confusion of methods in practise, a general distinction of purpose can be, and in a discussion of logical theory should be, clearly established. In Investigation, our primary interest lies in the direction of making discoveries and reaching important conclusions. In Exposition, we are interested primarily in stating those conclusions clearly and in systematic connection with other conclusions or with general knowledge, and especially in proving to others, as well as to ourselves, that our investigations have actually succeeded in establishing some principle or in solving some problem. Proof and the organisation of what we already know are thus the main goals of Exposition, though in attaining these ultimate aims there are also certain simpler forms which must first be considered.

(A) **Definition.**—The very first of all, a form of exposition presupposed by all organisation of knowledge for purposes of either explanation or proof, is definition. In the very beginnings of an enquiry or of an explanation, it is advisable to lay down and determine certain lines along which the enquiry or explanation is to proceed. This is the function

⁶ Cf. G. C. Cox, *The Case Method in Ethics*, *Journal of Philosophy, Psychology and Scientific Methods*, Vol. XI, 1914, pp. 16-23. Cf. Vol. XIII, 1916, pp. 212-213.

of a preliminary definition. In fact, definition means, drawing the lines which separate one direction of thought from another, delimiting a field of enquiry or explanation, stating what an object is in such a way that we can place it roughly within a system of problems or within some special department of knowledge. This latter portion of the meaning of definition is exemplified especially at the end of an enquiry or explanation, when we conclude with a clear-cut and more final idea of the object studied. Such a result of final definition is known as a concept. Definition, then, has a place both at the beginning and at the end of a process of enquiry or explanation. A preliminary definition tends to emphasise rather the elements which together make up the object studied, and the concluding definition tends to emphasise rather the principle of construction, in accordance with which the elements can be put together. In any case, however, the function of definition is fundamental in exposition. In order to classify, we must know definitely what the elements to be classified are—*i. e.*, we presuppose that they are defined. So too in order to prove any proposition, we must know definitely what we wish to prove and what elements play a part in the argument—that is to say, 'proof also presupposes definition. In general, then, definition is a form of exposition which we must regard as fundamental, as a necessary pre-requisite of all the more complex forms of exposition.

(B) Classification.—A second form of exposition which is closely connected with definition is classification. In investigating or in expounding, we find it helpful to group together a number of kindred elements so as to form a single group or class. When we can place some object of study in its proper context, when we can assign it to some class of which something is already known, we at once feel that we know a great deal about that object. If we know that a Mr. Smith is to deliver a public lecture, we do not feel more than mildly interested. He is only "a Mr. Smith" to us. But if we learn that he is a distinguished author, and a prominent Democrat or Republican, we at once know much more about him. If we learn further that he is an official representative of a certain group of interests, and that he is to speak on behalf of these interests, his meaning and value for us as an authority on his subject are increased, and we may go and listen to his address. In this way, then, we classify or arrange

objects in groups, because they thus throw light upon one another, whether for purposes of investigation or for purposes of exposition. We find that to put together objects which logically belong together—*i. e.*, objects which belong to the same universe of discourse, or have kindred meanings—is a great help both in acquiring and in transmitting knowledge.

(C) **Proof.**—A third form of systematic exposition is proof. To prove, whether to ourselves or to others, is so important a part of logic, that some of the older text books did not hesitate to define their study as the “science of inference and proof.” In a sense, every method of investigation which reaches a valid conclusion is a kind of proof, and it is sometimes stated that in Euclidean geometry the genuine proof is already given in the construction. But the formal proof which justifies the construction to others—or to oneself—usually follows other paths, and tends to consist in showing that what seems strange or novel about the theorem in question really follows from, or is logically of a piece with theorems or propositions previously understood and accepted as valid. If direct insight has not been attained, a form of argument known as indirect proof is sometimes used. This resembles the *reductio ad absurdum* arguments. We show that the opposite of the proposition which we wish to prove leads to conclusions which do not fit in with what we already believe on the subject in question, as Herbert Spencer seeks to prove that pleasure is an important ethical good by showing that its opposite, pain, is universally regarded as an evil, and that to suggest that we should seriously pursue pain is inconsistent with all that we believe of human motives.⁷ Indirect proof is seldom as satisfactory as direct proof, but there are almost always many lines along which we can advance to a definite proof, and sometimes methods which seem more roundabout and indirect are found more satisfactory and convincing in the end. This is the case especially where some science is still in its earlier stages, and insight into its subject-matter is still largely to seek. Needless to say, proof uses all the methods of analysis and synthesis already considered, and may be either deductive or inductive, according to circumstances.

⁷ See Spencer's *Data of Ethics*, chapter lli, esp. p. 22. Spencer's test of the truth of any proposition is known as “the inconceivability of the opposite.”

(D) System of the Sciences.—These three, then—definition, classification, and proof—are the chief forms of systematic exposition. It is usual, however, to add to a study of these forms some consideration of the ideal towards which exposition tends—*viz.*, the natural system of the sciences—as a concrete account of the kind of knowledge at which scientific method, in the forms of investigation and exposition, aims—the knowledge which in its applied forms aims at enriching our practical life with all the resources which intellect can muster for the service of society.

Summary.—The general characteristics of scientific method, then, vary according as the aim is investigation or exposition. The methods of investigation are analysis and synthesis, abstraction and determination, induction and deduction. The chief forms of systematic exposition are definition, classification, proof, and the system of the sciences. It remains to study each one of these general characteristics of scientific method, in the chapters which follow.

FOR FURTHER READING

W. Wundt, *Logik*, (3rd Edit.), Vol. II, pp. 1–2, 38–40.

EXERCISES

1. What are the chief differences between scientific and unscientific method in the following cases: (a) In purchasing household furniture. (b) In receiving and answering letters. (c) In studying a foreign language?

2. Show how analysis, abstraction, and the other methods of investigation might be applied in the following cases: (a) In choosing a career. (b) In estimating and judging character. (c) In research work (in a laboratory science, or in a literary or historical science).

3. Show how definition, classification, and proof could be used in the following cases: (a) In writing an essay on the value of travel in broadening the mind, or on the unaccountability of tastes. (b) In teaching elementary algebra or geometry to a high school class. (c) In explaining to a friend how to repair a minor injury to his automobile, or how to study efficiently for examinations.

CHAPTER XIX

ANALYSIS

Divide et impera is the motto of analysis. A music student who wishes to master the difficulties of some study does not practise the study as a whole, but splits it up into subjects or even phrases and practises these, one by one, first with the right hand alone, then with the left hand alone, and finally with both hands together, paying the utmost attention to every detail. So too a recruit learning the manual of arms divides the more complex movements into a number of elementary unit-movements, and practises every detail of these movements separately in order to present a good appearance at drill. So also in carrying through a nation-wide campaign, whether for political, economic, or religious purposes, the country is divided into a number of divisions, each of these divisions into districts, each of these districts into sub-districts, until finally the whole territory is parcelled out in such a way that one or two campaigners are responsible for each block or ward. In short, whether the aim is success in winning battles, or success in any other large and complex purpose, analysis is the first and most universal method adopted.

Nature of Analysis.—What exactly *is* analysis? Perhaps we can obtain the best mental picture of what is meant in the following way. Imagine a bundle of sticks, fastened together by a rope. If we untie the rope, the bundle falls apart into its constituent elements—the sticks. Analysis means precisely that—untieing or loosening up—and no more. Analysis, as such, introduces no new purposes, and is concerned with no ulterior motives. It simply looses the bonds of connection and lets the material thus set free from what held it together fall apart as it will, whatever elements happen to be there. It is absolutely vital to the method as scientific that this should be so—that the material should be permitted to fall apart in its own way, into its own elements, and that there should be no interference in the way of suppression

or addition, no re-arrangement, no selection or introduction of a prejudiced or arbitrary viewpoint. The sole aim of analysis is that the connection should be loosed and the material fall apart into its constituent elements. The bundle must cease to exist as a bundle. We must have in its place a number of sticks and a piece of rope—preferably without the slightest suggestion of their ever having formed a bundle or being usable in any special way, in order that the subsequent examination may be absolutely free from subjective coloring or prejudice, and can proceed to deal with the objects objectively.

The Aim of Analysis.—There is thus an aspect of will, purpose, or design, in analysis. We intend in the first place that the material shall actually fall apart, and shall fall apart into elements or units, portions which can be regarded as—at least for *our* purposes—ultimate. We intend, that is, that the analysis shall not in any way fail, but shall be carried right through to its logical conclusion. Analysis is to be complete. There must be no residuum which obstinately defies our efforts at analysis, but the whole must be taken apart without remainder. In the second place, we intend that the material shall fall apart into elements which are its own, that the bond of connection which we loose shall really be the bond which in fact holds together the elements in question. We intend to follow natural lines of cleavage, to divide up the object in accordance with its own nature, its own law of connection. Analysis is to be objective. We intend that analysis shall not fail by containing elements which are fictitious, mental fictions, fanciful, arbitrary, subjective, but that it shall cross the line which divides the fictitious from real, the subjective from the objective, and shall deal with the actual nature of the actual object. Completeness and objectivity—these represent the aim of analysis.

How far Realisable? (A) In Man-Made Structures and Mental Models.—How far can this aim be realised? Let us consider. A recruit is shown how to take his rifle to pieces. By turning here and pulling there he can take it to pieces in a way which is objective—for the striker, follower, main spring, *etc.*, are actual parts of the actual rifle. But his analysis is not complete. By the use of a screw-driver, the armorer will take the rifle still further apart, until he has reduced it to its last elements. His analysis is both objec-

tive and complete. So too, the use of a screw-driver and a wrench, *plus* the methods of pulling, pushing, and turning, will suffice to take to pieces a type-writer or piano, complex though these objects are, and such an analysis can be not only objective, but also as complete as we please. In fact, any machinery which the mind of man can devise, the mind of man can also take to pieces. Artefacts, then, *i. e.*, man-made objects fashioned in accordance with some rational plan, can be analysed in a way which is both objective and complete.

Let us consider another class of cases. A complex arithmetical example can be reduced to a number of simple operations with units. A complex geometrical figure can be reduced to a combination of simple lines and points. And generally speaking, mathematics presents us with a host of complexes which the mind of man can take apart because the mind of man has put them together, and put them together in accordance with a rational plan,—a plan which the mind of man can understand. These cases are, in fact, typical of the vast number of thought-structures to which we have referred as mental models. Whatever reason has constructed in accordance with its own laws, reason can take to pieces again in a way which is both objective and complete. Whatever has been constructed in accordance with intellectual standards or intellectual demands, can be analysed in accordance with intellectual demands, and speaking generally we can say: Whatever is rational can be analysed, and can be analysed just so far as it is rational, whether it is a mathematical problem, a carefully thought-out plan of life, an esthetical composition, or a piece of mere machinery. In such cases, where thought apprehends the structure which thought has itself introduced in accordance with its own laws, *i. e.*, where thought is dealing ultimately with itself—analysis, as we have described it, is possible. In such cases we can realise the aim of objectivity and completeness.

It remains to consider a further point. Whatever is rational can *theoretically* be analysed. But in order that this theoretical possibility should be realised, we must know something more. It is necessary to understand the special law of connection which binds together the elements in question so as to constitute a rational complex. Without insight into this law, we cannot know what *are* elements and what are not,

in a given case. We do not know what to look for. Let us suppose that we receive a message in cipher: "AAABBBBAAB A BBBAAAAABBBBAAABBAABBBBAABBAABABABBBAA BBABABAAABAAABBBAAAA." Here is something constructed in accordance with a rational plan. But without the key, what can we do? Suppose we analyse it into letters. That will not help us. For unless the cipher is constructed in such a way that each single letter corresponds to some single letter of the alphabet, our analysis is irrelevant. We are analysing into elements which are not elements of the cipher, but are arbitrary. They are, indeed, in a sense complete; for nothing is omitted. But such an analysis is certainly not objective—*i. e.*, does not analyse the object, the cipher itself, at all. As there are only two letters, A and B, it is plain that the single letter idea must be abandoned. We try again. Perhaps A represents one letter of the alphabet, AA another letter, AAA yet another, and so also with B, BB, BBB, *etc.* The most frequently occurring of these combinations are AA and AAA. But here again we fail, and our failure shows that the principle in terms of which we are trying to analyse is not the right one. We try again. Perhaps some combination of A's and B's is equivalent to each letter of the alphabet. We may remember having read of Bacon's bi-literal cipher, or we may add up the total number of letters and find that they will divide by five, and thus, even without knowledge of Bacon, assume that a group of five letters, consisting only of A's and B's, corresponds to a single letter of the alphabet. Our assumption may be correct, but of itself does not suffice for the solution of our problem. We cannot be sure that the analysis is correct, and that AAABB, BAABA, *etc.*, are the genuine elements of the cipher, until we have discovered more. We try constructing a cipher in accordance with the formula with which we are now experimenting. Let $a = AAAAB$, $b = AAABA$, $c = AABAA$, *etc.* There are, of course, many possibilities here, but we try the first one, and find—after a few more mistakes and trials—that it fits. Now that we can finally read off the message, and thoroughly understand the principle of construction, we can analyse our problem into its elements in a way which is both objective and complete. What is needed in order to realise the aim of analysis is (1) that the object

should be constructed in accordance with some rational plan, and (2) that we should have insight into the plan. Otherwise, our efforts at analysis are irrelevant, and fail from the viewpoint of objectivity at least. Such attempts are imperfect and artificial.

(B) **In Dealing With Natural Phenomena.**—Our procedure, in such cases as the above, is like that of a person who stands before a locked door with a bunch of keys, one of which is the right one. He tries key after key, until he comes upon the right one. So too we tried one mental model after another, until we hit upon the one which exactly fitted. But we must now advance to a further consideration. In the cases hitherto considered, thought is dealing with its own constructions, its own instruments, forged by itself in accordance with its own laws. In actual practise, however, it is very rarely that thought either needs or wishes to analyse itself and its own constructions. Thought-structures are instruments intended to solve problems which face us in life—problems in the objective world, and in dealing with concrete situations it is a question whether any mental model will “fit” except very roughly and approximately. Analysis is imperfect—*i. e.*, we have to use models arbitrarily chosen, and thus not perfectly appropriate—(1) when we do not know the principle which governs the situation before us. In actual practise, this is almost always the case. A specialist is asked to psycho-analyse a hysterical patient, with a view to finding out what is wrong. No one knows. He proceeds to test for all the frequent “complexes,” until he hits upon a group of associations which seems to be causing the trouble. This is a lengthy process, and the preliminary attempts at analysis are largely irrelevant and, in some cases, even misleading. But further, analysis is imperfect (2) when our instruments are not perfectly adapted to their work. The various diseases, physical and mental, which a physician has to diagnose, are cases in point. Every one knows that the most terrible mistakes sometimes take place. Where the symptoms are not accurately studied, it is possible for the physician to analyse in terms of a mental model which seems partially to fit the case, but in fact is tragically beside the point. Thus typhoid patients have been actually treated for appendicitis, cancer has been treated as indigestion, and mental disease has

received any and every kind of treatment, from religious veneration to imprisonment or a sound thrashing.¹

In dealing, then, with objects other than our own thought-constructions,—that is to say, in the face of nature with its infinite variety of problems—we are necessarily restricted to the trial-and-error method, and to experimenting with analytical models which we know to be more or less imperfect. The result is, that our conclusions never perfectly apply to their material, and in consequence, our analysis is hardly ever complete. There is almost always some residuum which obstinately defies further analysis. A chemist, analysing given material for traces of poison, knows that there is always a certain residuum for which he cannot perfectly account. Different chemists analyse the same material but come to different conclusions: A finds 5% of the poison, B finds 6%, and in certain cases such differences are important. Where our instruments are not perfectly adapted to their material, our analysis cannot but be imperfect, and, as contrasted with our perfect success in dealing with mental constructions, we can say that scientific method, when applied to physical or natural-science problems, is always partly incomplete, and never perfectly objective. We deal, not with things as they are in their nature, but with mental models whose structure we understand, and the difference between the mental model and the actual given situation is the unanalysed residuum which is the measure of our practical success or failure. If the difference is large, we fail. If it is negligible, we succeed in practice. But a theoretically perfect analysis of natural phenomena is an unattainable ideal.

METHODS OF SCIENTIFIC ANALYSIS

(A) *Mathematical.*—But because our efforts at analysis are empirical and imperfect, it does not follow that we cannot at least approximate to results which we can accept as satisfactory. Even mistaken experiments succeed, as a rule, in assisting us to *some* insight into the problem studied. In the case of the cipher, which we analysed above, we gradually worked our way to a method which was satisfactory. So also in natural science. We almost always begin with some method of

¹ Cf. Bernard Hart, *Psychology of Insanity*, chapter 1.

analysis which we know to be merely preliminary, because experience shows that it is helpful in preparing the material for a more final kind of analysis, and in giving us a kind of insight which, without such preliminary analysis, we lack. Thus, in attacking many a psychological or sociological problem, while we know that the only kind of analysis which can yield us final satisfaction will be specifically psychological or sociological, we begin with an analysis which is mathematical. In all the natural sciences, mathematical analysis is an auxiliary method of the greatest importance. So far as it goes, it is sufficiently exact, and it tends to leave the material in better shape for a more final analysis. For instance, how long should a Dachshund of given girth, head, and tail, be in the body in order to give the most esthetical satisfaction? For experimental purposes, a model Dachshund is used, made of celluloid, the body of which can be elongated or contracted by means of an apparatus which admits of exact measurement in terms of a millimeter scale. Starting with a short Dachshund, we observe it as it is gradually lengthened, until we feel that it is just right. A record is taken of the measurement, and we try again. After many such attempts, we commence with a Dachshund which is much too long, observe it as it grows shorter, until we feel that it is just right. Finally, the averages of the various measurements are taken, and, after a number of mathematical manipulations of the mathematical data, we reach certain conclusions as to the variability of the esthetical judgment, and also as to certain of the conditions which influence it. Such an analysis does not take us very far, and experimental esthetics generally is still in a very preliminary stage, but still, a beginning has been made, and has been made by using methods which are, at least in part, mathematical.

So too with problems which are sociological. Is there really any definite connection between drink and crime? Is imbecility hereditary? What factors govern the increase or decrease of population? In studying such problems, mathematical analysis is vital. Without statistical methods of considerable refinement, the material could never be reduced to a form which the scientist could use. This is true also of intelligence tests, tests of bodily efficiency, and generally, in all cases which admit of the application of mathematical methods of analysis. Breaking up a vast and complex situa-

tion into elements which can be counted and shifted around in accordance with quantitative methods is almost always helpful in adding to our insight into the structure of a concrete problem.²

(B) **Causal.**—A second mental model which experience similarly shows to be helpful in approaching the analysis of a concrete situation is the cause-and-effect principle. This is found helpful in analysing processes, events, and generally any phenomena which occupy time. Mathematical models are also employed as a rule, because of their value as auxiliary methods. But in dealing with processes and events, we tend to split the phenomena up into groups which are not so much 1, 2, 3, . . . , as before-and-after groups, antecedents and consequents, cause-and-effect groups. The physician called in to diagnose a case begins, it is true, by listing the symptoms. But his analysis throughout has less of mathematical, and more of causal reference. The enumeration of symptoms is less important as enumeration—*i. e.*, mathematically—and more important as throwing light upon causes, as a sore throat followed at a certain interval by a rash implies scarlet fever or measles, or as certain disturbances of digestion imply decayed teeth, *etc.* So too a professional man, in analysing the noises which disturb him at his work, is not content with a mere enumeration, but analyses with a very definite causal reference. Thus, freight-train disturbances point to increasing traffic, and thus to prosperity; passing automobiles, whether for business or pleasure, seem to point in the same direction; while certain other noises point to causes which should, in his opinion, be eliminated. In fact, during the greater part of our waking life we are analysing with this kind of causal reference all events which attract our attention, and all persons with whom we are brought into contact. On the whole, then, it seems legitimate to regard these two classes of mental patterns, the mathematical and the

² Experience with card-index methods will justify this statement. It is stated by some logicians (*e. g.*, Bosanquet and Wundt), that the body of science which constitutes Law does not use mathematical methods of analysis, as it is not quantitative. This does not seem to be exactly the case. In analysing a concept under various heads, it is usual to enumerate the heads of characteristics 1, 2, 3, *etc.*, and in all analysis in which card-index methods or anything of the sort are employed, there is a tendency to treat each record as a "unit," and in the shufflings and regroupings of these records, certain methods are used which are, to some extent at least, mathematical. It is true, however, that in theory and practise of Jurisprudence, mathematical methods play a part which is at best only subordinate.

causal, as the most universal and the most helpful of all our preliminary and imperfect methods of scientific analysis.

Validity of Methods of Scientific Analysis.—The mental patterns employed in scientific analysis, as exemplified in quantitative and causal explanations, are in part inadequate. But on the whole, they are found helpful in giving us insight into the structure of natural phenomena. On what does this helpfulness depend, and how far can we feel justified in approaching nature in the attitude of a judge, and compelling her to answer our carefully prepared questions? To this query, there is only one answer which we can regard as admissible. Our analytical methods are justified only so far as they are found intelligible on the one hand, and found to “work” on the other. They should not only be intelligible in themselves, and form part of a consistent system of similar mental models, but should also be justified in terms of sensory experience.³ They should approximate to the completeness and objectivity which are such conspicuous features of our analysis when thought examines only its own constructions. So far as our empirical analyses fall short of these standards of completeness and objectivity, so far these serve as encouragements to pursue and follow our path yet further, and it is only by continued and unwearied experimentation that we come to adjust our mental models more closely to the empirical facts, and thus approximate more and more to a validity which, in dealing with natural phenomena, appears to be beyond our reach.

Summary.—So far then, we have seen that analysis is a preliminary portion of scientific method, and that its application is universal. Its aim is, to loosen the bond of connection which holds a problem together, and let it fall apart into its elements along natural lines of cleavage, in a way which is not only objective, but also complete. This aim can be realised in dealing with mind-made structures, and with these only.⁴ In dealing with natural phenomena, we can only

³ For an example of what to avoid, the student with grounding in psychology is recommended to glance over J. Chr. Wolff's *Psychologia Empirica*. The mental patterns are clear and form a wonderfully consistent group as a whole, and yet there is perhaps not a single problem or theorem in the whole book which would be accepted by a present-day empirical psychologist. It is weak on the empirical and sensory side.

⁴ Machines are regarded, from a logical viewpoint, as mind-made structures. It is only so far as they are made strictly in accordance with our mental patterns that we fully understand them. Cf. Bosanquet, *Essentials of Logic*, p. 125, who treats a portion of a railway track as a materialised disjunctive judgment.

approximate to an accurate analysis by experimenting, and trying whether this mental model or that will apply. The most universally usable and the most generally helpful of such mental patterns are the mathematical and the cause-and-effect models. Though not perfectly satisfactory, they can still be accepted as trustworthy so far as they are found to "work," and to bring us into closer touch with, and understanding of, concrete problems. By such means we can hope for a gradually increasing insight into the structure of physical phenomena, and, indeed of all phenomena whatever which can be analysed in this way.

FOR FURTHER READING

W. Wundt, *Logik*, (3rd Edit.), Vol. II, pp. 2-8.

EXERCISES

Show what part is played by analysis in dealing with the following cases: (1) In learning to play tennis or golf. (2) In making oneself popular. (3) In finding out what is wrong with an automobile which suddenly refuses to work. (4) In finding one's way about a new city. (5) In looking up facts or dates in the encyclopedia. (6) In securing a position.

CHAPTER XX

SYNTHESIS

"Putting two and two together" is one of the most important of all our intellectual activities. Without this function, we should be imbeciles, unable to hold more than a single idea in our minds at a time and incapable of performing the slightest service for others, or even for ourselves. With it, we have an instrument which is indispensable in every walk of life. By its aid we construct our daily plans, our science, our art, and our religion. Our whole political and economic life rests upon it, and so vital does it appear in our reasoning processes, that philosophers like Aristotle and Kant have regarded synthesis as the most characteristic function of mind, and psychologists like Binet and Yerkes rank synthetic ability as evidence of advanced intelligence.¹

Nature of Synthesis.—What precisely do we understand by "synthesis"? It is the opposite of analysis. Analysis takes apart. Synthesis puts together. Given a concrete situation, analysis reduces it to separate elements. Given separate elements, synthesis reduces them to—what? Let us consider. Defends, a, his, dog, master, good, bravely. Here we have a number of separate elements. Synthesis puts them together in such a way that they make sense—*i. e.*, in such a way that they express a certain unity of meaning. "Two *N*'s, two *O*'s an *L* and a *D*," says the child, "Put them together and spell them to me." Here again, it is a question of putting them together so that they form a rational unity. Analysis, we remember, meant, not merely taking apart, but loosening a bond of connection, the principle which held together the phenomenon studied. Synthesis, then, means, not merely putting together, but introducing some unifying plan, some rational principle of connection, so that in place of a mere aggregate or disconnected heap we have a genuine totality,

¹ Cf. Yerkes-Bridges-Chadwick, *A Point-Scale for Measuring Mental Ability*, 1915.

an organised whole composed of elements which are united—*i. e.*, belong together and constitute a single entity, an individual.

Aim of Synthesis (A) Objectivity.—Synthesis means, then, no merely mechanical juxtaposition, but a putting together which really puts *together*, so that the elements which are synthesised enjoy a genuine *togetherness*, and constitute a unity which is rational. It means intelligent construction, rational organisation, and involves the application of intellectual principles to elementary data. Synthesis aims, then, at *organisation* of the data, and at an organisation which is *rational*. It also aims at *organisation of the data*. That is to say, like analysis, it aims at objectivity. No rational person would attempt to construct a triangle out of tones or a melody out of straight lines. In every case the principle of unity which synthesis introduces must unify the elements with which we start, and must bind them together in a way which is suited to their nature. Otherwise we fail in respect of objectivity.

(B) Completeness.—Can we say that, like analysis, synthesis also aims at completeness? Let us consider. At a *rational* completeness it certainly does aim. For this is involved in the very notion of objectivity. Thus, given the barrel, stock, magazine, cocking-piece, *etc.*,—*i. e.*, the elements out of which a Springfield rifle is constructed—there is only one rational way of assembling the parts. If they are to form the unity for which they are adapted, it will be necessary to omit nothing, but to include every single element. So too, given the requisite number of automobile parts, there is only one way in which these also can be assembled so as to constitute a genuine unity—the unity for which they are objectively adapted. No single part can be omitted. The synthesis must be complete.

That is to say, where the objective elements are such as to render it possible, the synthesis should be complete. But there are two other possibilities. The given material may contain elements which are too many or too few. Out of an assortment of materials taken from several old cars, a skilful mechanic will assemble a single car which can be used. But he will not have used up *all* the material. It contains parts enough, and more than enough, for one good car, but not enough for two. The surplus parts are thus omitted, and

the synthesis is not, in this sense, complete. But if we understand by "completeness" a due regard for objectivity and for what is reasonable in the particular situation, we can say that even in such cases our synthesis aims at all the completeness which could reasonably be demanded. So again where the elements are too few. In such cases, it may be impossible to put them together objectively—*e. g.*, if some vital connecting portion is missing. But in all such cases our aim is at completeness. Wherever possible, we piece out the imperfections of our material, and construct, as well as we can, in all its completeness, the totality, of which our data constitute fragmentary portions.

We may here note a certain difference between analysis and synthesis. Analysis seems to be confined exactly to its data. Its aim is thus to omit no element, whether relevant or irrelevant, and to add nothing, whether some vital element has been omitted or not. Synthesis, on the contrary, seems to be a more flexible and a more developed method than analysis. It can omit what is irrelevant or superfluous, and can add what is missing, or at least is imperatively demanded by the data in order to put them together. That is to say, synthesis can take account, to a greater extent, of rational considerations, and is not so tied down to its material. But in general its aim resembles the aim of analysis, in that it desires above all things objectivity and completeness.

How Far Realisable? (A) With Mind-Made Entities.—Give a child the parts of a jig-saw puzzle, or such letters as *N, A, T, H, S, O, G, W, I, N*, and ask him to put them together. Give an adolescent a box with various compartments, of which one contains the parts of a simple bell, another the parts of a simple lock, *etc.* Within a reasonable time, the child will have put together the parts of the jig-saw puzzle or the letters of the name, and the adolescent will show you a complete bell, lock, *etc.* So also a clock maker will assemble the parts of a clock, a trained mechanician will assemble the parts of an automobile, *etc.* In a word, wherever we have the parts of some mechanism devised by the human mind, the human mind can learn to put those parts together in a way which is both objective and complete. The history of invention further shows that synthesis can in many cases improve upon the original principle of construc-

tion, by designing models which use less material, fewer and simpler parts, and in a word are more economical and efficient.

Let us consider other cases which are mind-made, but less closely connected with physical matter. Given three straight lines, of which any two are together greater than the third, it is possible, upon a plane surface, to construct a triangle. Given the elements essential for the solution of a problem in simultaneous equations, it is possible to solve that problem. Given the concept of Man as finite, imperfect, and dependent, and the concept of God with the traditional attributes of absolute power, absolute knowledge, absolute wisdom, *etc.*, it is possible to construct a whole system of ethics based upon the relation of Man to God. If these instances are typical of mental models, we can state that whatever elements are capable of being put together in accordance with a rational principle, admit of a synthesis which is objective and complete.

In such cases, then, it is theoretically possible to realise the aim of synthesis. But before the aim can in fact be actually realised, something more is necessary than elements which are rationally unifiable. Take any college graduate and show him one of the standard puzzle-boxes. Give him the following instructions:—"Pull out this lever as far as possible. Then pull out this second one. Then stand the box upon the side which is painted white. Then turn the combination lock twice to the right, to the number 47, then twice to the left, to the number 36, then once more to the right, to the number 14, and the door will open." The data here are the particular instructions, and the problem is, to put them together correctly. In theory, the synthesis can be both objective and complete. In practice, however, the average college graduate will be unable to put together the elements of the instructions and hold them together in his mind. Either he omits to stand the box upon its white side, or he fails to turn the combination lock *twice* to the left, *etc.* That is to say, he has failed to grasp the rational principle, in terms of which the instructions form a unity. In order to succeed, it is essential to grasp the principle thoroughly, and to apply it exactly. So too, after hearing an address, we find that we can perhaps remember parts of what we have heard, but that we cannot put the parts together so as to form a rational unity—we have lost touch with the principle

which made the connection and sequence of thoughts clear. So too in carrying through our life-plans, there are times when we lose sight of our guiding principles. In such cases we find ourselves unable to make sense of our experience, and we blunder along at haphazard. In order, then, that the aim of synthesis may be realised, it is necessary, not only (1) that the data shall be rationally unifiable, but also (2) that we thoroughly understand the rational principle which is the key to their synthesis in practise. Then, and then only, can we advance to a synthesis which shall be both objective and complete.

Just what is our procedure when we are without insight into the principle of connection? Let us consider an instance. Hour, for, we, early, at, park, an, started, the, of, morning. Here we have a number of elements which can be put together so as to make sense. How do we synthesise them? We read over the given words, and try out various plans for connecting them. Something about *Park* and *Morning*. . . . "The park in the morning. . . ."? No, we cannot make sense that way. We try again, bringing in *Hour* and *We*. "In an hour we started for the park at early morning. . . ."? No—our synthesis is wrong. We have added the word *In*. We try again, joining up *Hour* and *Morning*, and this time—perhaps after one other mistake—we have it. That is to say, we use the trial-and-error method. We adopt tentatively one mental pattern after another, until we find one which fits. Then, and then only, when we have acquired insight into the principle of connection, do we succeed.

(B) **With Natural Phenomena.**—In dealing with subjects of study other than our own thought-made structures, we are almost always in this difficulty. We do not have an exact insight into laws of connection, and the greater part of the scientific work is directed towards finding out, as nearly as possible, what these laws are. We experiment with our mental models, one after another, until we gradually attain to a certain degree of insight. Thus, when brought into contact with an interesting stranger, we note all his peculiarities, and then try to put these together in a way which will give us insight into his character. The optimist tends to synthesise in terms of prevailing bias, and to see everyone as better, perhaps, than he is. The pessimist similarly sees people somewhat worse than they are, and in

general, a little consideration will assure us that, when faced with a problem for synthesis, we combine the data experimentally, in terms of mental patterns which we understand and regard as helpful.

There is, however, a certain difference between thought-structures and natural phenomena. In dealing with thought-structures we know that there *is* a key, and the sole problem is to find it. In dealing with natural phenomena, on the other hand, we assume that there *may* be a key, and a study of the history of science will convince us that we can, at best, only approximate to discovering a genuine law of connection. The process of experimentation is more prolonged, and we must not expect it to lead to a conclusion which will be perfectly satisfactory. We use the best mental models which we know, and there is no doubt that we find these helpful in putting together the data of our various problems, but when all is said and done, the mental model differs in its structure from the actual phenomena, and this difference represents an unknown amount of marginal error. Thus, in dealing with our fellow-men, we can proceed on the hypothesis that they are all self-centered—egoistic hedonism is the technical name for this model—and in general our constructions based upon this principle will be sufficiently like the structure of actual motive-complexes to “work.” But a wide experience of men as well as a less crude psychological theory will show us that the self-interested man who always acts upon calculation of what will be most to his advantage is a myth—*i. e.*, a mental fiction, and is not found in nature.² Our models, then, are not perfectly reliable, and in point of objectivity we can hardly hope to realise the full aim of synthesis. We construct something which always differs from what we wish to understand, and our mental model never quite fits into the world of actual phenomena.

So much for objectivity. What are we to say of completeness? The case resembles what we discovered in dealing with analysis. Without insight into the principle according to which the material can be unified, we do not really know what parts of the material are relevant and what are superfluous and negligible. In consequence of this lack of insight, we may unguardedly omit something which is vital, or add

² Cf. W. MacDougall, *Social Psychology*, preface.

something which is unnecessary or even misleading. In studying intelligence, for instance, what factors should be taken into account in our synthesis, and which elements should be omitted? Is ability in mathematical work, in logical tests, and generally in solving problems, to be estimated highly, while ability to reproduce strings of figures or nonsense-syllables, and similar tests of rote memory, to be estimated as somewhat of a minus factor, if anything? And what are we to say of visual and auditory acuity, and in general of good powers of sense-perception? Is their possession a sign of intelligence, or not? Given, as data, answers to tests in all these fields, and the problem being to synthesise these data in such a way as to rank in order of intelligence the individuals who have been tested, we clearly need insight into some principle which will tell us which tests are to be regarded as important, and which are to be entirely omitted, as of zero or minus value in estimating intelligence. Without such insight, we may blindly assign to sensory acuity a value equal to that assigned to the logical tests, or even assign a high value to the memory tests. In other words, we may include in our synthesis elements which are irrelevant or even contradictory, and thus may seriously vitiate our conclusions. In such cases we can learn to avoid an irrational and external completeness, and to approximate to a completeness which is reasonable, only after much experimentation with mental models, testing the tests themselves, until we find out with fair accuracy which tests constitute genuine elements, and which tests have a negligible value.

So also, on the other hand, when evidence is scanty, and it is necessary to add elements in order to construct the required totality, as nearly as may be. Without insight into the principle in question, we shall not know what elements should be added, and may go seriously astray. In interpreting the conduct of other persons, we have, as data to be synthesised, a number of observed actions. Our aim is, so to sum up these actions as to throw light upon the whole system of purposes and motives underlying a given individual's actions. Consider, for example, the character of the emperor Tiberius. On the evidence of the bare actions recorded in history, we should say, with certain critics, that he was a great administrator with poor social qualities, but of a character which, in the main, was highly valuable. Other critics, however, agree

with Tacitus in attributing to him a duplicity of purpose which is almost without parallel in the world's history.³ Historical reconstruction, in general, lends itself to additions here, and special emphases there, which may or may not be justified. How far they are scientifically correct, is a matter for careful weighing of the evidence. In cases, however, where the evidence still remains indecisive, suspension of judgment seems to be the only scientific course.

In dealing, then, with natural phenomena—*i. e.*, with data other than mental models—we seem unable to attain to full insight into a principle for unifying our data, and thus, in respect of both objectivity and completeness, our synthesis cannot be entirely successful. There is, however, no doubt that we can approximate to a synthesis which would be above reproach by using as mental models the most approved patterns.

Methods of Scientific Synthesis (A) Mathematical.—The first and most universal of such patterns as are approved on the basis of experience, is the mathematical group of models. Whatever can be counted, can be added or synthesised, and if we can do nothing else, it is at least something if we can regard each of our data as approximately=1, 2, 3 . . . , and can thus proceed to add them, or subject them to mathematical manipulation of some more advanced kind. A student's record in college, for instance, is expressed largely in terms of this kind of synthesis. In a given course there are (1) a number of papers, (2) a number of recitations, and (3) a final examination, as data. In most cases, the marking has been qualitative rather than quantitative. Recitations, for instance, are good, poor, or fair average. Papers are A, B, C. . . . By assigning numerical values to these data in accordance with a definite rule, these elements can be added, weighted,⁴ and averaged in such a way as to satisfy the demands of an elementary synthesis. Such marking is admittedly never quite perfect, but the introduction of the mathematical type of synthesis is at least a beginning in the right direction, and is far more objective and complete than a mere arbitrary "general impression" would be. So also in the case of the intelligence tests mentioned above, and in gen-

³ Cf. Furneaux, edition of *Tacitus' Annals*, and Boissier, *Tacite*.

⁴ For what is meant by a "weighted" average, see A. L. Jones, *Logic*, p. 199.

eral, it may be stated that in scientific constructions we have "science" in exact proportion as our constructions follow mathematical models.

Such a synthesis in terms of mathematical models is always correct as far as it goes. In most cases, however, it must be admitted that it goes only a short way. It is preliminary to a more specific type of synthesis. Its function is, so to prepare the material, that we can manipulate our data more easily and with a certain approximation towards insight into the requisite principle of unification in the specific case. Thus, in estimating the state of prosperity of the country, if the data—*e. g.*, market values of various staple commodities—have already been reduced to mathematical form and subjected to a manipulation which is mathematical and reduces them all to a common basis—we have already advanced a long way towards a synthesis which would satisfy economists. But in order to advance the whole way, we should have to go further and effect a synthesis which is specifically economical. The mathematician as such can handle data, but is without the specific insight into economical principles which is requisite for effecting a synthesis which shall fall within the province of economics. So also in the case of physics, or psychology, or sociology. The mathematical synthesis is a necessary preliminary to further work, the final synthesis being effected by a physicist, psychologist, or sociologist. In the more preliminary stages of such sciences, the mathematical form of synthesis is almost the only one which is regarded as legitimate. But as such sciences progress further, the merely mathematical synthesis tends to be regarded as a method which is, indeed, universal and necessary, but is auxiliary and preliminary to the specific synthesis which it is the aim of the specific science in question to effect.

(B) *Causal*.—A second mental pattern which, on the basis of experience, is thoroughly approved for scientific purposes, is the cause-and-effect model. Given as data,—as elements to be synthesised or put together in a way which will make sense—events, processes, and generally data with a temporal reference, it is found helpful to assume, as a mental pattern in terms of which they can be put together, a rule according to which one event or datum in time follows another. Mathematical models are employed as subsidiary methods, in the way explained above, but in dealing with events, the model

which exercises a controlling influence is the causal. The diagnosis of a case begins with an exact analysis and tabulation of the data in the form of symptoms. But, this analysis being completed, the next step is to put these data together in terms of some mental model which will throw light on the concrete situation which is the disease in question. The sore throat, high temperature, *etc.*, are regarded not as mere units which can be added and subtracted, but as *symptoms*—*i. e.*, as *effects* produced by some central *cause*, the disease whose nature is to be diagnosed. Certain phenomena of human growth are explained as *caused by* the direct activity of certain glands, the rapid movement by which *Dionaea Muscipula* secures its prey is explained as *caused by* the series of changes initiated by the lever-like action of the contact-hairs, and in general, the adoption of this mental pattern has served to unify, in a way which makes sense, phenomena the most diverse in appearance, and data whose connection had for centuries remained an unsolved problem. The mathematical and causal types, then, are among the most universal and most valuable of the mental models by the aid of which we endeavor to make sense of the world in which we live.

Validity of These Methods.—Mental patterns for explaining physical objects are seldom perfectly adequate in point of either objectivity or completeness. We have a bias in the direction of certain numbers, as when we imagine that every *third* wave, or every *seventh* or *ninth*, has the largest crest, or when we suppose that there are certain hidden rhythms in nature, which our poetic fancy can discover. So far as causal patterns are concerned, there is no human being but frequently believes causation to be at work when there is, in fact, nothing but hallucination. The belief in “ghosts” as causal factors in human affairs, in the influence upon our lives of “the stars which shone at our nativity,” in our ability to propitiate the forces of nature by appropriate ceremonials—all such phenomena indicate a hasty and improper use of causal models,⁵ and there is no mental model which is not liable to such unmeaning or misleading usage. In themselves, then, such methods of synthesis are neither valid nor invalid. In respect of validity, the sole question which can be raised is, as to how we use them. To this question, as to the similar ques-

⁵ Cf. Herbert Spencer, *Data of Ethics*, chapter iv.

tion in respect of analytical methods, we can give only one answer. The sole test of such mental patterns is— Do they “work”? Do they actually help us towards acquiring insight into the principles in accordance with which the world in which we live seems to be constructed? Do they tend to diminish the margin of difference which separates the way in which we think of things as behaving from the way in which things are proved to behave, when considered more objectively? Do they, that is, lead towards a progressive insight into the workings of nature, and thus help us to understand and solve our concrete problems? If so, they are so far valid. If not, they are worthless, except as sources of amusement—intellectual games which at best do no harm, but also do not bring us into touch with the objective world.⁶

Summary.—So far, then, we have seen that synthesis is a preliminary portion of scientific method, and that it is of universal application. Its aim is, so to make use of the bonds of connection which hold data together, as to put together elements in a way which is both objective and complete. This aim can be fully realised when thought is dealing with its own constructions, and in such cases only. In dealing with natural phenomena, we can only approximate to such a synthesis, by experimenting with mental models until we find one which seems to apply. Of such models, the most universally helpful are the mathematical and causal types. These can be regarded as valid so far as they are found to “work”—*i. e.*, to help us to escape from the vagaries of subjective imaginings, and to get into closer touch with the objective world. By such methods the way is opened for a synthesis which shall be progressively satisfactory.

⁶ The reference here is to the “dialectical” method, by which one or more persons, by consulting their own thoughts, would try to spin out of their own heads a philosophy of nature. The method originated with Plato (Cf. the last half of the *Timaeus*), and is well developed in mediaeval science. But it is not unknown in the history of modern thought. The best known instances are Schelling’s *Naturphilosophie* and certain of the speculations of Hegel.

FOR FURTHER READING

W. Wundt, *Logik*, (3rd Edit.), Vol. II, pp. 8–11.

EXERCISES

Show what part is played by synthesis in dealing with the following cases: (1) In learning to use the typewriter. (2) If accidentally locked into a room, in getting out. (3) In finding the address of a friend the other side of town. (4) In estimating character in the case of strangers. (5) In laying out the plan for a small vegetable garden. (6) In keeping accounts.

CHAPTER XXI

ANALYSIS AND SYNTHESIS

The Problem.—So far we have discussed analysis and synthesis in separation from one another, as though they were two sharply distinct and independent methods, and as though each could be applied by itself. In fact, they have seemed not merely independent, but even opposed. Analysis means taking apart by loosening a bond of connection. Synthesis means putting together by introducing a bond of connection. And yet, there have been indications that perhaps our distinction was a little sharper in theory than is warranted by the actual use of scientific method in practise. It is doubtful whether any of the instances we have given can be regarded as pure cases of analysis or pure cases of synthesis. For example, in deciphering the message *AAABB* we certainly *constructed* a complete cipher as part of our process of analysis. So also, in attempting to put together the elements hour, for, we, early . . . , we gave up one attempted synthesis after another, on the ground that they omitted certain of the data. But that certain of the data were being omitted could be known only by our analysing the totality which we were attempting to introduce, and then comparing its elements with the given elements. That is to say, so far as our examples go, they seem to indicate a certain interconnection of analysis and synthesis.

Again, if we compare the procedure, rather than the examples, we find that analysis and synthesis agree in a number of essential points. Both proceed by the trial-and-error method, and by introducing mental models—models constructed after a pattern which we can understand, or take to pieces and put together again. That is to say, the mental models which we use in both cases as a necessary and integral part of the method are themselves products of both analysis and synthesis—so that the analytical method uses models which are partly synthetical, and the synthetical method uses models which are partly analytical.

Finally, both methods have fundamentally the same aim. In order to take apart, analysis must first discover, as nearly as may be, what it is which holds the elements together. In order to loosen the bond of connection, we must learn, if possible, what that bond is. So too with synthesis. In order to put together, we must have some unifying principle, some bond of connection, with which to go to work. Given elements which are separate, they will not fall together of themselves. Some organising principle must be introduced, and in order to put together, we must have insight into some such principle. The primary aim, then, of both methods appears to be identical. Whether we wish to analyse or to synthesise, in both cases the discovery of some unifying principle constitutes our primary aim.

If we now put this evidence together, and realise that (1) our examples seem to employ both methods, (2) the mental models employed in both procedures are both analytical and synthetical, and (3) the primary aim of both methods is identical, it seems more than doubtful whether the previous suggestion of the independence of the methods is not to be given up. It looks as though analysis and synthesis are thoroughly interdependent—as though they should be regarded, not as two methods, but rather as two correlative aspects of a single method of scientific investigation. The problem of the present chapter is to examine more closely into the relations of analysis and synthesis, in order to discover whether there are two methods, separate and independent, or whether there is only a single method with two correlative and interdependent aspects.

Is Analysis Synthetical? (A) With Mind-Made Entities.—Let us begin by taking the analytical method and enquiring how far it essentially involves the use of methods which can properly be called synthetical. Suppose we find it necessary to take the lawn-mower to pieces. The ordinary procedure is to look it over, and to examine it somewhat as follows:—“These bolts are held in place by those nuts. If I loosen one, then this roller will fall out. If I loosen both, then I can remove the cutting blade also. To remove these revolving blades it will be necessary also to take off the nuts which hold that bar in place, *etc., etc.*” That is to say, in our ordinary and natural procedure with machinery, we certainly look it over *in order to form an idea as to how it is put together.*

In this way we obtain an insight into its structure which can only be called synthetical. Even if we fail to look it over as a whole, and proceed merely by unscrewing every nut and screw in sight, and then by pulling out every bolt until the whole falls apart, we are using an insight which is synthetical. We act under the impression that it is screws and bolts which are holding the machine together—*i. e.*, that *it has been put together* by these means. In dealing with machinery, then, with a view to taking it to pieces, an essential part of our analytical method appears to consist in considering how the machine has been put together—*i. e.*, in considerations which are synthetical.

Let us proceed to consider a second group of entities constructed in accordance with a mental plan. We can analyse a triangle, for instance, by taking it apart into three straight lines. Is there any aspect of our method here which should be considered synthetical? It seems to resemble the machinery case just considered. We do not know how to go to work unless we understand the principle which holds the three angles together, unless, that is, we realise that three straight lines must meet so as to form angles in such a way that the whole is a closed figure. It is only when we understand what it is that *holds the triangle together* that we can take it apart in a way which is both objective and complete.

If the above cases may be regarded as typical, we can state that, in analysing structures put together in accordance with a plan devised by the mind of man,—*i. e.*, in cases where thought is dealing with itself—it is necessary for us to acquire insight into the principle of construction—a synthetical insight into the way in which such structures have been put together—in order to analyse in a way which shall be both objective and complete. That is to say, in such cases synthesis constitutes an integral portion of the analytical method.

(B) With Natural Phenomena.—In endeavoring to analyse natural phenomena, our method is, as we have seen, to try out one mental model after another, with a view to finding out whether the structure of such mental models in any adequate way corresponds to the structure of the natural object which we are investigating. That is to say, we make guesses at the plan of structure of our object—we try to think out how it might have been put together, and then see whether our guess was correct, by trying to take the natural phenomena apart

in accordance with what we assume to be its structural plan. For example, in trying to understand the structure of the organ of hearing, the most widely accepted analysis is that which regards the basilar membrane as modelled on the structure of the wires in a grand piano, and in fact an artificial model of just such a membrane has been made in accordance with this prescription. Other authorities try to analyse it along the lines of a telephone-like model.¹ So too in analysing out the mechanical elements which enable a plant such as the bulrush or corn to withstand the pressure of the wind, it is usual to analyse such cases in terms of trusses and other engineering models. That is to say, in all such cases, we guess at *the way in which the phenomenon in question may have been put together*, as a necessary preliminary to taking it apart. Our conclusion, then, is, that whether we are dealing with the structures of thought or with the phenomena of nature, our analytical method contains, as an integral portion of its procedure, the construction of a model which is taken to represent the way in which the object-to-be-analysed has been put together. Analysis necessarily involves synthesis.

Is Synthesis Analytical? (A) Mind-Made Entities.—Let us now proceed to study a little more closely the method of synthesis. We have before us as data a number of differently shaped pieces of wood, *plus* a frame—technically known as a “form-board.” Our problem is, to put these pieces of wood together in such a way that they will fit into the frame. We look over the pieces of wood, and then at the place into which they are to be fitted. We think:—“This piece could go here, these two pieces could be joined together so as to go there, and perhaps in this remaining space we could put the three or four remaining pieces.” That is to say, we compare the space to be filled with the elements which are to fill it. On the one hand, we see that “these pieces could fill that space,” and on the other, that “this remaining space could be filled by those three remaining pieces.” In other words, when we attend more to the pieces and how they might be put together, our procedure is more synthetical. But when we attend more to the space and ask by what sort and number of pieces it could be filled, our procedure is more analytical. We proceed in this double way, and there is always some occasion for comparing (1) what we want to do and (2) the means at our disposal. When we make such a comparison, we necessarily

¹ Cf. W. B. Pillsbury, *Fundamentals of Psychology*, pp. 169–170.

analyse what we want to do, and see how the elements yielded by this analysis compare with the materials which have been put at our disposal. "If this piece were only a little shorter, it could be fitted in." That is to say, in putting together mechanical apparatus, our synthetical method includes an analysis of the totality which we wish to construct.

So too in other cases of the same general type. In solving a problem by means of simultaneous equations, we start by stating our data in algebraical form. "Let x = this, and let y =that. . . ." But when we come to the work of synthesising our x 's and y 's so as to represent in equational form the conditions which form our data, there is no doubt that we proceed in the two-fold way which we have noted above. We keep one eye upon the data, and the other upon the equational form in which we are trying to express those data. In the whole process of trial and error which culminates in the requisite equations, we are suggesting one equational content after another and rejecting it, if its elements differ from the elements which form our data, until in the end we find one which seems to satisfy all the conditions. That is to say, we analyse each suggested equational content into its elements in order to compare them with our data and see how far they coincide. So also if we are called upon to invent a plan which will satisfy certain given conditions, and provide us with a complete solution to some examination problem in ethics or economics. We try one suggested plan after another, analysing it in order to discover whether the conditions which it will really satisfy are the same as the conditions given in the examination paper. In other words, here also our synthetical method includes an analysis of the totality which we wish to construct, and if these instances may be regarded as typical, we can state that in dealing with mind-made entities we always, as an integral portion of our synthetical method, analyse the totality which we are attempting to construct. In such cases, then, synthesis necessarily involves analysis.

(B) **With Natural Phenomena.**—Where thought is not confined to its own constructions, but is attempting to get into touch with natural phenomena, what is our synthetical procedure? We try, as we have already seen, to put together our data in terms of some mental model,—primarily of a mathematical or causal type—which is especially designed to fit the

concrete situation as nearly as possible. In order to discover whether it *does* reasonably fulfil the requirements of the situation, it is, of course, necessary to analyse the mental model, and compare the elements into which we dissect it, with the elements which constitute our concrete data. For example, given as data the various species of animals, and the problem being to put them together in such a way as to obtain insight into the principle of specification and thus make sense of the infinite variety of nature—how have men proceeded? The older theory, that each species had been especially created, resolved the whole question into “the inscrutable will of the Creator,” and thus, in effect, gave up the problem. The more modern attempts at a solution in terms of the evolutionist hypothesis very definitely consist in introducing some mental model of what evolution is, and what results it would produce, if analysed carefully. These results of theoretical analysis are then carefully compared with the empirical facts, and this is the regular scientific method used in dealing with scientific phenomena.² In other words, in dealing with either mind-made entities or the phenomena studied in natural science, our attempts at synthesis always contain, as an integral portion of their constructive nature, some degree of analysis of the totalities which we wish to construct. Synthesis always involves analysis.

Summary.—So far, our examination has borne out the suggestions with which we started. Analysis involves synthesis, and synthesis involves analysis. There is thus no sharp distinction to be drawn. There are not two independent methods, each of which can be applied singly and separately. Analysis and synthesis are thoroughly interdependent, and constitute a single method. There is a single analytic-synthetic method, whose function is to enable us to obtain insight into the world in which we live by means of taking apart and putting together again phenomena in which we are interested. It is by constructing experimental models and trying them out in relation to the actual phenomena, that our mental grasp of the world grows, and it is in this feature—the construction of mental models for purposes of experimentation—that we should look for the chief characteristic of the method. For this is what both analysis and synthesis have in common. (1) In respect of ultimate aim—to understand the world in which

² Cf. Huxley's lecture on *The Demonstrative Evidence of Evolution*, quoted in A. L. Jones, *Logic*, pp. 287–300.

we live,—(2) in respect of proximate aim—to grasp the particular principle of connection which holds together a system of elements or concrete situation in which we are interested,—and (3) in respect of method—the construction of mental models which can be taken apart and put together again—analysis and synthesis are indistinguishable, and are so far to be regarded as identical.

Differences Between Analysis and Synthesis.—And yet, we must be careful to avoid a too hasty conclusion. Perhaps the view which we are taking may turn out to be too one-sided. Analysis and synthesis can hardly be regarded as wholly identical. Analysis has seemed more preliminary, more rudimentary, less flexible, less able to take account of rational considerations. It is apparently tied down to the given material, and appears to be without power of discrimination, either in respect of omission of what is irrelevant, or in the way of addition if some further element is plainly demanded to complete a datum which is fragmentary. Synthesis, on the other hand, has seemed to be more a completing of the investigatory process, a summing up of the results of the inquiry, and a final reconstruction of the principles which have been studied. It appears largely to come *after* the work of abstraction and determination, and thus to be more discriminative than mere analysis. It can apparently omit what is not required, improve upon its data in the way of economy and efficiency, and even, to a large extent, supplement its data by completing curves, filling up gaps, extending applications. It seems of greater efficacy in the positive or more constructive work of organisation, and generally to be a more advanced and final representative of what we mean by scientific method.

Again, analysis appears to be easier than synthesis. More people can take a machine to pieces than can put it together again. Nearly every recruit can follow instructions so far as taking his rifle apart is concerned. But when it comes to putting it together again, a large percentage of the new men are driven to seek assistance from the sergeants. So too any boy can take a clock to pieces. But when it comes to putting it together again, it has to go back to the clock-maker, unless the boy has a quite exceptional mechanical ingenuity. So too many students can follow a lecture in class or read an assignment at home, and can take it apart well enough. But when it comes to putting together again what they thus

studied, they frequently come upon unsuspected difficulties. It looks, then, as though it must be one thing to take apart, and another and more difficult thing to put together.

(A) **Apparent Starting-Point.**—Let us consider, then, somewhat more closely the apparent differences between analysis and synthesis. In the first place they differ in respect of their apparent starting-point. The datum for analysis appears to be a complex totality, and the problem is, to take this to pieces. More exactly, the problem is, to find out what *are* the pieces. In the case of the cipher *AAABBB*, for instance, we might suppose the elements in question to be combinations of *A*, combinations of *B*, or even simply and ultimately *A* and *B*. But as these suppositions are false, we should in this way be analysing, not the *cipher*, which is our real datum, but certain aspects of its mechanism of expression. In fact, the cipher is expressed in a way which is intended to conceal its real elements from all but the initiated. It is not certain what *is* the datum before the analysis has proceeded some little distance. The real datum is a significant message expressed in a form intended to conceal its real nature and mislead the analytical investigator, and it is *this* datum which analysis has to take apart. It is the significance which holds the elements together, and it is only when our thought has penetrated some way into this significance—*i. e.*, only when we have apprehended what the datum *is*, that we can avoid being misled, and can pick out the genuine elements. For analysis, then, the *apparent* datum is a (rational) complex, the product of synthesis.

For synthesis, on the other hand, the apparent starting-point is a number of disconnected elements, which have already been analysed out, such as *N, O, T, H, S, A, G, W, I, N*, and the problem is, to put these together in such a way that they make sense. More exactly, the problem is, to reconstruct the original totality, to discover what was the rational principle which united into a connected and meaningful form, these elements which are apparently given as disconnected. Here again, however, there is a difference between the apparent and the real datum. It is not at first apparent what the genuine datum *is*. It is not just a group of letters, but it is ten very definite letters which are given as elements which together constitute a totality, *i. e.*, as elements which can be rationally connected so as to spell out a single name, elements

which are significant, and it is only so far as our thought penetrates into their significance—the meaningful principle which connects them even in this apparently disconnected form—that we can escape misleading associations, and can hope to discover the totality in question. For synthesis, then, the *apparent* datum is a (rational) group of elements, the product of analysis.

Thus we see, that while both analysis and synthesis are dealing with a datum which in an ultimate sense is one and the same—*viz.*, a significant totality whose significance is partially concealed beneath its outward form—yet this ultimate datum is *given* in two different-appearing aspects. For analysis, the aspect which is prominent is the totality or complex unity. For synthesis, the aspect which is prominent is the scattered, severed, plural, disconnected form of the data. Thus we see that, while both methods are ultimately dealing with one and the same reality, the aspects of this datum which are most apparent, differ for analysis and synthesis.

(B) **Apparent Conclusion.**—In the second place, the conclusion to which the use of these methods leads, seems to differ according as we emphasise the analytical or the synthetical aspect. Analysis starts with a totality and concludes with a disconnected group of elements, with elements whose plurality and distinct nature is sharply emphasised. And yet, is this the *whole* conclusion? It is certainly the apparent conclusion, but, if we examine further, we find that we think of our elements not entirely as *disconnected*, but partly also as *inter-connected* in the totality. In analysing an argument, for instance, we tend to group its different steps as (1), (2), (3), . . . , thus emphasising, not merely their distinct character, but also the fact that they are each and all steps in one and the same argument. That is to say, the elements with which we conclude are not just elements-in-general, but definite elements, elements of the particular complex which furnished our starting-point. The genuine conclusion, then, is not so much a group of elements as the real nature of our datum. Our conclusion is an analysed complex—a totality broken up into its elements and held over against those elements by the mind, in the general form $x = (a + b + c \dots)$. In the same way the real conclusion of our analysis of the bundle is *not* a number of sticks, but the very same sticks which constituted the bundle, *plus* the now untied rope by

which they had been held together. There is, then, a sharp distinction between the genuine conclusion and the conclusion which is apparent. The apparent conclusion of analysis is a number of disconnected elements.

What is the conclusion of the operation of synthesis? We start with elements and we end—at least apparently—with a totality. At least it is the unity, significance, and complex individuality of our conclusion which are emphasised. With an old pack of cards, if we use a careful method of inter-weaving, we can create a card-house which will not fall apart, and which will be the admiration of children. The whole emphasis is on the unity, strength, and consistency of the totality. The *whole* emphasis? Let us examine further. Is not the emphasis always an affair of contrasts? Surely it is as contrasted with such beginnings that we admire the conclusion. Out of such elements to constitute such a totality—that is rather the idea in our minds as we contemplate the result. The genuine conclusion, then, as not the bare result, but rather the resultant-of-a-method-applied-to-these-definite-data. Our conclusion is no mere x , but an x which has been constructed out of $(a+b+c \dots)$. That is to say, while superficially it might seem as though the conclusion of our synthetic process was the bare totality, the genuine conclusion is the totality-as-constructed-out-of-the-data. Expressed in a general formula, it is $(a+b+c \dots) = x$.

Thus we see that the genuine conclusion at which both analysis and synthesis arrive is the same. Both are equally dealing with an organic unity, a totality or individuality which has two aspects. We can regard it as a one-in-many, as a unity which has also an aspect of plurality, as a totality which is composed of elements. Or on the other hand we can regard it as a many-in-one, as a plurality which has also an aspect of unity, as elements which together make up a totality. We can regard it either in the form $x = (a + b + c \dots)$, or in the form $(a+b+c \dots) = x$. The reality underlying both aspects is one and the same, and it is with this reality that our scientific method is trying to bring us in touch. But analysis aims more at expressing this reality in the one form, and synthesis at expressing it in the other. So far as we are dealing with conclusions which thus *appear* different, we can distinguish analysis from synthesis. But so far as these are aspects of an underlying reality which is

one and the same, analysis and synthesis do not differ in any fundamental logical respect.

(C) **Apparent Method.**—In the second place, the method we use seems to differ according as we emphasise its analytical or its synthetical aspect. So far as we are conscious of our method, we believe that in analysis we are taking to pieces. The recruit is apparently taking his rifle apart—and to a mind which is satisfied with appearances, with aspects which are prominent, that is *all* that he is doing. And yet, if we look a little further, it is not difficult to realise that he is doing more than *that*. The whole point of teaching a recruit to take his rifle apart is, not merely that he shall clean his weapon, but that he shall acquire an insight into its mechanism which he could acquire in no other way. He takes it apart in order to see how it is put together, and how it works. The method has, in fact, synthetical as well as analytical aspects. The true method of analysis, then, is analysis *plus* synthesis, usually in terms of some mental model which has both aspects. The apparent method, however, may be regarded as merely analytical, merely taking to pieces.

So too in synthesis. So far as we are conscious of our method, we believe that in synthesis we are putting together and constructing. The chief stress is on the positive, building-up characteristic of the method. So far as appearances go, we are putting together the pieces of wood so that they will fit into the space in the form-board, and if we are satisfied with the appearances which are prominent, this—the putting together—seems to be all. And yet, if we look a little more closely, it is not difficult to realise that at the same time as we are putting together the pieces of wood, we are *analysing* the space which we expect to fill with them. The true method is both synthetical and analytical. We analyse the totality which we wish to construct, as well as synthesising the means at our disposal for purposes of construction. The apparent method, however, may be treated as merely synthetical, merely putting together.

Thus we see that, so far as the real method is concerned, it is the same, whether we call it analytical or synthetical. In both cases the method consists in the analysis of a totality into its elements and the synthesis of elements into their totality. The logical relation involves both the relation of parts to their whole, and of a whole to its parts, and this

relation is fundamentally the same for both analysis and synthesis. The recruit who puts his rifle together is at the same time learning to take it apart, just as in taking it apart he is at the same time learning how to put it together. In both cases, what he is studying is the same—the relation of the whole rifle to its parts, and of the parts to the whole rifle. He is learning how the rifle works, and the analytical and synthetical methods are two ways of doing the same thing. So far, then, they are not to be distinguished. There is only one method. But it has two aspects, either of which can be made prominent. According as the one or the other aspect is the more prominent, we can distinguish the taking-apart aspect from the putting together aspect. In respect, then, of the method apparently employed, we can distinguish analysis from synthesis.

Is Synthesis a More Advanced Method?—In the light of these considerations, we should note that it is only when speaking of the “apparent” methods, that we can regard synthesis as more advanced and analysis as more preliminary. For example, analysis is not really confined to its data, but in virtue of the synthesis inherent in its method, can omit or supplement, as far as is desirable. Thus, in analysing the message “The supply of game for London is steadily going up,” if we wish to discover the elements which are really elements of the concealed message, nothing whatever can be done with “supply,” “of,” “for,” “London,” *etc.*. These are just words added in order to mislead, and form no part of the genuine message. It is necessary to construct a mental model for message-sending which shall emphasise every third word only, filling up the intervals with words which seem to make sense but really conceal the meaning which is intended. This mental model, which is constructed by synthesis, is however the principle of the cipher, and thus an analysis which really analyses *the cipher*—the genuine datum—will be an analysis which omits such words as “supply,” “of,” *etc.*, and picks out the first, fourth, seventh, *etc.*, words as genuine elements. Analysis is thus not restricted to its *apparent* datum, but can transcend this either by way of omission, or by way of completion, if some element is clearly demanded by the context which our synthetic mental model gives us. At the same time, analysis is restricted to the *true* datum. But then, the true datum is the same for synthesis

also. Synthesis transcends only the apparent datum, and is rigidly bound by the true datum. Thus, in assembling an automobile from a miscellaneous collection of elements, only those are selected which really belong together, which can be utilised in assembling a single machine. The others are rejected, as forming no part of the true totality. In this way, then, we see that synthesis is not more advanced than analysis. Both aspects of the analytic-synthetic method stand on the same level.

Summary.—So far, then, we realise that from a standpoint which is satisfied with appearance, analysis and synthesis can be sharply distinguished in respect of starting-point, conclusion, and method. But if we persist in looking beneath appearances, we must reinforce our earlier conclusions, and believe that there is only one method. Our final conclusion, then, is that strictly speaking there is only one fundamental method, the analytic-synthetic method, but that this method has two interdependent and correlative aspects, of which the one consists in taking apart, and the other in putting together, and that for practical purposes it may be advisable to emphasise now one, now the other, of these aspects. For strict logical theory, however, it is essential to realise that, whether we wish to analyse or to put together, we are employing a single fundamental method, which aims at understanding the world in terms of the part-whole relation, the mental model which gives us the structure of organisation. Analysis and synthesis are, then, the two correlative aspects of a method which endeavors to understand the world in which we live as a rationally organised whole.

FOR FURTHER READING

F. H. Bradley, *Principles of Logic*, Bk. III, Part I, chapter vi.
H. Lotze, *Logic*, Bk. III. Introduction, pp. 406–409.

EXERCISES

Show the inter-relation of analysis and synthesis in dealing with the following cases: (1) In learning to sing or to play on a musical instrument. (2) In choosing a new suit. (3) In escaping from the top floor of a building in case of fire. (4) In considering whether to insure one's life or not. (5) In translating from a foreign language. (6) In estimating one's probable expenditure for the next month or year.

CHAPTER XXII

ABSTRACTION

"One thing at a time" is the motto of abstraction. By its aid we can consider an acquaintance from the sole standpoint of personal appearance, or commercial standing, or political leanings, or church membership. We can select friends on the basis of golf-playing, to the total neglect of other characteristics, or of interest in music, art, public service, or what not. In every day life, its use is sometimes helpful, sometimes harmful. But for purposes of scientific investigation its importance is vital. Unless we could deal with one aspect at one time, any subject at all complex would be hopelessly beyond our understanding. But when we divide it up, and consider a single aspect at a time, something can be done. It is by the aid of abstraction that the division of labor is possible, one worker tackling one part of a task, and another concentrating his energies upon another part, and it is by this division of labor that complex and highly developed results are possible, in commercial life or in science. Abstraction is thus one of the watchwords of scientific management. It is one of the chief elements in organisation.

Nature of Abstraction.—Abstraction is no preliminary method. It presupposes both analysis and synthesis, and may be regarded as a further development of the analytic-synthetic method. Its starting-point is no concrete situation given in nature, but a situation which has already been analysed out into a number of different factors. Let the starting-point be symbolised by $x = (a + b + c)$. Abstraction proceeds by taking out of its context any one of these elements, a or b or c , and deliberately neglecting the others. Abstraction means *taking out of its context*. It is thus a highly artificial process and leads to a highly artificial result. When we say that a statement is "abstract," we usually mean that it is artificial and cut off from the actual flow of experience as experience is given to us.

Abstraction, then, is the exclusive concentration of atten-

tion upon some one element of a complex whole. Its nature is thus negative as well as positive. Negatively regarded, it excludes rigidly every suggestion of natural context. The statistician considers people merely as so many numbers in the "population." The politician considers them merely as so many potential votes for or against himself or his party. Both exclude from consideration the rich professional and domestic interests and activities of the individuals in question. So too a salesman's chief business is to "get orders," an artist's chief business is to paint pictures, *etc.* Abstraction thus excludes breadth of vision and confines itself to a rigidly narrow outlook. On the other hand, however, this narrowness has certain positive merits. The outlook of abstraction is intensive, and makes for efficiency. It not only takes away, but it gives something new. When we abstract, we do not merely cut off an element from its context. It is, of course, cut off, but the mind proceeds to fix it, to give it a certain self-subsistence, to give it an independent nature and unity of its own—which, amongst other things, can be named. "Whiteness," "softness," "depth," "ticklishness," "individuality," *etc.*, are abstract qualities, qualities taken out of their original contexts and endowed with a certain kind of being peculiar to themselves. The entities to which our scientific concepts and laws in the first instance apply, are one and all of this abstract and artificial, half mind-made kind. They are taken out of their natural contexts in experience and are endowed with a new nature and unity. The nature of abstraction, then, is to take out of its context a single element, or group of elements, and to endow this entity with an artificial kind of self-subsistent being.

Aim of Abstraction: (A) Objectivity.—What precisely is our aim or intention in abstracting aspects or elements from their contexts? In the first place, we aim at objectivity. The whole point of abstracting is to bring ourselves into still closer contact with objective reality than is possible by the use of analysis and synthesis alone. It is because we are helpless in the face of nature taken as a whole, that we endeavor to proceed piecemeal, taking one small problem at a time and breaking up our analysed totality into elements which we can examine separately, and thus proceed gradually to understand the whole. If we wish, for instance, to study

memory, we immediately discover that the subject is far too wide to be grasped all at once. Accordingly, we narrow it down, first to mechanical or rote memory, then still further, until finally we begin to direct our experimental attention upon some such problem as "How many repetitions does it take to learn a series of twelve nonsense-syllables"? or "What part does such and such a rhythm play in helping us to learn such a series"? or "Does learning a series in the forward direction help or hinder learning it in the reverse direction"? It is only after solving an enormous number of such narrow and special problems that we can begin to approach the more general questions connected with remembering. That is to say, abstraction brings us into closer and more intimate contact with the subject which we wish to study. So too if we wish to acquire mastery over some musical instrument, we can, it is true, attain to a certain level of general accomplishment by just playing anything and everything in which we are interested. But there comes a time when we have to choose between remaining at this low level, and giving up our interesting show-pieces in favor of technical studies, each of which is designed to assist us in acquiring some small and special point of execution. We practise a school of *velocity*, an art of *finger-dexterity*, a volume of *octave-studies*, etc., and this concentration of attention upon one point at a time helps us to work our way into *the subject itself*, to become acquainted with *its* difficulties and *its* peculiarities. But for the fact that our scientific insight and artistic execution sensibly improve by the use of this method, we should never abstract and substitute a study which is dull and dry for the living reality. If it were not that we actually feel our way better and more surely into the subject studied by this method, we should never apply ourselves to mathematics, grammar, or logic, but should be like the young Deities of romance, knowing all things by the sheer might of our minds. Our primary aim, then, in abstracting, is objectivity.

(B) **Completeness.**—A secondary aim, which is always before us when we abstract, is a certain kind of completeness. At first sight it might seem as though abstraction explicitly avoided completeness, because it aims at fragmentariness, at producing incompleteness, at isolating some element and separating it from the context which completes it. This is, indeed, true, but is not quite what is meant. It is

at the complete isolation of this element that abstraction aims. Negatively, the context or contexts in which such an element is found must be stripped off and excluded from consideration, and this is to be carried through completely. Positively, the element which is thus isolated is to be fully represented in its own nature, in complete independence of its original context or contexts. The element thus abstracted is to be understood and studied in its complete nature, so far as this is possible. This means in practise, that the element in question must be abstracted from a variety of different contexts, in order that we may escape from one-sidedness and may take a complete view of the element which we are abstracting. Thus, in studying the question as to whether Yellow is "simple" like red, green, and blue, or "composite" like purple, it is usual to experiment with sun-light, with gas-light, with electric light, *etc.*—*i. e.*, with a Yellow drawn from as great a variety of sources as possible. So too in studying in what sense an author uses a particular expression—*e. g.*, what Aristotle understands by "nature," or Kant by "consciousness-in-general"—it is usual to put together all the instances, taken from a wide range of contexts, in order to avoid one-sided errors and to attain to a complete view.

How Far Realisable? (A) With Mind-Made Entities.—How far can this aim be realised? That is to say, how far can we, while completely singling out and separating from its context some aspect of a larger whole, attain to objectivity? Let us see. In dealing with a complicated electrical machine which refuses to work, we try to discover what is wrong by the method of abstraction. We single out for special testing one contact after another, until we find where it is that the current is broken. Each case of singling out is here a single abstraction, and our method is certainly objective. For it deals with special elements which analysis discovers in the object itself. The contacts are parts of the machine, and they are the parts which it is important to investigate. Whether our abstraction is complete or not, depends upon the way in which we deal with each individual contact. It is certainly possible to isolate it from the rest of the machine and to test it completely with full reference to its objective nature. If this case may be regarded as typical, then, we can state that in dealing with machines our abstraction can be both objective and complete.

Let us consider another example. In looking over a system of shorthand, or in estimating the claims of some new artificial language, it is possible to concentrate attention exclusively upon some particular feature—*e. g.*, with a view to improving and simplifying it. In all such cases, *i. e.*, where thought is dealing with its own constructions, our abstraction can be objective—for it is directed upon some particular sign in shorthand, or upon some particular rule in the artificial language, *etc.* Whether it is complete or not, appears also to be in our power. It is merely a question of studying our subject carefully, avoiding misleading associations and concentrating our attention upon what is before us. If these instances are typical, we can say that, in dealing with mind-made entities, our abstraction can realise its aim in point of objectivity and completeness.

(B) With Natural Phenomena.—As has been already stated, abstraction is no preliminary method, but starts with a datum which has already been analysed, and proceeds to concentrate attention upon some one of the elements revealed by analysis. It is therefore with a mental model, rather than with the natural phenomenon itself, that we are more immediately concerned in abstraction. But even here, the chief value of the method is that it brings us into closer connection with the objective facts than would be possible without its aid. Let us consider an example. J. Chr. Wolff's *Psychologia Empirica* is a product of analysis and synthesis. If, however, we wish to test its objective significance, we concentrate attention upon some special "proposition" or group of "propositions." Let us take one. Wolff asserts that the laws of waking life are diametrically opposed to the laws of dream-life, so that, whereas waking life is rational, deliberate, *etc.*, the dream-life is irrational, and does not admit of deliberation, choice, *etc.* We test this case by bringing special experiments to bear. We test the laws of association—they seem to be fundamentally the same in both cases. We make experiments upon reasoning. These also give similar results, for many dreamers solve mathematical problems. We test deliberation and choice. Many dreamers deliberate, and many dreamers exercise choice and volition. After making a large number of experiments of this general type, we find ourselves much closer to the objective facts than we were by reading and accepting the system of Wolff. In dealing with natural

phenomena, it is precisely by such specialised observations and experiments that we bring ourselves into closer relation to the objective facts. That we can attain to complete objectivity in dealing with such facts, is not claimed. But that we can improve our objective grasp upon the workings of nature, and can improve it indefinitely—*i. e.*, progressively and without limit—by the help of abstraction, there can be no doubt.

Completeness in any final sense, when we are dealing with natural phenomena, is perhaps out of the question. We collect as great a variety of instances of the aspect of our phenomenon under study, as is possible for us, and endeavor, by comparing these with one another, to escape the danger of one-sidedness and to make our abstraction complete. But that we can do more than approximate to this aim, appears improbable. For example, in psychology, we try to experiment upon attention, or memory or reasoning, and upon some special point in one of these fields, to the complete exclusion of every point other than the one we wish to study. This complete exclusion cannot however, as a rule be accomplished, and thus we have to content ourselves with taking a number of instances of our phenomenon from different angles of approach, and, by putting these together, to eliminate the elements peculiar to each case and thus obtain something like a concentrated, intensive view of the common elements present in each case—elements which are more directly connected with the problem under study. By studying the phenomenon now in this context, now in that, we are enabled to loosen somewhat its connections with any particular context, and gradually to frame an idea of it apart from the contexts in which we have experienced it. This is an indirect method of abstraction, and is found helpful in direct proportion to the variety of the instances compared. But a complete abstraction is hardly attainable by such means. However, by concentrating our attention, as far as possible, upon a single aspect at a time—as in some of our more specialised intelligence tests—it is possible to improve our methods of experimentation, and gradually to revise the generally adopted mental models with which work is done in such fields. Abstraction, then, though not perfectly complete, is a method which leads to progressive improvement in dealing with natural phenomena, in respect of objectivity.

Types of Abstraction (A) Isolation.—So far we have seen

that abstraction is a method of separating from its context some aspect of a phenomenon in which we are interested, and it might seem as though abstraction, as such, *must* be a method of isolation. It is usual, however, to distinguish two main types, (1) isolation, and (2) generalisation. When a machine such as a telephone refuses to work, the repairer takes one fuse at a time, and proceeds from one contact to the next, testing each one until he discovers the cause of the difficulty in question. That is to say, he proceeds very definitely by a method of isolation. He takes each contact by itself, and tests it to see whether it is in working order, without any reference to the rest of the mechanism. That is an example of the method of isolating abstraction. So too, if in trying to play a difficult study a music student finds there is some point of technique which is beyond his immediate powers, he practises that particular point of technique—*e. g.*, the trill or mordent—by means of specialised studies until he has mastered it. It is by specialisation, or by isolating abstraction, that he overcomes his difficulties. This instance differs slightly from the telephone case. In dealing with machinery, we isolate a single instance, and are satisfied with that. In practising so as to learn to execute the trill, we specialise on that general kind of finger-movement, and practise as great a variety of trill-studies as possible, so as to be able to play the trill in whatever kind of context we may happen to meet it. That is to say, in this type of case we isolate by taking as many varieties of the phenomenon under study as possible. There is, however, no difference in principle between the two types of instance. All isolation is with a view to specialised study, and both varieties of isolation illustrate the principle of abstraction.

(B) **Generalisation.**—Generalisation is not, in principle, different from isolating abstraction. But from a superficial standpoint it certainly *appears* different. For instance, it seems to lead to a different kind of result. Most empirical *laws*, *e. g.*, are generalisations from a number of experiences. We find that graduate students in psychology can, as a general rule, learn a series of twelve nonsense-syllables in from seven to nine repetitions. We put together the results of several distinct observations based upon certain further experiments of this general type, and gradually construct a memory curve, a learning curve, a practise curve, *etc.* Each

of these is a generalisation on the basis of repeated experience. Thus we see that not only the practical wisdom which accumulated experience of men and things brings with it, but also most of the concepts and laws of empirical science, are products of generalising abstraction. We arrive at such generalisations by isolating, so far as we can, instances of the special phenomenon under study, in order to eliminate what is irrelevant to our purposes, and thus from a variety of instances extract the composite result which we call a generalisation. Like the more complex cases of isolation, generalisations are thus products of an abstraction which works by isolating special aspects of phenomena in which we are interested.

Validity of These Methods.—There is nothing sacrosanct about abstraction in any of its forms. Isolation always has its dangers. In separating some particular aspect from its context, it is peculiarly easy to omit something vital or to add something which is irrelevant. Too much attention to detail, *e. g.*, in literature, prevents our appreciating the whole, and it is often that we fail to see the wood because we direct our attention too exclusively upon particular trees. So too generalisations are often hasty and erroneous, the products, as we say, of a "vicious" abstraction. Thus, in what is known as "Faculty" psychology—which represents a general tendency, rather than any definite school—we regard our minds as consisting somehow of a number of distinct "faculties," such as Memory, Reasoning, Will, Emotion, Sensation, *etc.*¹ That is to say, we come to regard what *we* have taken apart into elements as though *in itself* it consisted of a number of distinct elements. Such "hypostatisation," as it is called, is the peculiar danger of abstraction, and is a common source of error in all the mental sciences.

Abstraction, then, is sometimes invalid. On what does its validity, when it is valid, depend? It is valid only so far as it leads to progress in obtaining insight into the workings of the phenomena in which we are interested. So far as it leads to a definite advance in scientific knowledge, it is valid. But it must always be remembered that what *we* separate for purposes of special experimentation, is *given* together

¹ Cf. Locke's *Essay on Human Understanding*, Bk. II, chapter xxi, sects. 17–20, and G. F. Stout, *Manual of Psychology*, 1899, Bk. I, chapter iii.

with various other elements, and that it may be fatal to regard as separate *in itself* what we have found useful for our special purposes to regard as *so far* separate and distinct. In dealing with natural phenomena especially, abstraction is very seldom *fully* valid, but may be regarded as justifiable so far as it leads to greater knowledge of the objective facts studied.

Summary.—In all its typical forms, abstraction is a method of taking, from the context in which it is embedded, some aspect of a phenomenon in which we are interested. Negatively, the context is eliminated and stripped off, so far as this is possible. Positively, the special aspect studied is given a certain unity and self-subsistency by the mind, which not only cuts it off from its context, but fixes and names it.² Although the action of our mind is to some extent arbitrary—for we select for study any aspect whatever in which *we* happen to feel interested—we yet aim at objectivity, and endeavor to make our abstraction complete. In dealing with mind-made entities, this aim can be realised. In dealing with natural phenomena, it can be realised only approximately. In all cases, however, the method of taking one thing at a time leads, or tends to lead, towards attaining to greater insight into the workings of the phenomenon under study, and abstraction is thus, as a scientific method, progressive and fruitful in results.

² Hence the danger of hypostatisation.

FOR FURTHER READING

B. Bosanquet, *Logic*, Vol. II, pp. 21–24. B. Erdmann, *Logik*, (2nd Edit.), pp. 65–78, 88–92. Chr. Sigwart, *Logic*, Vol. I, pp. 248–259; Vol. II, pp. 66–69. W. Wundt, *Logik*, (3rd Edit.), pp. 11–17.

EXERCISES

Show what part is played by abstraction in dealing with the following cases: (1) In finding out what has caused a headache. (2) In learning a new dance. (3) In translating from a foreign language. (4) In correcting papers in a high school subject. (5) In selecting a dinner from a menu-card. (6) In choosing a career.

CHAPTER XXIII

DETERMINATION

Nature of Determination.—Determination is the reverse of abstraction. Abstraction takes a single element out of its context and isolates it as strictly as may be, treating it by itself. Determination takes an isolated element out of its isolation and places it, as far as possible, in a context to which it is adapted. In rare cases determination exactly reverses an abstraction, and replaces an element in the precise context from which abstraction had taken it. But as a general rule, starting with an element which has been thoroughly loosed from its original context, we proceed to determine it by placing it in a variety of contexts which happen to suit our purposes, whether the element in question has previously formed part of such contexts or not. For instance, starting with a single muscle, which abstraction has isolated from its place in *e. g.*, the frog, we proceed to determine the nature of muscular action by placing it upon an electrical machine and galvanising it into activity until it is thoroughly fatigued. In this case we place it in a context in which it has never formed a part, but which throws considerable light upon its workings. So too in studying memory. We start with abstract entities such as nonsense syllables, and by learning these in various types of context of which it is safe to say they never naturally formed part, we proceed to determine our memory of them in a way which has led to a number of remarkable discoveries as to the workings of memory.

Results of this type are, however, in themselves still somewhat abstract; and determination proceeds to make them more concrete by placing them in contexts which more nearly approach the conditions of actual life. Thus, we consider how muscular action becomes modified which we have, not an isolated muscle functioning by itself, but when we have several muscles working together in the organism in a whole. Our earlier results with the electrical machine become considerably modified. We learn, for instance, that in the living

organism no muscle ever becomes completely fatigued, and that fatigue seems to be a matter of attention rather than of the muscles as such. So too the nonsense-syllable results are not, as such, immediately applicable to material which has meaning for us, and become determined in a variety of ways as we experiment with more concrete contexts. Determination is thus a process of rendering ever more concrete the results produced by the method of abstraction.

Negatively, then, determination deprives a given element of its splendid isolation, and in so doing usually deprives it of part of its clearness and simplicity. But positively, by fitting it into a context which is adapted to it, determination usually supplies the element in question with accretions of meaning, and the new context may make an enormous difference to the significance of such an element. For instance, a system of ethics constructed in the abstract world of pure concepts may be very clear in itself, and very explicit as to our duty and our highest good. We must, we are told, aim at so acting as to treat others, not as means, but as ends in themselves. Another similar precept is to act from pure reverence for the moral law, or to aim at loyalty to loyalty itself. In the abstract conceptual realm—*i. e.*, in what Kant calls the *metaphysic* of ethics—these ideas may be extremely clear. But when we attempt to apply them in more concrete cases, *i. e.*, to determine them in reference to a special context, they tend to lose much of their abstract clearness, and to become mixed up with all kinds of interests, feelings, and instinctive desires. Every motive-complex into which they enter endows them with new shades of meaning until, in many cases, it is almost impossible to discover a trace of the original ethical motive.

So far, we have been considering determination purely from the side of the abstract element which we seek to determine. But we must also consider it from the side of the context into which we seek to introduce the element in question. This context, in order to admit of the addition of such an element, must itself be incomplete and abstract. If the element is made more concrete by being placed in the context, the context must be made more concrete by the addition of the element in question. Determination is two-sided and reciprocal. If x is determined in relation to $a, b, c \dots$, $a, b, c \dots$, are determined in relation to x . Thus,

in the case mentioned above, if the ethical motive is altered by its union with the psychological motives, psychological motives are often transformed into something higher by union with ethical motives—*e. g.*, in the case of love.¹ In fact, such a “context” may itself consist of not more than one element, and may be as isolated and abstract as the element with which we start. In such cases it is easy to see that both become more concrete in their inter-relation, and that they determine each other. For example, the concept of “Becoming” arises, according to Hegel, as a reciprocal determination of the two abstract concepts of “Being” and “Not-being.” Becoming, that is to say, means the passage from one to the other, and is the only way in which such abstract concepts can be unified and given concrete meaning.

One further point remains to be considered. We have spoken so far as though only “elements” were capable of being determined. But we saw in the case of analysis that it was possible to isolate not only an element but also an aspect of a concrete situation. So too with determination. We can determine, not only elements, but also aspects. We may, for instance, start with a general aspect or view of a person’s character. This is abstract, for it is divorced from knowledge of his ways of acting in specific cases. We may, for example, have heard in a general sort of way, that “X is a good kind of fellow.” If now we are brought into much contact with X, it is possible to determine further this general, vague, and abstract outline of his character, and fill it out with a wealth of concrete detail. We note his behavior in matters regarding himself and his self-interest. We note his behavior in matters regarding others, in public as well as in private—how he meets his obligations, how he goes beyond his strict obligations and likes to do something for the other fellow. In each case we acquire a more definite and detailed knowledge of his character, or—to express it otherwise—the vague indeterminate concept “good kind of fellow” becomes thoroughly determinate and clear-cut. After a year’s close association with X, our idea of his character has become very definite and specific.

Determination, then, consists in the synthesis of two or more relatively abstract elements or aspects, in the bringing these in relation to one another in such a way that, in virtue of this relation, each takes on new qualities or characteristics,

¹ Cf. Dewey and Tufts, *Ethics*, pp. 578 ff.

and becomes more specific and concrete—as (1) the moral law becomes concrete by applying it to a special case of choice, and the choice itself becomes ethical by our reference to the moral law, or (2) as our insight into *X*'s character becomes more concrete by studying his behavior in special circumstances, and our understanding of his behavior in special circumstances is made more profound because of our general insight into his character as a whole.

Aim of Determination (A) Objectivity.—We determine, then, by bringing *x* in relation to *a*, *b*, *c* . . . and noting carefully the resulting modifications. In so proceeding we aim, in the first place, at objectivity. It is because we find determination an indispensable method in dealing with the objective world, that we make use of it. Abstractions may be correct, but we tend to find them unsatisfying. They seem to belong to the twilight world of mere theory, whereas we seek the golden tree of life. How many a student of abstractions has felt, like Faust, that he is getting out of touch with vital things, and must plunge into the concrete in order to recover his mental and moral balance? Mental models are of value only so far as they serve to bring us into closer and more accurate contact with the objective world. We theorise in order to practise, and abstract in order to determine; and what we wish to determine is always what we take to be what is most real in our experiences. We wish to understand reality, not only in principle, but also in detail, in the concrete. And the first aim of determination is to ensure that the concrete detail which our methods bring to light shall be objective, and shall bring us into yet closer intimacy with nature and ourselves.

Starting, then, as we do, with elements or aspects which are the products of a partially objective analysis and abstraction, we hope by comparing these one with another, by turning them over and over, inspecting them from all sides and in all kinds of relations, to arrive at a still more objective understanding of the world in which we live.

(B) Completeness.—In the second place, we aim at completeness, not perhaps in any absolute or final sense, but at least at the greatest degree of completeness which can be attained. We are never satisfied with acquiring one or two new determinations for our concepts. We wish our determination to be as complete, as many-sided, and as methodical

as is possible. The complete and final determination of the moral law, for instance, or of a person's character, seems out of the question for any understanding short of Omniscience. But we try to fit the rule to the case as methodically and as completely as the nature of the case allows, and by varying our viewpoint as much as possible, to take into consideration a wide variety of possibilities.

How Far Realisable? (A) With Mind-Made Entities.—The aim, then, of determination is, by bringing x in relation to $a, b, c \dots$, to discover new and concrete modifications of x —*and also of $a, b, c \dots$* —which are of objective significance, and to make our list of such modifications as complete as possible. How far can this aim be realised? Let us first consider cases from the world of mind-made constructions. An engineer who is interested in inventing an air-plane motor, for example, has a fair general idea of what conditions are to be met, and of what type of engine is capable of meeting such conditions. But his idea is still in part indeterminate. He accordingly studies a number of models in actual use, $a, b, c \dots$, and by comparing closely his ideas with a , perceives both excellencies and defects in that motor, in such a way that his general idea becomes yet further modified, both positively and negatively. He next studies b , and obtains yet further notions from that study. At the end of his study of actual models, the model he has been shaping in his mind has become very sharply determined, both negatively and positively, and he is now prepared to make and test a new model of his own. The determinations which his idea has undergone are of some objective significance—for they have been made in relation to actual working models, and with express reference to the conditions which should be satisfied by an ideal model. At the same time, if his comparison in the case of each individual model has been exhaustive, it is possible for each determination to be reasonably complete, and if, further, the number of motor-types in actual existence is small, it is possible for the determination as a whole to be reasonably complete. Absolute finality in point of objectivity and completeness is hardly to be expected, as the history of invention shows—for there is always room for yet further improvements. But in general, in dealing with machines it is possible for determination to assist in producing marked improvement over (1) one's own previous ideas or mental

models, and (2) the actualised mental models of other inventors. It is, in short, possible to approach a solution of the objective problems rather better than has been accomplished by any invention up to date. That is to say, in all such cases determination can be both objective and reasonably complete.

Let us consider another case. We all have a certain vague general idea as to the nature of the beautiful. This general idea, which is indeterminate and abstract,—*i. e.*, not in close contact with the world of actual art-creations—we can make more determinate and concrete, by bringing it in relation to actual concrete attempts to realise the idea of the beautiful. We can study pictures, poems, music, *etc.* That is to say, we can determine our general idea by bringing it into close contact with actual art-works and asking (1) how our ideal of beauty applies to such cases, and (2) how far such works satisfy our ideal. In this way we gradually educate our artistic sense until, from only knowing in more extreme examples what we like and what we do not like, we have developed a thoroughly concrete and determinate artistic appreciation and artistic theory. Such a determinate theory is certainly (1) more objective than the general idea with which we started—for it has been developed by the closest possible observation of artistic models, and has become modified by taking account of the various theories which have enjoyed a currency in the history of art, so that it is no longer a merely subjective imagining which has fed only upon itself—and (2) may be reasonably complete, if we have been careful and exhaustive in our study of each separate art-work—though, taken as a whole, it can never be fully complete, for the number of actual models is very great, while life is short. But a certain well-roundedness of artistic insight can be attained, just as a certain degree of completeness in respect of education in general can be attained. On the whole, then, we can state that, in the case of mind-made entities, determination can be both objective and reasonably complete. That is to say, it can bring us into closer and more complete contact with objective conditions and objective facts than is possible apart from such determination.

(B) **With Natural Phenomena.**—In dealing with phenomena other than mind-made entities, what is our procedure? In the cases from biology and psychology already mentioned, we noticed that an abstract theory of muscular action or of the

laws of memory is made determinate by being brought in relation to a more specific and concrete context. So too the diagnosing physician tends to have in mind a general hypothesis as to the nature of the case which he is examining, and makes his general hypothesis more specific and determinate by relating it to each symptom in turn. That is to say, he has in mind a schematic and indeterminate mental model, which he makes more specific and definite by applying it, point by point, to the case before him. How far such procedure is objective, depends partly upon the objective nature of the preceding analysis which has divided the case before him into points or symptoms. Assuming this to be partially objective, we can say that the method of taking a single element of the situation and bringing it in relation to the other elements of the situation, one at a time—or of taking a mental model of the whole and making it more concrete and specific by comparison with each element revealed by the analysis—we can say that this method is at least as objective as the analysis which has preceded it, and that, in practise, it tends to be more objective—*i. e.*, to bring those who use it into closer and more intimate contact with the objective facts.

As to completeness, a glance at the progress of science will be enough to show us that completeness in any final sense is entirely out of the question. For each new generation of scientists pushes further, inquiries which were in many cases regarded as sufficient by previous generations. In determining the concept of *arthropod*, for example, or *protozoon*, all that can be done is to bring together as carefully as possible all the viewpoints and all the knowledge of the period, and thus bring the concepts in question up to date. Our textbooks of natural science, in fact, represent a kind of cross-section through the research and knowledge of the period in which they are written, and all that we can ask of the various determinations carried through by writers of a given period is that they should be up to date. But the process of scientific inquiry does not stand still, and determinations which held good ten years ago are now out of date. Some of them have been passed by in the race for objectivity, but almost all are now regarded as incomplete. Still, as in art and education, so also in science. A reasonable degree of completeness can be attained.

Thus we see that in the case of mind-made entities, and also

in the case of natural phenomena, progress in objective understanding is always possible, and a certain degree of completeness can be attained, so that the aim of determination can in both cases be approximately realised. In neither case, however, can it be completely and finally realised.

Validity of Determination.—That some determinations are fantastic and quite arbitrary, is beyond reasonable doubt. The view of the planets as a concrete system of spheres, set in motion by “epicycles” or excentrics of a crystalline nature, is determinate, and has even some slight objective value. But as a serious scientific explanation it has long since been discredited. The number of epicycles, in particular, which the system assumed in order to account for the observed phenomena, seems to have been quite arbitrary, and for their “crystalline” nature the only evidence offered was, that they *must* be of some transparent substance, in order to account for their invisibility to human eyes. So too the philosophy of nature developed by the speculative philosopher Schelling is determinate, but is generally regarded as subjective and arbitrary to the last degree. There is, then, nothing sacrosanct about determination as such, and its validity must accordingly be a matter of how the method is used. On what conditions is determination to be regarded as valid?

To this question there is only one answer which we can give. As a scientific method, determination is valid, if and so far as it brings us into touch with objective laws and objective facts in a way which leads to an advance in some specific science. If it “works”—*i. e.*, if it fits in with the system of mental models approved by experience in that particular branch of inquiry, and if it also helps to solve our particular concrete problem in a satisfactory manner—so far it can be regarded as valid. But if it is clear and concrete from the viewpoint of intellect alone, or if imagination alone,—as in the case of Wolff’s *Psychologia Empirica* or Schelling’s *Naturphilosophie*—and fails to bring us into contact with the objective world and the realities of experience, it is worthless. How far it is valid, then, is a matter which can only be decided by the gradual advance of science. There is no easy and simple criterion.

Summary.—Determination, then, is a synthesis of x with a , b , c . . . , in such a way that, as thus interrelated, both x and a , b , c . . . take on new shades of significance and

become more specific and concrete. The aim of determination is to be objective and reasonably complete, that is, to add to our insight into objective laws and facts in a way which can be accomplished by no other method. In the case of mental models and of natural phenomena this aim can, to a reasonable extent, be realised. But the extent to which it can be realised, and in general the validity of our use of the method, is a matter only to be decided in the light of scientific progress. So far as determination leads to insight into objective structures, and helps us to solve our concrete problems and thus advance the science into whose department the problems in question fall, its use as a scientific method is justified.

Abstraction and Determination.—It remains to consider briefly the relation of abstraction to determination. Apparently, abstraction is analytical rather than synthetical, while determination is synthetical rather than analytical. Abstraction takes an element or aspect out of its concrete context. Determination puts an element or aspect into a concrete context. Abstraction and determination thus appear different in starting-point, in result, and in method. But, remembering as we do that analysis and synthesis were found to involve each other, and to be two sides of a single method, we must ask whether abstraction does not similarly involve determination, and determination involve abstraction, so that here also, perhaps, we are dealing with two sides of a single method.

Does Abstraction Involve Determination?—Let us begin by asking whether abstraction, as such, involves determination as an essential part of its method. In abstraction, we take an element out of its context. Does our procedure, however, render it absolutely contextless? Let us consider. If we deprived an element of all context—of all connection with and relation to other elements—would it not be cut off from the intelligible world and become meaningless? It looks, then, as though in taking it out of context a we are perhaps putting it into context b; and a little consideration will convince us that this is precisely what we are, in fact, doing. We are always placing it, at least, in a universe of meaning, in a system of thought which is intelligible. There is a world of concrete realities, and there is a world of abstract meanings, and abstraction consists largely in taking an element out of its place in a particular concrete situation, and giving it a different place in the world of meanings. For example,

the music-student isolates a point of technique for study by practising it in a context composed of finger-exercises. He ceases to play a certain passage in Liszt or Chopin until he has practised that kind of passage in its contexts in the finger-studies of Czerny or Hanon. The psychologist isolates the "rote" element for special study of memory-factors by using the artificial context of nonsense-syllables. That is, in fact, how we isolate. We extract the "rote" element by using nonsense-syllables. We manage to extract the "double-sixths" element by playing the specialised studies in Czerny. So too with generalisations. When we generalise, we very evidently leave the world of concrete fact and enter the world of mental models, the world of scientific hypotheses and laws. In this way, then, we see that abstraction involves determination, and involves it as an integral part of its method. Abstraction from the concrete is at the same time determination in terms of mental models, and it is, in fact, by determining in terms of mental models that we manage to abstract from the concrete.

Does Determination Involve Abstraction?—Let us now reverse our inquiry, and ask whether determination as such involves abstraction as an integral portion of its method. If we consider the question closely, we see that we are not taking an entity which is devoid of context, but rather an entity which belongs at least to the world of intelligible meanings, in order to give it a context which shall be concrete rather than abstract. That is to say, we abstract it, not from a pure isolation, but from its meaning-context in order to determine it in a more sensory context. As we saw, an element, in becoming determinate, tends to lose something. The moral law loses much of the clearness and sharpness of outline which belongs to it in the metaphysical-ethical world of pure meanings, and takes on something of the confusion and irregularity of our psychological impulses. Our psychological impulses, on the other hand, lose much of their merely mechanical nature in becoming transformed into something ethical. So too our notion of *X*'s character loses its generality and schematic nature in becoming specific, and our understanding of *X*'s specific acts loses much of its narrowness when envisaged from the general standpoint of insight into *X*'s character as a whole. It is plain, then, that determination involves abstraction as an integral part of its method, and that abstraction

and determination must accordingly be regarded, not as two methods, but as two one-sided aspects of a single method.

Comparison With Analysis and Synthesis.—What is the relation of this method to the analytic-synthetic method? In the first place, it is a less elementary method. It presupposes both analysis and synthesis, and starts, not with a concrete situation, but with a context already analysed out into elements, or with elements already analysed out of a context. In the second place, it takes us further in our investigation of objective laws and facts. By taking one thing at a time and concentrating our whole powers in a single direction, we are able gradually to make further progress than would otherwise be possible. As a scientific method whose aim is to place us in touch with the objective world, abstraction and determination represent a considerable advance upon analysis and synthesis, a carrying further of the work which the more preliminary method had begun.

FOR FURTHER READING

B. Bosanquet, *Logic*, Vol. II, pp 168–172. Chr. Sigwart, *Logic*, Vol. I, pp. 265–270. W. Wundt, *Logik*, (3rd Edit.), Vol. II, pp. 17–19.

EXERCISES

Show what part is played by determination in dealing with the following cases: (1) In looking up the meaning of a word in the dictionary. (2) In selecting courses of study in one's junior year in college. (3) In ascertaining the grounds for a gloomy, depressed feeling. (4) In choosing pictures for one's room. (5) In choosing one's friends. (6) In writing a "report" on an assigned book.

CHAPTER XXIV

INDUCTION

Nature of Induction.—Induction is a highly complex method of scientific investigation. It makes use of analysis and synthesis, abstraction and determination, in all their varieties and in any order which suits its problems. What holds together this use of these various methods and gives to induction an existence in its own right, and specific significance as one of the most important of the scientific methods, is its goal. Analysis takes apart, abstraction isolates or generalises, but *induction discovers laws*. It is this which distinguishes induction from abstraction and determination, which are concerned with concepts rather than with laws, though they also establish certain empirical generalisations. It is this also which marks its importance for science. Analysis and synthesis are more preliminary, and deal more with concrete situations. Abstraction and determination are more advanced, and carry further the work started by analysis and synthesis. But induction—and deduction—are the most advanced of all the scientific methods, and carry to its full completion the work of these preliminary methods. The chief aim of investigation is the discovery of laws, and induction thus sums up all the other methods in itself, and carries their work to its completion, so far as this is possible. The nature of induction, then, is to discover laws.

1.—How is this done? There are two main ways, which at first sight seem sharply distinct. In the first place, we have the kind of generalisation which we have already studied under the name of abstraction. In induction this method is largely used, but is carried a little further. By taking a variety of instances of the problem under investigation, it is possible, if we compare them carefully in respect of all the points already made clear by analysis, to eliminate a large number as immaterial, and thus to narrow the field to such an extent that an explanation in terms of some working hypothesis is more easy to discover. Similarly, by taking both posi-

tive and negative instances—that is to say, (1) instances in which the particular point in which we are interested is present, and (2) instances closely resembling the former, but differing in that the point in question is absent—it is possible, by a careful comparison, to eliminate a great deal which is immaterial, and thus considerably to simplify our problem.

For example, if a large number of cases of ptomaine poisoning are reported from a single district, it is possible, by careful comparison of the chief circumstances revealed by the preliminary analysis, to eliminate points which vary in the various cases, and to narrow the problem to a consideration of the points which remain constant throughout the whole series of cases. If *A, B, and C* were at work on drains, while *D, E, F . . .* were not *and yet were poisoned*, the drains as a possible factor can as a rule be eliminated. Similarly if *A, B, and C* were poisoned, while *H, I, J . . .* were also at work on the drains *but were not poisoned*—this is a negative instance—the drains as a factor to be considered would probably be eliminated, at least provisionally. As a rule a problem of this kind becomes narrowed down to such a point that it is discovered that all were taken ill *after* their evening meal, and that the materials for the evening meal were almost all purchased at a single store. It is then possible to carry on the same method of elimination of variables until only the elements common to every meal are left—such as canned food and one or two other items.¹ When the problem is thus narrowed down, a hypothesis at once suggests itself, and leads to the immediate impounding of samples of the food in question for purposes of chemical analysis.

So too in the famous case of Sir Isaac Newton's experiments² to discover the conditions on which the colors of the solar spectrum depend. Newton so narrowed the problem that he was able to use positive and negative instances which were already considerably simplified. A ray of sunlight entered the dark room of his physical laboratory, and shone upon a screen, making a whitish or yellowish circle of light. The spectral colors not being present, this furnished a negative instance. By inserting a glass prism into the path of

¹ Such cases are only too common. In the particular case which came under the writer's notice—in Berlin in 1913—the *causa mali* was German sausage.

² A careful discussion of the inductive methods involved will be found in S. H. Mellone's *Logic*.

the ray, Newton was able to produce the colors upon the screen. This furnished a positive instance. The production of spectral colors from the ray of sunlight was thus due to something in the nature of the glass prism. Analysis showed that prisms might vary in size, in shape, and in material. Abstraction suggested devising special experiments to test each one of these special possibilities, both positively and negatively. In these experiments, size was totally eliminated, material was partly eliminated, and certain kinds of shape were eliminated, other kinds being thus made more determinate. The problem being thus narrowed down, the hypothesis of the composite character of sunlight suggested itself, and was subsequently verified.

II.—In the second place, we have a method which seems more akin to determination. We frame a possible explanation of the case before us, and then proceed to modify or determine this hypothesis by bringing it into connection with the points revealed by analysis, one by one. What is superfluous or erroneous in the hypothesis or mental model becomes eliminated and corrected, until finally it fits the facts as closely as we can make it. For example, in the work of criminal investigation, a detective constructs a hypothesis to account for the facts, and proceeds to determine it further by following up specific clues, until a satisfactory conclusion is reached. So too in prospecting for minerals, it is usual to start with a general hypothesis based partly on general mineralogical information, partly on special local knowledge, and to determine this hypothesis by testing for ore here and there until a completely satisfactory conclusion is reached. More particularly, the method consists in framing a hypothesis, and then deducing from the nature of the hypothesis or mental model certain specific consequences, which can be brought into more immediate relation to the facts of the case. If these do not fit the facts, the hypothesis is modified until the deduced consequences and the observed facts more nearly coincide. This is brought out with especial clearness in Huxley's marshalling of the evidence for the evolution hypothesis. He first deduced what, assuming the hypothesis to be correct, one would expect the facts to be in a specific case—the case being concerning certain anatomical details to be expected in the ancestors of the horse—and then showed that the actual examples discovered and placed in the Yale museum agreed with the deduc-

tions in question. He even proceeded to make further prophecies in the shape of deductions along the same general lines, and these deductions have been verified by more recent discoveries.³

This second method of induction, which proceeds by framing mental models and then determining these, does not seriously differ, when we come to look closely at both, from the method of generalising by means of abstraction. In the first place, they look like two phases or halves of one and the same method. Thus, when the problem of poisoning had been simplified to a certain extent, the subsequent procedure consisted in framing a hypothesis about the canned food, and then determining this further by means of chemical analysis. So too in the case of Newton's experiment—the hypothesis of the composite nature of sunlight was suggested after the problem had been partially simplified, and was subsequently further determined. That is to say, it looks as though the method of induction consisted of approximately three phases:—(1) analysis and simplification of the problem, (2) construction of some hypothesis on the basis of the simplifying analysis, and (3) determination of the mental model thus suggested, by careful comparison with the facts. This last stage is sometimes known as Verification.

But, in the second place, it is possible to look into the procedure still more closely, and to realise that it is less with two successive phases, and more with two simultaneous aspects, of the inductive method, that we have to do. The method of abstraction and generalisation, which results in eliminating certain factors, really proceeds itself by the help of mental models, and thus may reasonably be regarded as itself a determination of some general hypothesis, or making it more specific. In the poison case, the general hypothesis concerns the source of the poison, and this hypothesis certainly is present in the beginning and becomes determined and narrowed down with each step in the inquiry. In fact, the whole problem may reasonably be regarded as an attempt to determine the source of the poison. So too Newton had a general idea that there must be something in the nature of sunlight which made it possible for the spectral colors to be

³ See Huxley's lecture, *The Demonstrative Evidence of Evolution*. It is quoted, with a few omissions, by A. L. Jones, *Logic, inductive and deductive*, pp. 287–300.

produced in his experimental laboratory, and each step in the experiment may legitimately be regarded as making this general idea or hypothesis more determinate and specific. So too, on the other hand, by following up special clues, the detective is learning what may be eliminated, and by his special tests the prospector is discovering what sites may be regarded as unpromising and negligible. There is thus *one* complex method, which results in the discovery of some principle or law.

Aim of Induction (A) Objectivity.—Induction, then, is a highly complex method, which employs all the resources of analysis and synthesis, of abstraction and determination, until our mental models approximately fit the facts under consideration, and we are thus able to discover some principle or law. The aim of induction is, first and foremost, to be objective. We do not wish to discover some hypothesis which satisfies *us*, but turns out to be fantastic or unverifiable, when brought into connection with the actual facts. We seek to discover objective laws, laws which really seem to apply to the facts. Sometimes it is in terms of causal models, sometimes in terms of mathematical models, sometimes in terms of models which belong to a more specific branch of scientific inquiry, as when we seek to discover the principle which holds a particular botanical theory together, or which unifies the various ways in which a particular term is used in literature. The use of the various scientific methods in order to discover a principle or law which is of objective significance, is the primary aim of induction, whether the principle is causal, mathematical, or what not.

(B) Completeness.—In the second place, induction aims at completeness. As we saw, the characteristic which distinguishes induction from the methods previously considered, is that it carries their investigation further. That is to say, it aims, not only at a greater degree of objectivity, but also at a greater completeness of investigation. Abstraction is satisfied with isolating some special element or aspect. Determination is satisfied with making some such element or aspect concrete and specific. Induction aims at carrying the investigation further, and abstracting or determining the law which governs the case in question—and at discovering it as completely as may be possible. It is usually thought that induction is never perfectly complete, and that science aims at progressiveness

and fruitfulness in its solutions, rather than at formulations which are to be accepted as final. But there can be no doubt that it is satisfied with nothing less than the fullest determination of which the evidence admits. Objectivity, then, and completeness represent our aim in using the method of induction.

How Far Realisable? (A) With Mind-Made Entities.—Give a college graduate one of the psychological puzzle-boxes, and ask him to find out the principle in accordance with which it can be opened and shut. For convenience of reference, let us consider the box to which we previously referred.⁴ A little experimenting completes the work of analysis and synthesis. The factors are learnt to be (1) the two levers, (2) the bolt which falls by its own weight when both levers are drawn out and the box is placed upon the white side, and (3) the combination lock. Abstraction and determination further are brought to bear upon (1) and (2), until the principle which governs them is discovered. Then the combination lock is experimented with, in terms of one mental model after another, until the limits within which it works are finally determined. These items of knowledge thus ascertained, their application, in the proper order, to the different elements of which the works of the box are composed, will open the door. The student is now possessed of the principle. His induction is certainly objective, for it works. He can actually open the box before him. He understands the principle of the thing. His insight can reasonably be regarded as complete. For what remains to be discovered? Once the box is opened, the structure, at which he had previously guessed, becomes visible, and even the combination lock can be taken to pieces without much difficulty. It is true that certain ultimate problems in physical science—*e. g.*, as to the nature of levers, including the question why one end should move when we move the other⁵—remain unsolved. But those go beyond his problem. His problem was to open the box, and to find out the principle of its works. He has opened the box, and has found out the principle. His problem is solved, and his induction can thus be regarded as not only objective, but also complete.

Let us consider another example. In working out a text-

⁴ Cf. p. 221 above.

⁵ Cf. Sir Oliver Lodge, *The Survival of Man*, p. 81.

book problem in simultaneous quadratics, the first thing to do is to get the conditions stated in terms of x^2 and y^2 . This represents the work done by analysis and synthesis, and in the case of most text-book examples is practically done for us. Then follows the work of abstraction and elimination, and this complex procedure is continued until eventually we have eliminated everything on one side of our equation but x , and on the other side everything but a numerical value—say 5. We then go back to our first mental model, *e. g.*, an equation of the form $2x^2 - 4xy + y^2 = 50$ and, by substituting for x its numerical value 5, are able to eliminate x and thus determine y , eventually discovering that $y = 20$. In this way we have discovered the principle or law which we were seeking. The result is objective, for it is obtained by rigorous operations upon the analytic-synthetic reconstruction of the data. It is also complete, for the problem is solved. It is true that the solution of equations presupposes answers to a number of mathematical, logical, and even metaphysical questionings, but these lead beyond the problem with which we were dealing. *That* has been definitely solved. If such cases are to be considered as typical, we can say that in the case of mind-made entities, where thought is dealing with itself and its own constructions, our inductions can be both objective and complete.

(B) With Natural Phenomena.—In cases where thought has to deal with phenomena other than its own constructions, the path to objectivity and completeness does not seem so certain. We proceed, by constructing mental models, and by a process of trial and error which eliminates the less promising of these, to simplify our problem in various ways, using analysis and synthesis, abstraction and determination, until in the end we have brought our mental model as close to the objective laws and facts as seems in our power, and have made it as complete as we can. There is, however, as we have seen before, always some gap between our mental models and the empirical facts. For instance, in using the mathematical type of model, we might even use one in the form of simultaneous quadratic equations—thus to some extent resembling the mind-made case mentioned above. Once our data are expressed in algebraical form, a conclusion can be worked out which is both objective and complete *so far as the mathematical conditions are concerned*. But between a given school-book example and a real problem in physical science, there is all the difference in the

world. The school-book example is mind-made, and can be fitted completely and without remainder into the equational form in question. But the phenomena dealt with in physical science are seldom, if ever, of this exact form. There is in practise always a certain marginal error, and our examples, as a rule, refuse to work out nicely. We cannot quite overcome the gap which separates our models from the phenomena which we are studying, though we can, by improved methods of determination, reduce this gap to very small proportions. In the case of the poisoned workmen, for instance, the methods employed rarely *prove* in a way which could be regarded as completely satisfactory that the poison came from the canned food. We need the further chemical analysis, and till that has been performed, we tend to suspend judgment. Even the chemical analysis is not *perfectly* satisfactory. So too in criminal cases, circumstantial evidence may point strongly in one direction, and may still be entirely misleading. In fact, in dealing with such phenomena, our methods seem to be like those of indirect proof; where direct insight is not to be had. Inductive methods, in such cases, seem like an attempt to substitute painstaking completeness for direct insight. We try, by proceeding both positively and negatively, so to manipulate the analysed data, that one mental model seems highly probable and all alternative explanations which could reasonably be framed are shown to be improbable, and are thus eliminated. But nothing short of direct insight is perfectly satisfactory, and in dealing with natural phenomena we have put up with something short of direct insight, and can thus only approximate to realising the aim of objectivity and completeness.

Types of Inductive Method.—The chief types of inductive method are the mathematical and the causal. That is to say, we can distinguish inductive types in terms of the type of mental model employed. In practise, however, these represent only the two chief forms. Strictly speaking, there are at least as many types of inductive method as there are specifically distinct sciences, and as these are not only indefinite, but also increasing in number, it seems unprofitable to attempt in any way to sum them up. Attempts have been made to distinguish inductive methods according as positive instances alone are employed, or a mixture of positive and negative instances, *etc.*, but in practise all instances are partly positive

and partly negative,⁶ so that it is unwise to attempt to introduce distinctions which are without practical—*i. e.*, objective—significance. The only safe statement for us to make is, that induction is a highly complex method of scientific investigation, which feels its way by means of analysis and synthesis, abstraction and determination, until it reaches some approximation to insight into the law for which it is seeking.

Validity of Induction.—In dealing with mental structures—*i. e.*, with cases in which it is possible to attain to an insight which is both objective and complete—it is easy to test the validity of an induction. An induction is valid which really leads to such insight. When we have reached the principle which makes clear the fastening of our puzzle-box, or the solution of the problem in simultaneous quadratics, we know that our induction has been valid, because it works in a way which we can test directly, and can understand. But in dealing with natural phenomena, where such direct insight is out of our power, our inductions can only approximate to full validity. In such cases, we can say that they are valid so far as they work—*i. e.*, so far as they enable us to understand the law in question. For example, if we can predict, from our approximate insight into some such law, what will happen in a given case, *and experience really fulfills our prediction*,—if we can handle phenomena much as though they were mental models—as we seem able to do in calculating eclipses—so far we can regard our inductions as valid. But for approximation to a progressively more objective and more complete insight into the structure of natural law, we must look to indefinitely continued inductive inquiries—*i. e.*, to the development of empirical science itself. It is only in the light of wider and deeper knowledge that we can judge whether our present experiments are leading in the right direction or not. The “right” direction is the one which leads to a more objective and more complete knowledge of the laws which seem to underlie natural phenomena.

Summary.—Induction is a complex method of investigation which sums up in itself the work of analysis and synthesis, abstraction and determination, and carries this work further, until it discovers some law. Its aim is objectivity and com-

⁶ For example, the “positive” instances agree in the presence of the phenomenon in question but disagree by varying in many other respects. That is to say, in respect of points other than the phenomenon in question, such instances are “negative.”

pleteness, and this aim can be realised with mental structures, but not precisely with natural phenomena. The validity of induction is tested by inquiring how it "works," and can thus be settled only by more prolonged experimentation, *i. e.*, by the progress of science itself.

FOR FURTHER READING

B. Bosanquet, *Logic*, Vol. II, pp. 175-179. F. H. Bradley, *Principles of Logic*, pp. 329-342. B. Erdmann, *Logik*, (2nd Edit.), pp. 742-754, 774-784. H. W. B. Joseph, *Introduction to Logic*, chapters xviii-xix. Chr. Sigwart, *Logic*, Vol. II, pp. 288-311. W. Wundt, *Logik* (3rd Edit.), Vol. II, pp. 20-30.

EXERCISES

How might the following cases be investigated inductively: (1) The theft of a sum of money from a room in a hospital to which only three nurses have access. (2) The comparative teaching efficiency of three grade school teachers. (3) The meaning of "Imagination" in Aristotelian psychology. (4) The comparative efficiency of party government and non-partisan government. (5) The degree of accuracy with which we can localise touches on the forearm. (6) The effect of wave-action upon rock. (7) The rate of growth of the different bodily organs. (8) The chemical constitution of a given substance. (9) The significance of the Biblical version of Nebuchadnezzar's defeat. (10) The value of memory-training in increasing intellectual ability?

CHAPTER XXV

DEDUCTION

Nature of Deduction.—Deduction is the reverse of induction. Induction reasons from specific instances to a law or general principle exemplified in those instances. Deduction starts from a law or general principle, and reasons from the principle to its consequences, or from the law to its workings in some special case. Deduction is in every respect fully as complex as induction, and may make use of experimental abstraction and determination, as well as of the more general and simple methods of analysis and synthesis. Thus, starting with the general principle that life insurance is a good thing and that we ought to insure our lives with the best possible company, it takes a great deal of analysis and synthesis, abstraction and determination, before we can be certain which is the best possible company, and can thus apply our principle to a definite instance. Or we may start with an ethical principle such as "All rational beings as such are to be treated as ends-in-themselves, and not as mere means to *our* ends," and deduce from it as a consequence the specific principle that children and imbeciles, criminals and animals, not being fully rational, cannot be treated in this way, but must have their "ends" set for them by the more rational elements of society. It has even been argued by men of high education, such as Aristotle and even Plato, not to mention more modern instances, such as Nietzsche, that people of relatively low education, and all animals, are definitely to be exploited, and find their highest development in serving the higher ends of more rational people. Just how far this principle is to be applied in practise, is doubtful. But in any case, the deduction of specific consequences requires special experience, and a considerable amount of abstraction and determination.

One of the aspects of deduction which makes it of special value as a method of scientific investigation, is that it does not require a law to be already established in order to have a starting-point, but can assume a hypothesis provisionally, and

then ask, Given such and such a law, what would follow from it? This experimenting with mental models is of the greatest value when we are dealing with complex situations into which we have little direct insight. For example, the Copernican hypothesis was first worked out in this provisional and experimental way, until the coincidence of its deduced consequences with the observed facts made it universally accepted. So also in dealing with complex geometrical problems, we tend to assume various hypotheses and ask what follows from them, until we hit upon one which seems acceptable. The same procedure is adopted in dealing with causal models also. It is even possible, in cases where our actual knowledge is extremely slight, for us to assume successively (1) a certain provisional hypothesis, and (2) its opposite, and then deduce consequences from each of these hypotheses until, by means of these consequences, we succeed in bringing our assumptions into contact with actual or possible experience. We can then decide directly between the groups of consequences, and thus indirectly between the opposed hypotheses. This we do frequently when it is a question which of two alternative plans we should follow in some complex problem of conduct, but the "method of hypotheses," as it is called, has been employed in philosophy since the time of Zeno and Plato, and under the name of "multiple working hypotheses" is employed, in a slightly more complex form, in modern science.

Perhaps the most noticeable characteristic of deduction is its logical consistency. By means of deduction we build up mental patterns which are all of a piece. We cannot deny a single consequence without denying the principle with which we started, and without at the same time invalidating all other consequences which have been deduced from the same principle. If we can deny that two *plus* two make four, we are at the same time denying, not only the truth of all the principles upon which arithmetic rests, but also the whole body of the science, so far as it depends upon these principles. We are denying, for instance, that $2+4=6$, that $20-10=10$, *etc.* On the other hand, each consequence which proves acceptable, strengthens the main principle, and also an argument in favor of accepting other consequences. If this medicine was good for John, when he had the measles, and for Mary in similar circumstances, and for a number of other people whom we know, then it will be good for us too. There must be some

principle underlying the previous cases, and it is reasonable to assume that our case also will come under that principle. This mode of thinking is especially important in the science of healing, but applies also to almost all of the more developed sciences. Where observations have been correlated for a great number of years, a body of science gradually grows up which constitutes a system, and is all of a piece, in such a way that we can confidently apply its principles even beyond the hitherto observed types of case.

As a method of scientific investigation, deduction is used chiefly in drawing consequences from the mental models which we have constructed, whether these models are merely provisional, or have been made in accordance with principles securely established upon a basis of constant experience. These consequences may then be compared with the observed facts or with the known laws in the special science of which they form a part. In special cases, they may even be compared with the general principles of consistency. If the consequences do not agree with the known facts, we know that the model we have assumed for the purposes of understanding a concrete situation, does not apply to that situation. If they do not agree with what are regarded as known laws in their special field, we must conclude that there is something wrong, either with our model, or with the "known laws"—if not with both. If, finally, the consequences of a principle lead to noticeable inconsistencies among themselves, *that* is evidence that the model itself contains logical flaws, and has been wrongly constructed.¹ The nature of deduction, then, consists in reasoning consistently from principles to consequences, and in scientific investigation more particularly consists in inferring to the consequences involved in the mental models which we employ for our various scientific purposes.

Aim of Deduction (A) Objectivity.—The consistency which is the chief characteristic of deduction is not an end in itself. For purposes of scientific investigation our aim in deducing consequences is to get into touch with objective facts and laws. Spinning thought-webs is, in itself, valueless to the scientist. He is interested primarily in discovering laws and testing the objective validity of hypotheses in his special field, and thus the first and chief aim of scientific deduction is objectivity. Deduction is valuable as a scientific method of investigation precisely so far as it brings us into closer touch

¹ Cf. Plato, *Parmenides*, esp. 127E–128D.

with the objective world than would be possible without its aid. Without it, we might, perhaps, be able to construct brilliant hypotheses, but we should certainly remain unable to verify them in any way, or even to choose wisely between rival explanations of the same phenomenon, because we should not be able to connect them up with our empirical observations. A great portion of the usefulness of deduction to science, consists in its being the main highway along which we can pass from our mental models to the actual facts, and thus verify hypotheses and establish laws. Objectivity, then, is the first and greatest aim of deduction.

(B) Completeness.—In the second place, deduction aims at completeness, or at least at reasonable completeness, a completeness such as sufficiently meets our problem and safeguards our conclusions. We do not, as a rule, attempt to deduce *all* the possible consequences of a principle, but only the more striking types of consequence, and more especially such consequences as can be brought in definite relation to the concrete situation, and can furnish “test cases” for the mental model which we are applying. In order, however, to realise the aim of objectivity, it is necessary for us to avoid superficiality and one-sidedness, and thus to deduce a considerable variety of consequences. By “reasonable completeness,” then, is understood that deduction aims at drawing a variety of consequences, which on the one hand are appropriate to the concrete situation, and on the other are sufficient in number to strengthen our conviction of the objectivity of our procedure.

How Far Realisable? (A) With Mind-Made Entities.—How far can this aim be realised? How far is it possible for us to deduce consequences from principles in a way which shall be both objective and reasonably complete? Let us consider an example. If we understand the principle which governs the working of our psychological puzzle-box, we can deduce just what will take place if, *e. g.*, we pull the two levers in the wrong order, or if we turn the box on its white side before pulling the levers, and then turn it back again before pulling the second lever; or if we turn the combination lock twice to the right, as indicated, but only once to the left, or if we stop at the wrong figure, *etc.* So too if we understand the principles of harmony and the structure of the piano, we can apply our knowledge in such a way as

to enumerate the chief concords and discords, and explain just what physical phenomena—*e. g.*, in the way of “beats”—will take place when such and such notes are struck together. In such cases our deductions are certainly objective. For the object to which we refer is nothing but a concrete embodiment of the precise principles whose implications we are making explicit, and every one of the consequences drawn can be verified by reference to the facts. In dealing, then, with objects of this kind, where the object is constructed in terms of a plan which is rational, it is possible to deduce the consequences implied in the principle of the plan, and such consequences will be found to hold good of the object itself.

What are we to say of completeness? If such deduction is objectively correct, if, *e. g.*, we deduce precisely what happens when we turn the combination lock only once to the left, it could not well be more complete—at least for practical purposes. It is true that a physicist's understanding of the nature of the material employed and the various physical laws involved would theoretically be more complete, though even in physics any final completeness is out of the question. But the problem is not properly concerned with the *physical* nature of the material, but with the material only so far as it is the embodiment of a *rational* plan. Our deduction has solved its special problem completely. But if we regard deduction as reasoning, not to a single consequence of the principle, but rather to a reasonably complete variety of consequences, so that the truth of each will reflect credence upon the others and will also strengthen our confidence in the principle—we see that an absolute completeness is out of the question. The success of each of our inferences strengthens the conviction that we have really grasped the plan according to which the box has been put together, and our aim is not to deduce every last consequence, but only a sufficient variety and number to avoid one-sidedness and provide an adequate test of the principle. This, however, can be done, in the case of the box, and in the case of the piano, and it seems fair to state that, so far as mind-made apparatus is concerned, our aim can be realised, not only in respect of objectivity, but also in respect of completeness.

Let us examine another example. When once we have grasped the principle of the bi-literal cipher, and realise that a group of five letters is to represent a single letter of

the alphabet, and that in selecting the members of each group we are restricted to some combination of the two symbols *A* and *B*—we can deduce at least as many consequences as there are letters of the alphabet. Thus, we can deduce that $AAAA B = a$, $AAABA = b$, $ABAAA = c$, $ABAAA = d$, etc. In fact, the reading of the cipher message is itself a complex consequence of the principle, and the most convincing consequence too. The drawing of consequences is in such cases a kind of construction—for by this means we construct the details of the mental model in question—and if this construction agrees in its details with the details of the given problem $AAABBB$, the problem is solved. Such deduction is certainly objective—for it enables us to read the cipher message—and reasonably complete—for it enables us to construct a whole alphabet, and indeed, any number of cipher messages which could be understood by the transmitter of the original problem. If such examples are to be regarded as typical, we may state that in dealing with mental models, the aim of deduction can be realised.

(B) **With Natural Phenomena.**—We deal with natural phenomena, as we have seen, primarily through the medium of mental models. We construct a model in terms of some rational plan, and then deduce its detailed consequences with a view to comparing these with the details of the phenomenon in question. If the mental model fits the phenomenon in detail, we regard the principle of both as practically identical,¹ and are satisfied that we have practically discovered the law of the phenomenon in question. So far as deduction is confined to elaborating the consequences of our mental model, so far the account given under the head of “mind-made entities” holds good. But in dealing with natural phenomena, we mean something more. Does our deduction in any sense take us beyond the realm of mind-made models, and bring us into contact with the objective world? Is deduction objective in this sense? Let us consider an example. In trying to understand the principle of the basilar membrane, some scientists use the model of the wires in a grand piano, while another group use the model of the membrane in a telephone. In both cases, consequences are deduced from the nature of these models and compared with the experimental evidence.

¹ Cf. William James, *Pragmatism*, lecture I.

Has this kind of deduction a significance which is objective? Yes, it certainly does help to bring us into closer and more intimate contact with the meaning of the phenomena in question. It is not at the present day possible to state that either of these models is perfectly correct, but the deduction of consequences from both models, and the experimental attempts to verify the theoretical deductions in terms of the actual facts, have undoubtedly led to a great increase in our knowledge of the objective conditions and facts of the case. So far, then, as objectivity is concerned, the aim of deduction can be at least partially realised in this field.

In respect of completeness also something can be done. The deductions as such are from the principle of the mental model to the details of the mental model, and so far as our thought is confined to the realm of mental models, each such deduction can be regarded as complete. But when we transfer the question to the world of natural objects, this is no longer the case. There is, as we have already seen, always a certain gap which separates our models from the concrete realities. The reality is always something more detailed, for instance, than we seem quite able to grasp. Nature, as we say, has an infinite variety. This gap, this difference between what we seem able to construct on the one hand, and the objective phenomena on the other, is a measure of the extent by which we fall short of completeness. We cannot deduce *all* the details of the phenomenon because we have not *perfect* insight into the law in question. Certain of the more important and striking consequences we can deduce, where our knowledge is well developed. But completeness is more than we can expect. In dealing, then, with natural phenomena, the aim of deduction, in point of objectivity and completeness, can only partially be realised. This partial realisation, however, is certainly valuable, not only in itself, but also as leading to further progress in scientific discovery.

Types of Deduction.—There are two main types of deduction, the mathematical and the causal. That is to say, we can distinguish deductions in terms of the type of mental model whose consequences are deduced, and—as we have already seen—the most commonly found, and most generally valuable types of mental models are precisely those which belong to the mathematical and causal groups. But mental models are by no means confined to these two types. On

the contrary, there are at least as many possible types as there are specifically distinct scientific viewpoints. But these are indefinite in number, and are also tending to increase as science takes in more fields of inquiry, and it is thus not, perhaps, possible to confine within any rigid limits the number and types of mental models. In consequence, we cannot restrict in any way the number of typical forms which can be used in valid deduction. The types of deduction are innumerable. In the history of logic, attempts have, indeed, been made to work out all the forms of valid deduction which the mind could possibly use, but as the most notable attempt restricted thought to following the detailed developments of a single mental model, and that too neither quantitative nor causal²—we must regard it as bold but unprofitable. Thought cannot be restricted to the use of any single group of models, but is as various and complex as it finds advisable in dealing with the problems which arise. As it is clearly impossible to limit such problems, so it is impossible to limit the forms of deduction which can be used in solving those problems. Thought is free and unfettered. All we can safely say is that, at the present stage of science, the mathematical and causal types are the most frequent and the most universally valuable.

Validity of Deduction.—That deduction is not always objectively valid will be plain if we consider briefly a few typical cases. Hemlock Jones in the story deduces with convincing clearness that Potson has stolen his pipe—and the pipe is, in fact, in Potson's pocket. Potson realises that it would be quite impossible to convince Jones that his ingenious deduction is invalid, though he knows it to be wrong. So too the paranoiac deduces with almost flawless completeness that all the world is in a conspiracy against him—and indeed they do actually put him under restraint in the end. It would be quite impossible to persuade him that his deductions are unsound, and yet we know that they are. It is possible, in short, to explain one and the same fact in terms of a variety of mental models, and yet certain of those mental models may be radically mistaken, as when Bain tries to explain parental affection in terms of the benefits to be returned in after-years, or

² The reference is to the Aristotelian *sylogism*. Aristotle himself believed this to be a *causal* model, the "middle term" being identical with the cause. But this is today generally regarded as a mistake.

Mill tries to explain universal benevolence in terms of egoistic self-seeking. There are thus cases of deduction which seem convincing enough from the viewpoint of consistency alone, but are found to be inadequate when we apply them to the actual world. What, then, is the criterion? How are we to distinguish a valid from an invalid deduction? To this question we can only answer that there is no simple criterion. In order to test our deductions and discover whether they are objectively valid or not, our only final criterion is the progress of science itself. If the mental model which we have used is found to be helpful in the concrete situation, and also fruitful in suggesting and solving further problems—if, in short, it “works”—we may regard it as so far valid. If, however, like Bain’s insurance model applied to the case of family affection, it seems to solve the case before us, but brings us into hopeless conflict with the models of explanation accepted as verified in all kindred researches, and is of no avail to solve further problems—there is so far reason to regard it as invalid. In the case of natural science, then, the only adequate test of the objective validity of deduction is the further progress of the science itself. In the case of mind-made entities, however, we have already seen that deduction can be objective. In such cases it is a mere matter of consistency, and of *that* we are fairly good judges, especially in the simpler instances. In complex cases, analysis is necessary, and only the advanced thinker can come to valid decisions.

Summary.—Deduction, then, is a process which makes explicit the consequences implied by some principle or law, whether such law is already established, or is merely assumed as a provisional hypothesis. Its chief aim is to bring us into closer touch with the objective facts than would be possible without its aid, and to be not only objective, but also reasonably complete. This aim we can accomplish in dealing with mental models, but can only approximate to attaining in the case of natural phenomena. In such cases the validity of our deductions can be appreciated only as science itself advances by such means. In general, however, it may be confidently asserted that deduction is a valuable factor in securing such an advance.

FOR FURTHER READING

Chr. Sigwart, *Logic*, Vol. II, pp. 181–192. W. Wundt, *Logik*, (3rd Edit.), Vol. II, pp. 30–38.

EXERCISES

How might the following cases be investigated deductively: (1) The comparative efficiency of learning a long poem as a whole, or as a number of sections. (2) The value of the ergograph as a test of mental fatigue. (3) Whether black is or is not a positive sensation. (4) The best method of holding the hand in writing. (5) The most efficient method of practising at the piano or violin. (6) The scientific value of psychological or pedagogical study. (7) The best time for planting corn. (8) The means by which the automatic chess-player (described by E. A. Poe) was worked. (9) The practical value of a college education. (10) One of the Sherlock Holmes problems?

CHAPTER XXVI

INDUCTION AND DEDUCTION

The Problem.—Owing to historical associations, induction and deduction have become watchwords of scientific and logical thought. Many of our elementary manuals to this day are entitled "Logic, Deductive and Inductive," as though deduction and induction together exhausted the field of logical inquiry. There are even well-known books published on Deductive Logic alone, and on Inductive Logic alone, and many writers who treat of both, do so in separate volumes. We have, then, a tendency, owing to historical considerations,¹ to regard induction and deduction as separate, and even opposed, methods. Deduction, in our minds, stands for Aristotle and mediaeval thought with its emphasis on argumentation and clear ideas, and its almost total lack of experimentation and discovery. Induction similarly stands for modern science, with all its empiricism and distaste for arm-chair methods, and with its extreme insistence upon observation and experiment. We even tend to think of deduction as the method of mere proof, of organisation of knowledge obtained from some non-deductive source, while induction seems to us to be the method *par excellence* of scientific investigation, the method of discovery, the genuine source from which all knowledge comes. For the popular consciousness, then, the two methods are sharply distinct and even opposed.

For the scientific consciousness, on the other hand, they are at the present day recognised as working hand in hand, and indeed as a method of investigation deduction tends to be preferred, at least in the more advanced and exact sciences.² The popular view rests upon the idea that for deduction, the law which forms its starting-point must have been already established, whereas for the scientist the most fruitful characteristic of deduction consists precisely in the fact that its

¹ Cf. Goblot, *Traité de Logique*, pp. 82–83.

² Cf. Wundt, *Logik*, 3e Auflage, Vol. II, p. 31.

starting-point may equally well be a provisional hypothesis, assumed deliberately for purposes of experimentation. For the scientific consciousness, then, the old and sharp distinction of function which the popular consciousness still retains, has broken down, and it is conceivable, since both methods are found to work together in science, that they may not be so sharply distinct as is usually supposed.

The problem of the present chapter is to examine the relation of induction and deduction to one another, in order to discover whether they are two distinct methods, as the popular consciousness and even many scientists still suppose, or whether they are two correlative aspects of a single method of scientific investigation, as perhaps a majority of scientists and logicians at the present day tend to believe. We shall begin by inquiring whether induction necessarily involves deductive elements, and shall then pass on to the question as to whether deduction necessarily involves inductive elements as an integral portion of its method in investigation.

Does Induction Involve Deduction?—Let us examine a typical case of induction. We recognised provisionally two main types of induction, in the former of which abstraction seemed more prominent, while in the latter greater emphasis seemed to be laid upon determination. Subsequently, we threw doubt upon any attempt to draw a hard and fast distinction between these two types, but for the sake of completeness we shall here examine cases from each group. Let us consider first a case which belongs more to the abstraction-type. The problem being to discover whether there is, in point of fact, a law of "general intelligence" pervading our activities in different fields of work, we proceed by selecting a great variety of experimental tests, designed to probe our processes of attention, memory, reasoning, etc., and giving these to a group of subjects whose ranking, in respect of intelligence, is already ascertained, at least in part, by other means. Let us assume that the ascertained order of the subjects is A, B, C, D, . . . , and that the results of our experiments are as follows:

<i>No. of Test.</i>	1	2	3	4	5	6	7	8	9	10
Order of Subject A	2	1	1	1	1	1	1	2	1	1
" B	4	3	6	3	2	2	2	1	3	2
" C	7	5	3	9	3	3	5	3	2	3
" D	6	4	4	8	8	7	6	6	7	4

These figures represent the work of the preliminary analysis and synthesis. We proceed to abstract and determine the final ranking of the subjects by abstracting the average of the various test-results for each subject, and then arranging the subject accordingly. In the end, we find that our conclusions agree with the previously ascertained order. Our tests also rank the subjects in the order A, B, C, D, That is to say, the subject who is considered most intelligent by other standards really leads in most of the tests, and the subject who is considered poorest really is last in most of the tests, *etc.* So far, then, as these particular tests go, it looks as though there might conceivably be some such entity as "general intelligence" manifesting itself in solving problems of any and every kind.

Does deduction play any part in the above inquiry? Yes, it certainly does. We deduce that general intelligence—assuming that there is such a thing—must be such as to make its possessor do well in any kind of test. That is to say, we deduce the specific consequences, that A—supposed to be the most intelligent subject—will come out first in test 1, in test 2, in test 3 . . . , that the least intelligent subject will come out last in test 1, in test 2 . . . *etc.* It is because our experimental results on the whole bear out these deductions, that we consider it reasonable, so far as this evidence goes, to suppose that there may be such a thing as general intelligence. That is to say, without some such deductions, we should not be able to establish or even to reject the suggestion. For even if the results had not borne out our deductions, but had led to negative conclusions, the deduction would still have played an essential part. It is, in fact, only by reference to consequences deduced from an assumed principle, that we can be sure that the facts establish a law, or reject it, or leave it not proven. Deduction, then, is an essential and integral part of the inductive method by means of which we establish laws.

Let us take another example, in which the determination-aspect is more prominent than the abstraction-aspect. In playing the piano, or in writing with a type-writer, we are able to estimate certain spatial relations with great accuracy, as is shown by the way in which we can strike almost any key we wish, without looking at it. What is the principle or law underlying this accuracy? We can investigate the ques-

tion by first constructing a mental model in its general outlines, and then working out its details and seeing how these compare with the experimental results. Accuracy in motor localisation must depend upon something in our sense-organs. We must have some sensory organ capable of estimating spatial movements with accuracy. In the case of the arm, there are only three possible factors—(1) the skin, (2) the muscles and tendons, and (3) the joints at wrist, elbow, and shoulder. We assume that if one of these plays a great part in such localisation, treating it in such a way as to paralyse it for the time being will interfere with our accuracy, and conversely, if throwing one of these factors out of gear makes no appreciable difference to our accuracy, it is not seriously concerned in our estimation of movement. Thus if the skin is sprayed with ethyl chloride, it becomes frozen, and its sensitivity is much impaired. If the skin is a vital element in estimating movements, we should expect our accuracy to diminish, and conversely, if our accuracy does not diminish, we should regard the skin as not playing an important part in such localisation. That whose presence or absence makes no apparent difference to the phenomenon, cannot be causally connected with the phenomenon. Similarly with the muscles and the joints. Experiments are devised which impair the sensitivity of one of these factors for the time being, and our accuracy is tested in the same way as with the skin. It was discovered by experiments of this type that the muscles play a certain part, but not a very great one, and that the chief part is probably played by the joints.³ That is to say, the experiments bore out certain of our expectations, and refuted others.

In such cases there is no necessity for minute inquiry. We very obviously deduce the consequences of our assumed hypothesis, and this is, in fact, often referred to as a portion of the "deductive method of induction." So far, then, as the determination-type of induction is concerned, we may regard it as admitted that it is fundamentally deductive. We may also note that a precisely similar deduction of the consequences of a mental model plays a part also in the abstraction-type. We there deduced what consequences should

³ Cf. Goldscheider's paper in *Archiv. für Anatomie und Physiologie*, 1889, pp. 369, 540. Also James, *Principles of Psychology*, Vol. II, pp. 189 ff.

hold good if the theory of general intelligence was to be upheld. That is to say, we made explicit which was logically involved in such a theory—*viz.* that *A* would be first in tests 1-10, *etc.* If such cases may be regarded as typical, our conclusion is, that induction as such necessarily involves the use of deduction, as an essential part of the method of discovering laws.

Does Deduction Involve Induction?—Goldscheider's experiment with the skin, muscles, and joints illustrates the deductive method of induction, and in dealing with that aspect of deduction which is found useful as a method of investigation, it would be difficult to find a more typical case of deduction. The use of deduction, in actual practise, is to construct in its details a mental model already constructed in general outline—to deduce the detailed consequences which follow from the original plan—in order to see how these compare with the experimental or observed results. If the two coincide in detail, the principle exemplified in the mental model is regarded as so far established. In Euclid's well-known proof of Bk. I, prop 4, the method of superposition is employed. The deductive element in this method consists in the argument that if the triangle *ABC* is superimposed upon the triangle *DEF*, so that the side *AB* falls upon the side *DE*, and the side *AC* upon the side *DF*, the plan of structure is such that *B* must coincide with *E*, and *C* with *F*, and the base *BC* must—as a further deduced consequence—coincide with the base *EF*, so that the triangles will be found to be equal in all respects. The establishment of the principle consists in experimenting by means of superposition, and finding that the experimental result does actually bear out the consequences, in proportion as it is carried out correctly,—and indeed cannot do otherwise. So too in the cause-effect model, we reason that when, if *A* is present *X* is always present, and if *A* is absent *X* is always absent, and if *X* is present *A* is always present, and if *X* is absent *A* is always absent,—then *A* and *X* are causally connected. These specific deductions represent in fact the detailed nature of the cause-effect mental model. Whether causal connection is or is not established in a particular case, depends on whether the experimental or observed results coincide with the details of this model, or fail to coincide, respectively.

In scientific investigation, then, induction always contains

deduction of consequences as a part of the inductive discovery of laws. But does deduction always necessarily contain induction? Can we perhaps maintain that induction connects up a mental model with the facts, and thus comes into play only when we go beyond the mental model and try to bring it into contact with something which is not a mental model—the actual objective phenomena? If so, it looks as though when thought was merely concerned with itself, we might have a deduction of consequences without the slightest reference to the world of natural phenomena—*i. e.*, a case of *pure* deduction, unmixed with anything empirical and inductive. It is sometimes thought that Euclid's reasonings are of this kind. Can we do this, or is it impossible?

Let us consider the actual facts. In the first place, if this were possible, should we not have a case of "pure" thought—of thought thinking itself without reference to the sense-perceivable world? But this is the kind of thought sometimes attributed to infinite Beings—*i. e.*, is a transcendent mode of thought which human beings do not seem to possess. In the second place, we find that it is at least possible to use induction in dealing with mental models—as when we are seeking to discover the key to a cipher, or to make our way into the puzzle box, or to solve a geometrical or algebraical problem. And in the third place, if we examine the mathematical and causal models exemplified above, we find that there is an inductive element which appears essential to the deduction itself, so far at least as that deduction constitutes a method of investigation. In the case from Euclid, the inductive portion of the method consists in the experiment of superposition itself, and in seeing that when the triangle ABC is superimposed upon the triangle DEF , they do in fact coincide. Incidentally, in the deduction of each individual consequence there is involved a reference to the actual construction of the figure, and the verification of this reference is certainly inductive in nature. When we apply the side AB to the side DE so that the point A falls upon the point D , induction is needed to verify the deduced consequence that the point B coincides with the point E , and so also with the other deductions. To each deduction of consequences there is a corresponding induction, and in fact we always proceed in this double way, comparing the mental plan with the actual details, and the actual details with the mental plan. So too

in applying the causal model, both in respect of its nature as a model, and in respect of its application to the world of events and processes in time. So far as the model itself is concerned, each deduced consequence is accompanied by an induction which verifies it, and so far as the application to the empirical world is concerned, so well is it known that for each deduction there is a corresponding induction, that the causal model is frequently taken as the typical model which represent *inductive* procedure, and in fact is often regarded as though it belonged exclusively to *induction*. If these examples may be regarded as typical, then we can state that every deduction, as such, is necessarily accompanied by a corresponding induction which verifies and confirms it—that is to say, deduction necessarily involves induction.

The Inductive-Deductive Method.—Induction and deduction, then, necessarily involve one another, and thus turn out to be two aspects of a single fundamental method of investigation. In the first place, they are *two* aspects, correlative but distinct. In respect of starting-point, conclusion, and method, one special aspect stands out more prominently in the case which we call induction, and the other aspect is more prominent in the case which we call deduction. In induction we appear to be starting from an analysed and determined situation, and to conclude to a law. In deduction we appear to start with a law, and to conclude with a group of consequences which together constitute an analysed and determined situation. But these differences are only apparent. The appearance arises from one-sided emphasis, and if we look a little more closely into induction we see that it starts just as much with a mental model, provisionally assumed as incorporating the law which it is sought to establish, as with the situation to which such a model is assumed to apply. So too it concludes just as much with the application, to its data, of a set of consequences deduced from the mental model, as with insight into a law. In fact, application of deduced consequences to the data is the way in which we obtain insight into the law. So also in the case of deduction in its use as a method of investigation, if we look a little more closely, we see that its starting-point is not just some mental model in general, but a specific mental model which is adapted to the data. That is to say, we start just as much with the data of the problem as with the law or mental model, and we conclude

just as much with establishing the law as with deducing consequences. And in respect of method, we see that (1) viewed as inference from law to consequences it may be called deductive, but viewed as *verifiable* consequences, as consequences legitimately inferred from the model in question definitely applying to the data, there is an inductive reference to the detailed features of the situation. In other words, the method has both an inductive and a deductive aspect, and these are thus two correlative and interdependent aspects of a method which is, however, fundamentally one.

The function of the inductive-deductive method is, to sum up and carry through the work begun by analysis and synthesis, abstraction and determination. In fact these three methods, (1) analysis-synthesis, (2) abstraction-determination, and (3) induction-deduction, may be regarded as three successive stages or phases of one and the same general method—the method of scientific investigation. In the first place, they are successive. Analysis and synthesis are preliminary methods, and are content with being able to take apart and put together again the problem under investigation, which is usually a concrete situation. Abstraction and determination are more advanced methods, and, starting with a situation already analysed and synthesised, single out some element or aspect for special consideration, and by thus concentrating their activities in a single direction, are enabled to go further than was possible for analysis and synthesis. At the same time, they do not go so far as induction and deduction, but are satisfied if they take a single element out of one context and determine it by reference to another context. Induction and deduction are highly complex methods which make use of both the foregoing methods and carry their work further until we succeed in discovering some law and its application to a situation which has been analysed and determined.

In the second place, these methods are not merely successive, but are also phases of one and the same fundamental method, the method of scientific investigation. This we can realise from the following considerations:—(1) Analysis and synthesis involve, as we saw, insight into the law of the phenomenon to be analysed and synthesised. So too with abstraction and determination, while induction and deduction are concerned with insight into the same law. (2) The aim

of all three methods is one and the same—*viz.* objectivity and completeness. They are concerned with the same objects, for all alike are trying to bring us into contact with the world in which we actually live. (3) Finally, the method is fundamentally the same. In all three cases we proceed by the construction of mental models and their application to the facts of the situation.

Summary—Scientific Investigation.—There is thus one fundamental method of scientific investigation, in which we have distinguished three phases, (1) the analytic-synthetic, (2) the abstractive-determinative, and (3) the inductive-deductive. The function of this method is to enable us to understand the world in which we live, and to understand it objectively—*i. e.*, in its own nature—and as completely as is possible for us. This function is accomplished by the construction of mental models which we can fully understand, and their application to the special problems and situations which arise for us. A perfect application of our mental models to natural phenomena is not quite possible. There is always a certain incompleteness, a gap of some sort between the rational model and the actual concrete situation. But the gradual progress of science gives us assurance that by the continued and persistent application of this method—*i. e.*, by treating phenomena as if they were entirely rational and entirely resembled, at least in principle, our mathematical and other models—we can work our way towards a progressively more objective and more complete insight into the nature and workings of the world in which we live.

FOR FURTHER READING

B. Erdmann, *Logik*, (2nd Edit.), pp. 742–754, 774–784. W. R. Boyce Gibson, *The Problem of Logic*, pp. 326–327. J. G. Hibben, *Logic*, Part II, chapter i. H. Lotze, *Logic*, Bk. II, chapter vii. Chr. Sigwart, *Logic*, Vol. II, pp. 418–460.

EXERCISES

Point out the necessary inter-relation of inductive and deductive aspects of the method of scientific investigation in examining the following cases: (1) The psycho-analytical examination of a hysterical patient. (2) The value of circumstantial evidence in a criminal case. (3) Testing the accuracy of a chronoscope. (4) Studying the effects of alcohol upon quantity of muscular work. (5) Learning to sing, or to play upon some musical instrument. (6) Studying how far Shakespeare followed the details of his “sources.”

CHAPTER XXVII

DEFINITION

The Problem.—What are our habitual beliefs on the subject of definition? In the first place, we feel sure that for purposes of clearness in exposition, whether in discussion or in writing, it is well to have the exact signification of our terms laid down beforehand, and to use our terms only in the sense thus established. We feel that language is misleading, in that one and the same word is usually associated with many meanings, *a*, *b*, *c*, and that a speaker may intend sense *a*, but the hearer may understand in sense *b* or *c*. The misunderstandings which thus arise are sufficiently annoying, where they are not merely amusing, in ordinary social intercourse. But in scientific discussions, we feel sure, such variations of meaning are not to be tolerated. Each science thus tends to develop a technical language of its own, in which an endeavor is made to use each term only in one sense, and in order to have it known what that sense is, it is usual to fix it by an arbitrary definition. Carried to the extreme logical conclusion to which this feeling of ours points, science should establish an entirely artificial and technical system of signs—such as we find, *e. g.*, in algebra—in order to express its thoughts in a way which should be unmistakable. That is to say, our feeling for the necessity of clearly defined terms leads logically to the creation of a special scientific language—to what has been called an algebra of thought. Among thinkers, a number of attempts to create this symbolic language have been actually made, and there is no doubt that, with all their artificiality, they give expression to a natural tendency of our thought when it is dealing with problems of exposition.¹

In the second place, we have at the present day a certain

¹ In the history of thought, this attempt is associated especially with the names of Raymond Lully, and of Leibniz. But in modern times, symbolic logic furnishes an excellent example, and for general philosophical purposes the construction of a technical terminology is advocated by Professor Lovejoy, in his presidential address to the American Philosophical Association. See *Philosophical Review*, Vol. XXVI, 1917, pp. 123–163.

mistrust of definitions, even in the more abstract and technical sciences. We think of definition as somehow interfering with the life and movement of thought, as crystallising it into clear-cut forms which, with all their clearness, are devoid of life. We think of definition as somehow implying that the movement of thought has come to an end, and as constituting a check upon further development. It is unprogressive, and stands for mental stagnation. We think of it as a creation of mind which has somehow come to stand between us and the actual phenomena which it was originally designed to represent—a something which hinders rather than aids the pursuit of knowledge. From this viewpoint, then, we tend to regard exact definitions with considerable mistrust, and to feel that they are superficial. They may be clear, but they do not go far into their subject, and we tend to regard their function as, at best, provisional only.

There is, then, in our ordinary educated thinking, a certain confusion of ideas on the subject of definition. On the one hand, we tend to view it as extremely valuable, and on the other, we seem to regard it as almost harmful, from the standpoint of scientific exposition and scientific discovery. The problem of the present chapter is to study the subject of definition more closely, with a view to discovering what its uses are, and thus to remove the confusion in our ordinary ways of thinking.

Nature of Definition.—What exactly is definition? It is a statement of the nature of some subject under discussion. In the first place, it is a statement, a mind-made structure or mental model, of the subject of discourse. Thus a name, or any sort of designation which is fixed by the mind so as to refer to the subject of discourse, partakes of the nature of definition. "The subject before you is a pencil. It is used for drawing and writing. It is made of a hollow wooden cylinder with a lead core." Each sentence here contains a different sketch or mental model, and is so far to be regarded as a definition.

In the second place, it states the nature of a subject. It is an answer to the question "What is it?" or "What is its nature?" In the example just given, the first attempt at definition states merely that it is a thinkable and namable—*i. e.*, a subject of discourse. The statement of the name does not go very deeply into the nature of the pencil. The second

attempt defines it in terms of use, *i. e.*, treats it as a usable, or as something whose nature is to be understood by reference to the system of human purposes. Its nature is declared to be instrumental. It is a tool for drawing or writing. The third attempt defines it in terms of the material out of which it is composed, and also in terms of structure—a lead core surrounded by a wooden cylinder. These are three attempts to state the nature of the object in question, and are so far definitions. Let us take another instance. “This object is called grass. It is used for feeding cattle—and indeed for a hundred other purposes. For the botanist it is one thing, for the artist it is another, for the child it is another, and for the moralist it is yet another thing. It is not made, but grows from seed in the following way. . . . It belongs to the genus called *poa*. It is a monocotyledon.” In this example, the object, not being an artefact, does not seem to have a *single* nature. The botanist frames one mental model of it, the farmer another, and the moralist yet another. And there can be no doubt that, from his special viewpoint, each is justified.

In the third place, it is some subject which is under discussion, which we define. We do not form a mental model of the thing as it exists in itself, but rather of some aspect of it which interests us. In fact it is doubtful how far objects have what we could call a “nature” in themselves. What we define is always their nature in reference to some interest or purpose of ours. That is to say, our definitions have not only an objective, but also a subjective reference. For example, it is usually possible, given a number of definitions of one and the same object, to infer to the interest or purpose behind the definition. Thus, take the three definitions of Grass:—(1) the natural food of sheep and oxen, (2) the English equivalent of the Latin *poa pratensis*, (3) a species of the genus *poa*, tribe *festuceae*, family *gramineae*. It is easy to see that the first is a farmer’s definition, the second a scholar’s, and the third a botanist’s definition. Definition, then, has a subjective, as well as an objective reference.

Aim of Definition (A) Objectivity.—Definition thus presents us with a mental model of some object in relation to some interest of ours. What is our aim in framing such mental models or statements? In the first place, we aim at objectivity. We wish to represent the actual nature of the

object in which we are interested. To say that definition has a subjective reference, does not in any way invalidate this statement. It is true that it is from his own point of view that the farmer is interested in grass. But from that point of view he wants to know *what grass is*. The point of view does not swallow up the difference between natural and artificial feeds, or between one sort of feed and another. On the contrary, the special viewpoint leads to special refinements of insight into the nature of the subject under discussion—such as studying the chemical properties of grass which render it especially nutritive to cattle. It leads to analysis and synthesis, abstraction and determination, and may result in a considerable accession of knowledge. So too the student who is translating from Latin into English does not care whether *poa* is monocotyledonous or dicotyledonous, whether sheep feed on it or not—all he wishes to know is, by what English word it is best translated. But *this* he does wish to know; and wish to know correctly. The scholar's viewpoint, too, may lead to all kinds of refined and subtle research. The taking a special viewpoint, then, does not interfere with the search for objectivity, but on the whole rather assists it by concentration of effort in a special direction, and we can fairly say that our primary aim in defining is objectivity, or getting in touch with the real nature of the object under discussion.

(B) **Completeness.**—In the second place, we aim at completeness. We aim at so defining or stating the nature of the subject of discourse, that, from the viewpoint which interests us, nothing remains to be added, and nothing is to be taken away. Whatever the special question which definition asks, we aim at answering that question completely. Thus in the case of *poa pratensis*, the scholar's "What is it?" means, what is its correct English name?, and the complete answer to his question is contained in the word *Meadow-grass*. What we wish to avoid in definition is vagueness, indefiniteness, ambiguity, incompleteness in any shape or form. We want to be definite, precise, exact, clear, final. We want our definition to accomplish what it sets out to do. We want it to be complete.

How Far Realisable? (A) With Mind-made Entities—How far can we express the nature of some subject in which we are interested, in a way which shall be both objective and

complete? Let us consider first the case of mind-made entities. "A chair is a piece of furniture designed for a single person to sit on. A chair is made of:—4 legs, with supporting cross-pieces, 1 seat, 1 back, and possibly 1 or 2 arms. These materials are put together in such a way that the legs are fastened to the seat from beneath, and the arms and back from above, in this manner. . . ." Here we have three definitions of a mind-made object from the viewpoint (1) of function, (2) of materials, and (3) of structural plan. All three definitions are objective. For, in spite of the difference of viewpoint, each expresses the actual nature of the object from its special angle of approach. Thus, the function of a chair *is* to be sat upon, and to be a piece of furniture. That is what Aristotle would call its *final* cause, the idea which we seek to realise in constructing chairs. So also the materials out of which it is to be put together *are* legs, seat, back, *etc.*, just as much as the "Two *N*'s, two *O*'s, an *L* and a *D*" are the materials out of which the name *London* can be put together. This is what Aristotle calls the *material* cause of the chair. So also in the case of structural plan. If the materials are put together in accordance with the directions, we do actually have a chair. This is what Aristotle calls the *formal* cause. If Aristotle's viewpoint is here accepted, each of these definitions will be seen to belong to the causal type of mental model, though, in respect of the last two, mathematical aspects also enter in. In dealing, then, with artefacts, our definitions can be objective.

Can they also be complete? "A piece of furniture designed for a single person to sit upon"—is this a complete definition of a chair? Complete, that is, from its special standpoint? We can test its completeness by asking (1) are all chairs pieces of furniture designed with this purpose, and (2) are all pieces of furniture designed for a single person to sit upon—chairs? (1) seems to be correct enough, but (2) seems to include stools as well as what we should call chairs. However, if what we are interested in is not structure, but function, even (2) is correct. For the difference between stool and chair is chiefly structural. From the special viewpoint, then which is interested in function, our definition may be regarded as complete. So too in the case of the other two definitions. (1) All chairs are constructed out of material

such as legs,² a seat, a back, and possibly arms, and (2) all objects constructed of such materials are what we should call chairs. So too (1) all chairs are constructed in accordance with the plan mentioned, and (2) all objects constructed in accordance with such a plan would be called chairs. In dealing with artefacts, then, our definitions can be regarded, not only as objective, but also as complete, at least, from the special standpoint of each definition.

Let us take another example. "A triangle is what one learns to construct in a special form in the first proposition of the first book of Euclid. A triangle is a three-sided rectilinear closed figure. A triangle consists of three angles which together form a closed figure. A triangle is a rectilinear closed figure, the internal angles of which are together equal to two right angles. A triangle is a rectilinear closed figure such that any one of its external angles is equal to the sum of the interior opposite angles. *Etc., etc.*"

Each of the above definitions is objective. The construction of a triangle is the subject treated of in Euclid I. i, a triangle is a three-sided figure, a three-angled figure, and does possess all the other properties ascribed to it, along with very many more which might equally well have been used, and no doubt many of which have not yet been discovered. Of the objectivity of all of these definitions, there can be no possible doubt. Are they, however, all complete? Let us consider? (1) All triangles are what we learn to construct in a special form in Euclid I. i.—Yes, for we there do learn to construct the equilateral form, to construct *any* equilateral triangle. (2) All things which we learn to construct in the special form according to Euclid I. i, are triangles.—Yes, this also is true. All equilateral triangles are certainly triangles. The first definition, then, appears to be complete. Again, (1) all triangles *are* three-sided rectilinear closed figures, and (2) all three-sided rectilinear closed figures *are* triangles. The second definition is complete, and the same proves to be the case when we apply the same test to the others. If the above examples may be regarded as typical, then, we can state that in respect of mind-made entities our definition can be both

² The number of legs is relatively immaterial. We think of most chairs as possessing *four* legs. If, however, it is thought advisable to point out that some chairs have only *three* legs, and are still perfect as chairs, it is easy to alter the definition, by specifying the possible variation in the number of legs, as of arms.

objective and, from its special viewpoint, complete. This is especially recognised in the case of equations. An equation is recognised as the complete definition of the corresponding graph, and the corresponding graph might equally well be regarded as the complete definition (from a certain viewpoint) of the equation.³

(B) **With Natural Phenomena.**—As we have seen, we deal with natural phenomena through the medium of mental models, and thus our definition of a natural phenomenon is the mental model itself, in terms of which we are trying to understand the phenomenon in question. As there is, further, a gap which separates our mental model from the natural phenomenon, there can be no doubt that our definitions are, directly, statements of something different from the object to be defined, and refer to the object itself only indirectly. Thus, when we define an island as “a piece of land surrounded by water,” we are directly constructing the mental model of a circle—or of some such geometrical outline—and we can only apply this mathematical model to the natural phenomenon by a kind of mental *fiat*—“Let one side of this figure be regarded as land, and the other as water.” If we change the direction of this *fiat*, and regard the outside as land, and the inside as water, we have, with the same geometrical model, the definition of a lake. At the same time, although there is thus a gap between the mental model and the reality—a gap which has to be bridged by this *fiat*, a certain degree of objectivity cannot be denied to such definitions. If we define an island as “a hill-hop from the sea-floor projecting in part above the water-level,” we seem to be a little more nearly expressing its objective nature. So too in Zoology it is usual to define the spider or the horse in terms of the mental model of family relationship. In such cases, while there is no doubt that this specific model has an objective application, reference to a number of authorities will show that opinions differ as to what animals should be assigned to the arthropod or mammal group, and even to the invertebrate and vertebrate groups. From this evidence it would appear, that, so far as the concrete filling in of this mental model is concerned, we can hardly expect full objectivity. Still, there is no doubt that such definitions are partly objective, and that, as science

³ Cf. Goblots, *Traité de Logique*, pp. 121–122.

advances, definition of natural phenomena can become progressively more objective.

So also in respect of completeness. The work of science is never complete, and consequently the summing up of our knowledge at a definite stage, and its embodiment in a definition, can never be fully complete. In respect of completeness, then, as well as of objectivity, the aim of definition can in such cases be only approximately and progressively realised.

Types of Definition.—The general question of definition is "What is it?" To this question certain typical forms of response are given by certain typical forms of definition. Let us consider a few of the more prominent forms. In the first place, definition means, laying down the *fines* or boundaries of a subject, establishing its outlines in such a way as to distinguish it from all other subjects with which it might be confused. For example, "Pages 387-419 of Pillsbury's *Fundamentals of Psychology*" is an exact preliminary definition of an assignment on the subject of Reasoning—a statement of the boundaries of the subject to be studied, which is perfectly adequate to distinguish it from being confused with any kindred subject. So too, "all books numbered 160-199" is an exact preliminary definition of the books on philosophy in the Minnesota University library. It lays down the precise boundaries within which such books will be found, and serves to distinguish philosophy books from books on psychology—which are numbered 150-159—with all the precision to be expected in a library. For assistants in the library, and for such students as have access to the stacks, such definitions are not only useful, but also for many purposes perfectly adequate. For certain purposes they are not only needed, but are *all* that is needed.

What exactly is this kind of definition? It is not, of course, the object, but is rather a kind of outline sketch or model of its boundaries. It is even of a mathematical character, and indeed derives much of its exactness from the use of numbers. Its function is to *localise* the object with which we wish to get in touch, and it has even a directly spatial reference. This is true not only in such cases as "The third house on your right after you have crossed the park," as a definition of the locality where Mr. X lives, but also of the portion of the text-book set for a lesson on Reasoning, and even in the case of the philosophy books—for the numbers have a very

definitely spatial reference to certain shelves located in a certain room of the library. Such a definition tells us, not exactly what the object is, but rather where it is to be found. It is a mathematical model with a spatial reference.

Let us consider a second type of definition. "A pencil is something used for writing, when there is no ink. A typewriter is something you use when you want to write as clearly as print. A spoon is something you use for stirring liquids, or for eating food like soup or oatmeal. A meal is what you take to satisfy hunger. A dog is what you use to guard the house from tramps. Art and religion are the most ennobling things we have in life." For certain purposes, especially practical purposes, there can be no doubt that such definitions are perfectly adequate. They are sketches of the objects in terms of *use*, and thus refer partly to the nature of the object, and partly to the system of human purposes. On the one hand, they tell us what the object can be used for, and on the other, they tell us how *we* can use it. They are thus mental models of the objects under discussion, in terms of human uses, and as such are of the greatest importance to practical men.

There are many other types of definition. Thus, certain definitions refer less to what *we* human beings can do with the object, and more to what it itself does in a state of nature:—"A fire is something which burns. Water is something which flows, and wets whatever falls into it. Rain is something which comes down from clouds in the form of drops of water—is something which helps to make the crops grow, especially in the spring-time." This is a kind of causal model, and tells us what the object does. Another type of causal model tell us rather how an object is caused or produced:—"Thunder is a phenomenon due to lightning. Plant-rust is a phenomenon due to the action of bacteria. A rainbow is a phenomenon caused by our seeing the sunlight through water."

The typical forms of definition meet the general question "What is it?" by answering questions such as "Where is it, What can we do with it, What does it do, or make, or cause, How is it caused, or made, or produced, What is it made of, What is its law or principle of construction?" There is no limit to these questions. Each expresses a different interest, or represents a different angle of approach, and there are as

many possible types of definition as there are possible interests, or possible questions to be asked. As these are indefinite in number, and have never been classified, it is profitless to attempt to limit definition to any one form—as has been attempted in the history of logic. *Any* definition which answers its special question in a way which is objective and reasonably complete, represents a legitimate type of definition, and no definition absolutely exhausts the full nature of its subject.

Validity of Definition.—Definitions are mind-made entities in terms of which we try to understand the world around us. There is thus something experimental about them. They enter into the method of trial and error, and are frequently mistaken and thus false. In point of fact, this is true when we are defining mental models themselves, as well as in the case of natural phenomena. Thus, a common dictionary definition of a triangle is "A figure with three angles."¹ But if we test this by asking, are all figures with three angles triangles? we see at once that it is inadequate. For we might have an open figure, like a square with one corner missing—which would certainly be a figure with three angles, and would with equal certainty *not* be a triangle. So too a common dictionary definition of *chair* is "A movable seat."¹ But strip a chair of its legs and back, and it is still a movable seat. So also a stool or bench might be called a movable seat. But none of these would be called chairs. In the case of natural phenomena, it is not necessary to give specific instances. For as the history of science shows, all definitions of such phenomena are incomplete, and very many, if not all, are also partially false. Definitions, then, may be invalid. On what does their validity or invalidity depend?

In the case of mind-made entities, where the object may be created by mental construction, it is possible for a definition to be perfectly valid. Thus we can define the knight's move or bishop's move in chess with perfect accuracy, or "Alignment" and "Point of rest" in military science, provided that we follow the established conventions. So also in the case of triangles and other mathematical objects. If we accept the general view of mathematical space, we can define with sufficient accuracy. Mistakes are made here from time to time, but

¹ The definitions here are taken from *The American Popular Dictionary of the English Language*.

they can be rectified, and made satisfactory in point of objectivity and completeness. In the case of natural phenomena, however, complete validity is out of the question. But our definitions can, even in such cases, approximate to validity as scientific knowledge increases, and as our definition sums up correctly the knowledge of our own time. The only adequate test of such definitions is the progress of science itself. The further development of knowledge alone can show whether the views held at an earlier stage were pointed in the right direction or not. In general, then, a definition is valid, so far as from its special viewpoint it expresses the objective nature of the phenomenon under discussion.

Function of Definition in Exposition.—For scientific purposes, whether of inquiry or of exposition, definition has two main functions. In the first place, it is used in opening up an inquiry, in the form of a preliminary definition. Its use in such cases is to establish the general lines along which research or discussion is to be carried on. Thus, in scientific exposition, it is usual to start with a statement of the "problem," or to sum up the beliefs with which we approach a new subject, as Coleridge, before opening a new book, used to write down briefly his own thoughts on the subject. Used in this way, preliminary definitions are found very helpful in dealing clearly and objectively with a subject of discourse.

In the second place, definition is used in closing an inquiry, in the form of a concluding or final definition. In such cases its function is to sum up the results of inquiry, to answer the problem set at the beginning, or at least to sum up the beliefs with which we lay the subject on one side—as some thinkers write down briefly what they have learnt from each book they read, immediately after finishing it. Such concluding definitions are of the utmost value in promoting clearness, definiteness, and objectivity in our researches and explanations.

Summary and Conclusion.—So far we have seen that definition states, in terms of some mental model, the nature of some subject of discourse, and that its aim is objectivity and completeness. This aim can be attained in the case of mind-made entities, and we can at least progressively approximate to such objectivity and completeness in the case of natural phenomena. We have also seen that the chief use of definition in scientific exposition is to open or close an inquiry, in

the form of a preliminary, or of a concluding definition, as the case may be.

We should now be in a position to resolve the difficulty with which our inquiry opened. Is definition a help, or is it a hindrance, to scientific progress? It is certainly a creation of mind, a mental model, but its function is not to stand between us and the object, and thus obscure or mislead our vision. Its aim is to state the nature of *the object*, and to be a help towards clearing up our ideas and guiding our vision aright, and we can state that, in proportion as mental models prove of assistance to the progress of knowledge, so far definitions are helpful. On the whole, the preliminary definition seems of more assistance than the concluding definition—for it certainly leads to further progress. But if we adopt the modern progressive view of science, and do not regard *our* conclusion—the summing up of our inquiries as far as *we have gone*—as concluding the subject, and exhausting the nature of the subject itself, we can say more. If we regard our concluding definition not as in any way final, but rather as leading on to more determinate inquiries, both forms of definition may be equally fruitful and equally helpful in scientific exposition. What we objected to in definition, was not its clearness but its tendency to pass over into dogmatism, into the idea that it was more than an experimental mental model, always subject to revision. If, however, we avoid this error, we shall find definition always of assistance in understanding ourselves and in general the world in which we live.

FOR FURTHER READING

W. R. Boyce Gibson, *The Problem of Logic*, pp. 32–36. H. Lotze, *Logic*, Bk. II, chapter i. Chr. Sigwart, *Logic*, Vol. I, pp. 286–294. W. Wundt, *Logik*, (3rd Edit.), Vol. II, pp. 40–47.

EXERCISES

1. Give three definitions of each of the following, from three different viewpoints: Circle, Typewriter, Child, Apple, Air-plane, Diamond, Dandelion, Book, Ink, Student.

2. Is there any viewpoint from which the following definitions are strictly legitimate: (a) A lake is a water island in the land. (b) A lama is a woolly sort of fleecy hairy goat, with an indolent expression and an undulating throat. (c) A liar is a man who wilfully misplaces his ontological predicates. (d) A caterpillar is an emblem of life and a vision of joy. (e) A straight line is the arc of a circle

of infinite radius. (f) A useless life is a form of death. (g) God is a substance consisting in infinite attributes, of which each expresses eternal and infinite essentiality. (h) True wit is nature to advantage dressed, what oft was thought, but ne'er so well expressed. (i) A politician is not a saint. (j) A locomotive is something which moves from one place to another. (k) An honest man is the noblest work of God. (l) Architecture is frozen music?

CHAPTER XXVIII

CLASSIFICATION

The use of classification is universal. Not only in science, but also in every-day life, we classify everything and everybody. We classify a new acquaintance as someone we can or cannot get on with. We classify him as clever, stupid, or just average. We classify him as old, young, or middle-aged—as tall or short, thin or fat, plain or good-looking, rich or poor. In fact, there is no viewpoint—social, industrial, political, artistic, religious, *etc.*, from which we cannot put him into some class, into a group along with other people. In science, the importance of classification for investigation, as well as for exposition, can hardly be over-estimated. By grouping together a large number of phenomena or experiences which bear upon a single point, it is usually possible for a scientist to obtain insight into some law, or for a speaker or writer to transmit his own insight to others. In general, then, for every-day life as for science, classification is of universal use.

Nature of Classification.—Just what is classification? It means, quite simply, putting together so as to form a class. In the first place, it signifies *our* putting together. That is to say, the viewpoint from which we classify is ours, depends upon our interests and purposes, and is so far subjective and even arbitrary. For instance, to group together books on various subjects, in accordance with the first letter of the writer's surname, seems thoroughly artificial. It has very little to do with the nature of the book, and indeed seems largely accidental. There is no logical connection between the fact that this book, which happens to be on metaphysics, should have been written by a man whose family name began with *M*, and the fact that that book, which happens to be on metaphysics, should have been written by a man whose family name began with *B* or *T*. There is, in fact, no necessary connection between the first letter of a man's surname and the nature of his writings. Why, then, do we group books in this way?

It is because we find it convenient. There are only twenty-six letters in the alphabet we use, and we are thus able to assign any book whatever to some one of twenty-six recognised classes, and for purposes of reference this is found convenient. So too in a very small private library, it is quite common to find books assigned to different shelves in accordance with their size, so that all the smaller volumes go on this shelf, and all the ponderous tomes on that. This also has little or nothing to do with the contents of the books in question, but it suits the conveniences of the owner of the books. So too for purposes of shipping, books can be classified according to weight, or for esthetic purposes according to color and style of binding. They might also be classified according to place of publication, according to date of publication, according to the name of the publisher—or according to any one of a hundred different interests and purposes. Classification, then, in the first place is arbitrary. We put together in accordance with *our* interests.

To this arbitrariness there is, however, a limit. We put together only what *can* be put together. We do not group books according to their thermodynamic qualities, or according to their arboreal habits, or according to their qualities as chronoscopes, plethysmographs, or type-writers—because they have no such qualities. We put them together in accordance with characteristics which they really have. It may be accidental for a book on political economy to be written by a man whose family name began with *M*. There *are* books on political economy written by men whose names began with *S* or *V*. But there is no doubt that, in that particular case, it does begin with *M*, and as we find that convenient for our purposes, we make use of that characteristic. In the second place, then, we group together in accordance with some characteristic which the object to be classified really possesses.

In the third place, we put them together *so as to form a class*. What is a class? A class is a group of individuals held together by some law of connection, this law being some principle which all have in common. Thus students of biology form a class, being held together by the interest in biology which all have in common. Books written by men whose surnames begin with *M* form a class, being held together by the characteristic which all possess in common, and, in general, wherever objects have a single characteristic in common,

whatever that characteristic, and however superficial or accidental it may seem, such objects can be grouped together so as to form a class, provided always that some one happens to be interested in that special characteristic.

Finally, in order to obtain a clear idea of the nature of classification, we must compare it with what is known as Division. Classification groups together individuals so as to form a class. Division takes a class apart, so as to form sub-classes. Thus, we can group individuals together as Americans, as men interested in business, as millionaires, as politicians, or simply as men. This is classification. On the other hand, we can take the class *men*, and subdivide it into white-skinned, yellow-skinned, brown-skinned, red-skinned men, *etc.*,—i. e., we can divide the class up into sub-classes according to variations in respect of a single characteristic. So too we can divide politicians into progressives and reactionaries, into honest and corrupt, or millionaires into coal-kings, railroad-kings, *etc.*, according to variations in respect of the source of their wealth. On the whole, division and classification should be regarded as two aspects of a single method, like analysis and synthesis, abstraction and determination, *etc.*, and in order to grasp sufficiently the nature of classification, it is necessary to bear in mind this relation to division. Thus, in classifying individuals as millionaires, we are putting them into what is a sub-division of the class *men*, and in dividing the class *millionaires* into the sub-classes of coal-kings, railroad-kings, *etc.*, we are forming classes. Classification thus means, placing individuals together in a class which is usually itself to be regarded as a sub-class of some wider organisation.

Classification, then, is arbitrary—in that a class is formed from some particular viewpoint; yet not wholly arbitrary—for the individuals which are put into the class all possess some common characteristic, in virtue of which they can be unified and regarded as constituting a single group; and finally the class which we form is usually a part of some wider system, so that classification is a kind of organisation.

Aim of Classification (A) Objectivity.—What do we classify? What is our aim in grouping together all sorts of objects which possess a single characteristic in common? In the first place, we aim at objectivity. It is with a view to handling *the objects* more conveniently, and understanding *the*

objects more readily, that we form them into groups. Every science, and every complex research or exposition, has its classificatory stage, during which it is assembling material and getting it into usable shape. It is with the aim of getting into closer touch with the objective nature of this material, that science classifies it, or forms it into various groups which, for purposes of inquiry or exposition, seem to belong together. Thus in Botany, the otherwise enormous and unwieldy mass of material is grouped according to the family connections of the different small groups, so that the plan of systematic botany resembles a genealogical tree. But this classification, which is made primarily from the viewpoint of the evolutionary theory, is in a secondary way of use for other purposes also, so far as it makes the material more easily handled. Thus, in studying the reactions of plants to stimulation, the new viewpoint cuts across all the old class distinctions, and it is necessary to form an entirely new grouping, according as the type of reaction to light, for instance, or to contact, separates members of one and the same family group, and links them up with members from widely diverse branches of the genealogical tree. But until the re-grouping in accordance with the new interest has been effected, it is found convenient to investigate one family group at a time, in search of the new characteristics, following through the standardised family-group plan, until the whole ground has been covered.

So too with periodical literature. It is published in various magazines, each of which for purposes of reference tends to be regarded as forming a kind of standardised class of its own. But when we are interested in some single question—*e. g.*, the study of apparitions—we collect together into special groups all articles dealing with apparitions of animals, apparitions of mail-coaches, apparitions of houses and gardens, *etc.* But until the new grouping has been carried through, we find it convenient to make use of the old grouping in magazine-units, because in this way, by looking up the index of magazine *A*, of magazine *B*, *etc.*, we can conveniently cover the whole ground, and can get into touch with everything in the periodical literature which bears on our special interest. The primary aim, then, of classification is to reduce the material studied to such a form that we can handle it conveniently, and thus bring ourselves into closer

contact with the objective facts than would otherwise be possible.

(B) Completeness.—In the second place, we aim at a certain kind of completeness. We wish to cover the whole ground, and to leave nothing important out of account. Classification and division together always aim at covering the entire field. Division, for instance always tries to be exhaustive. We divide, *e. g.*, books in general into books on history, books on chemistry, books on philosophy, *etc.*, and at the end, when we have used up all the definite subjects of study or reference, we group together the remaining volumes as “miscellaneous.” So too in classifying. If we are making a collection of all books bearing upon some minute point in history or literature, or on a subject such as porcelain or indigo-dyeing, we want our class to be complete. In such cases, in order to ensure that nothing important shall escape us, we tend to go over the whole ground by the aid of division, so that all standard groups of books—literature, history, art, *etc.*—come in for consideration, and the whole field of literature is covered. We do not, of course, aim at including every single book bearing on our subject, but rather one book for each distinctive view point, *i. e.*, at including books each one of which definitely adds something new to the collection. We aim at including representatives of every variety, every distinct species, and thus to cover the entire ground in a way which shall do justice to its many-sidedness.

How Far Realisable? (A) With Mind-Made Entities.—In grouping objects together so as to form classes, how far can we realise this aim of objectivity and completeness? Let us consider first the case of mind-made entities. Can we form a class of musical instruments in a way that shall be objective and complete? Let us see. We take first of all instruments which produce musical tones by means of striking a vibrating cord—such as the monocord, clavicord, clavecin, piano, *etc.*, then instruments which produce musical tones by means of pulling or plucking a cord—such as the harp, guitar, the whole lyre family, the spinet, *etc.*, then instruments which produce the sound from cords by bowing—as the violin and the whole viol family—then the group of percussion-instruments—such as the drum—woodwind instruments—such as the flute family—reed instruments—such as the oboe, clarinet, *etc.*, on the one hand, and the harmonium on the other,

etc., etc. In short, byclassing instruments together in groups according to the way in which the tones are produced we could try to find a place for everything. Such a class would certainly be objective—for it would really group together actual types of musical instrument—and there is also no doubt that it can be reasonably complete. Indeed, so far back as we have historical records, it can be entirely complete.

Let us take another instance. Can we construct a class of curved lines which shall be both objective and complete? Let us see. A curve is anything from a circle on the one hand to almost a straight line on the other. We can put together a class of regular curves, composed of representatives of the circle-group—the circumference of a larger circle is less curved than the circumference of a smaller circle, and the arc of a circle of infinite radius would be a straight line—of the ellipse-group, of the parabola-group, *etc.*, and thus secure a class which is certainly objective, and is reasonably complete—in fact, entirely complete. For although the number of possible degrees of curvature is theoretically unlimited, the whole ground has been covered. So too with books, pictures, and tools of any and every sort. All mind-made entities can be classified in a way which is both objective and complete.

(B) With Natural Phenomena.—We classify natural phenomena, not directly, but through the medium of mental models. For instance, if faced with a collection of articles so heterogeneous that we can bring them under no other single head, we proceed to make an “inventory” of them. That is to say, we make a list of all the articles, with the numbering 1, 2, 3, 4, . . .—*i. e.*, put them together in terms of a mathematical type of mental model. This is not perfectly exact, from an objective viewpoint. For on the inventorial list every object listed counts for one, and none counts for more than one, and this standard, when applied to diverse objects, is often a travesty. But from the view point of completeness it leaves little to be desired, for it certainly covers the whole field, though in a very preliminary way. So too in geology we classify the various crystals in nature in terms of a mathematical group consisting of the tetrahedron, octahedron, dodecahedron, *etc.* But the mathematical group includes forms not found in nature, such as the eikosihedron, and the actual forms never perfectly correspond to the mathe-

mathematical models, so that, while the mathematical series is complete—for it certainly covers the whole possible field—it is not perfect in respect of objectivity. So too in chemistry the various elements, when arranged in relation to atomic weight, are found to lie on a spiral curve, and by investigating correspondencies suggested by this mathematical model, many important discoveries have been made. But here too, there are gaps, and the spiral is more perfect than the empirical facts which it puts together into a group. That is to say, it is perhaps too complete, and is not perfect in respect of objectivity.

At the same time it must be admitted that classification of natural phenomena in terms of such mental models is certainly of very great assistance in bringing us into objective contact with the actual varieties of natural objects, and that the progressive insight into the objective facts which is thus brought about, could probably not be brought about in any other way. Classification of natural objects, then, progressively approximates to objectivity, but is never perfectly objective. It is, however, complete, in the sense of covering the whole ground—that is to say, complete in a somewhat external way, as an inventory may be complete, whatever the objects thus assembled, and whether they have any inner relation to one another, or not.

Types of Classification.—The most elementary type of classification is the *inventory*, a simple mathematical model with no pretensions to going deeply into the nature of the subject studied. Objects are simply numbered, quite arbitrarily, as 1, 2, 3, 4, . . . , in the order in which the classifying clerk happens to come across them. The best known and perhaps most frequently used of all types of classification is a refinement upon this. It is called *index classification*. This also is largely accidental and arbitrary, and does not go far into the nature of the subject. But by confining the number of classes to twenty-six which are grouped from A to Z, a great step has been taken towards introducing order and system. Because of its very great convenience for purposes of reference, this alphabetical model is used in classifying all sorts of objects—*e. g.*, in filing away letters of all kinds, in libraries, in commercial offices, in administrative work, in research work, and generally, wherever it can be applied. There are many variants upon this principle, and we have

card-index systems based upon the days of the month, or of the week, or of the hours of the day, or of a series of years, *etc.* There is no absolute limit to its usefulness, or to the variations of model which may be employed.

Another common kind of classification is the diagnostic type. This resembles index classification, in that it is used largely for purposes of reference, but at the same time goes somewhat further into the nature of the subject studied. The characteristics used for forming the groups are selected upon the basis of being striking and immediately evident, and thus tend to be somewhat external and superficial. But they are not so superficial as the initials *A, B, C, . . .* Thus a physician readily classifies a disease by reference to the most striking symptoms, and many an amateur botanist finds out to what family the specimens he discovers belong, by looking them up in a book especially written from this viewpoint. So too the common way of judging character on the basis of general appearance and readiness in conversation belongs to this type, and in fact most of us carry around in our heads a ready-reference system of this general type, for dealing with any subject in which we are especially interested.

A further type of classification, common in pure science, goes more deeply into the general nature of the subject studied. Thus, the kind of classification which we find in zoology and botany attempts to group together animals, or plants, according to their family relationships, and generally to trace their descent, according as the various genera and species seem to have developed in nature. Because biologists thus follow lines of organisation established by nature, this type is sometimes referred to as *natural* classification, though there has been among logicians an attempt to extend the usage of this term so as to cover all cases of classification of a certain type. Thus classification tends to be called *natural*, where the grouping seems to deal with the subject less from isolated and arbitrary viewpoints which cut across all "natural" class-distinctions, and more from insight into some law which seems fundamental in explaining the various characteristics of the object as a whole, as the principle of evolution helps to explain a very great number of characteristics in biology and anthropology. We must admit, however, that from a strictly logical viewpoint this form of classification also is arbitrary; for the scientist has a special interest in tracing lines of

descent, and to group animals in terms of mere family relationships may be highly artificial. We see this especially when it comes to classing together objects whose structure is very different—such as (1) thriving members of a group and (2) degenerate parasites which are the suckers and hangers on of animal society, *sans* eyes, *sans* legs, *sans* almost everything except their great thirst and their family tree.

If, however, we set out to classify the various typical forms of classification, we soon find that they are too numerous, and based upon too great a variety of interests, to be fully classified. Some follow structural lines, others follow functional lines, and many of the special models employed defy any general naming. The fact is, types of classification may be, and should be, nearly as numerous as the interests and questionings with which we approach the phenomena of experience. These, however, are too numerous and too diverse to be classified profitably, at least at the present stage of knowledge.

Validity of Classification.—Not all classifications are correct. On library shelves, books are generally grouped together primarily in respect of content and secondarily in respect of alphabetical considerations. Thus, all encyclopedias are grouped together, and all books on logic are grouped together, although within the group the books written by authors whose surnames begin with *A* are placed first, *etc.* It so happens that a well-known work on logic was originally published as volume 1 of a projected "encyclopedia of the philosophical sciences." The other volumes have never appeared, but the volume on logic will, in many libraries, be sought in vain where it should be—among the books on logic, and will be found where it has no right to be—between two of the encyclopedias. Classifications are thus sometimes incorrect. On what does their validity or invalidity depend? It depends solely upon whether they serve their special purpose, and help us to understand the objective nature and objective connections of the phenomena studied. Thus a certain group of organisms is classified by the botanists as belonging to plant-life, under the name *Myxomycetes*, and by the zoologists as belonging to animal-life, under the name *Mycetozoa*, and among primitive organisms many are grouped in this two-fold way. Yet such classifications are perfectly legitimate, for they certainly help in understanding the phenomena under study,

and these certainly have connections with plant-life on the one hand and animal life on the other. *Any* method of classification, or any group of methods,¹ is valid, so far as it brings us into objective contact with the phenomena under study, in such a way as to help on the advance of science.

Function of Classification in Exposition.—In scientific exposition, classification and division exercise two main functions. In the first place, classification has the preliminary function of collecting and arranging the material as a preparation for proof. The importance of this function can hardly be over-estimated. It is only so far as the material has been well organised in this preliminary way that we can be sure, *e. g.*, that our proof has dealt with all the points which stand out as important, and also that it has covered the whole ground. That is to say, the objectivity and completeness of an exposition depend largely upon the efficiency of the preliminary classification. Incidentally a good classification adds to the clearness of our exposition. When we can see that a subject properly has three main divisions, each of which has two sub-divisions, *etc.*, that of itself assists us in seeing our way through the subject. The first function of classification, then, is the preliminary work of so organising the material for exposition that we can proceed to a proof which shall plainly be objective and shall patently cover the whole ground.

In the second place, classification and division are, as we have seen, a form of organisation which is of a certain general type. When we classify, we place individuals in a class which is itself part of an organised system, and the individuals receive a considerable increment of meaning from being placed in such a class. For instance, man as a bare individual, apart from his place in society, is a poor thing. Alexander Selkirk was monarch of all he surveyed, but he surveyed little which was of importance to him as a man, unless he gave it a distinctly social reference. Place the individual in the class "member of a family," and his significance at once increases proportionately. As a husband and father he is more of a man than when monarch of a desert island, and if we place him in the class "citizen"—*i. e.*, in the class of men who think for themselves on political questions, and vote

¹ *E. g.*, Bosanquet's application of botanical categories to logic. This "transgression into another kind" was deliberately practised by Royce in seminar-work with advanced students.

as they think right, his significance increases still more. He takes his place in the forward march of humanity. When further we place him in relation to science, art, and religion, we begin to realise something of his full stature, and to form a more adequate idea of man's place in nature. That is to say, this second function of classification consists in remedying, to some extent, the one-sidedness and arbitrariness of many of our preliminary classifications. Final classification endeavors to take a large view of the subject in *all* its more fundamental relations. In this way it leads gradually to placing the subject in its full setting, in its proper place in the system of scientific knowledge.

Summary.—Classification is thus a kind of organisation which assists in the advance of science. In exposition it gives us clearness, objectivity, and completeness, especially when we are dealing with mind-made entities, but also to a considerable extent when we are dealing with the world of natural phenomena. Its typical forms are valid so far as they lead to genuine insight, and classification as such on the one hand prepares the way for proof, and on the other leads logically to the construction of a system of the departmental sciences.

FOR FURTHER READING

H. Lotze, *Logic*, pp. 120–142. Chr. Sigwart, *Logic*, Vol. II, pp. 158–168. W. Wundt, *Logik*, (3rd Edit.), Vol. II, pp. 47–64.

EXERCISES

1. In how many ways might the following be usefully classified: Man, month, mountain, river, wind, nation, island, country, tree, state, city, girl, book, frog, bulrush, rhododendron, life, consciousness, Robinson Crusoe, Mt. Blanc, Ruskin, October, America, oak, Minneapolis, death, plant x^2 , dinner, penknife, prayer, New York, Mary, animal?

2. Is there any viewpoint from which the following classifications are strictly legitimate: (a) Clothiers, land-ladies, book-sellers. (b) Children, barking dogs, automobiles. (c) A candid friend, and a deadly enemy. (d) A lama (as defined in the exercises to the preceding chapter) and an unsuccessful literary man. (e) Poetry, painting, music, dancing. (f) Corn-stalks, wood shavings, and old newspapers?

CHAPTER XXIX

PROOF

Proof is considered such an important part of logic, that certain logicians have defined logic as the science of inference and proof, and in the ordinary consciousness there still lingers on, the medieval conception of the logician as the trained reasoner "who makes the schools ring with his *sic probo*." In modern logic, however, it is discovery which is regarded as the chief function of trained thought, and proof is relegated to a very secondary position. It is still regarded as important, and part of the training of every scientist consists of proving some thesis, usually in connection with the attainment of an academic degree. In exposition it is vital, and almost all the methods used in definition and classification are employed as preliminary to the real work of exposition—proving one's thesis.

Nature of Proof.—We prove by first constructing a hypothetical mental model and then testing it to see whether it is correct, *i. e.*, whether it actually applies in detail to the situation in which we are interested. In certain cases the mental model is a re-construction rather than a construction. Thus, given a long addition sum, we construct a mental model by adding from below upwards, from the bottom of each column to the top, in the ordinary way. But in order to prove whether our answer is correct or not, we re-construct the situation by adding again. We may add in precisely the same way as before, or we may start at the top of each column and proceed downwards. Another form of reconstruction in frequent use is to divide the whole column up into tens, and add up each ten lines separately, then each ten of those ten, and so on, until the whole is added. The second addition, or the addition in some other direction, is a way of guarding against misleading associations, and assuring ourselves that we have really counted all the figures and have omitted or misread nothing.

Let us examine another example. Let us prove that

$AB=BA$. We begin by constructing a mental model. Let
 $\cdot \cdot \cdot \cdot = A$, and let $\cdot \cdot \cdot \cdot = B$. Then $\cdot \cdot \cdot \cdot = 1$ row of B .

$\cdot \cdot \cdot \cdot = 2$ rows of B , and $\cdot \cdot \cdot \cdot = A$ rows of B .

We now proceed to *prove* that A rows of $B = B$ rows of A . If we turn the mental model which we have constructed, upon its side, we see that the top row makes 1 row of A , that the two top rows make 2 rows of A , and that the whole figure makes B rows of A . That is to say, our mental model is at one and the same time, (1) A rows of B , and (2) B rows of A , and we can realise this by counting vertically and horizontally. We then, by a reconstruction in which we use crosses or other symbols in place of the dots, come to realise that the symbols used, and also the actual number of the symbols, make no difference to the truth of the model—so that $A \times B = B \times A$ generally. Q. E. D.

The whole point of the proof in this case consists simply in our apprehending what we have done in our construction. We find that in constructing A rows of B , we were at the same time inevitably constructing B rows of A . That is, we find that AB and BA are two aspects of a single construction. It is a case of reason apprehending what it has itself put into the figure, and becoming perfectly conscious of the implications of its own procedure. This example may be regarded as representative of all algebraical proofs of the kind we use in solving problems by means of equations—let $x =$ this and $y =$ that, *etc.*—and indeed as representative of all mathematical proofs generally.¹

Let us take a non-mathematical case. In order to prove whether *black* is or is not a positive sensation, we first construct the appropriate situation, and then observe whatever is to be observed. The “construction” here consists in entering the laboratory dark room and closing the door. The conditions being experimentally controlled, we may be certain that no ray of light will enter to stimulate the eye, and that accordingly here, if anywhere, the sensation of black is to be experienced. The “proof” consists simply in observing closely our visual sensations, from the first confused blur of after-

¹ The student is advised to look up a few of the Euclidean proofs, with their three stages, (1) statement of the problem, (2) construction of an appropriate situation, and (3) “proof,” or insight into the relations involved in the constructed figure, in order to verify this statement.

images which we find on first entering the room, to—whatever ultimately results. When we reach this ultimate state of the visual organs—say in thirty or forty minutes—we perform the experiment over again, and persuade other persons to join us. We also try various other kinds of construction—such as looking at a piece of “Hering black” paper, at a piece of black velvet, *etc.* If the results agree, we regard our answer to the problem as proved, much as in the case of the addition sum considered above. The stringency of the proof depends largely upon the appropriateness of the situation and the strictness with which the conditions are experimentally controlled. Thus the velvet is better than the paper, and the dark room is better than either, and here also, the whole point of the proof seems to consist in our apprehending what we have ourselves brought about by means of the “construction.”

Aim of Proof (A) Objectivity.—The aim of proof in logic is always, in the first place, objectivity. It is from the structure of the phenomenon under study that we try to prove that things must be so and not otherwise. A mental model which was not a model of the phenomenon under consideration would be so far irrelevant, and worthless as evidence. We proceed by constructing the phenomenon itself, or at least a mental model which is as objective as may be, in order to see our way into the case actually before us. In proving his thesis, a scientist always endeavors so to arrange and marshal the objective evidence as to make it plain to his colleagues that he has kept in closest touch with the objective facts throughout, and that his construction is of objective significance. Anything else would be recognised as being beside the point, and thus, from a logical point of view, entirely worthless.²

(B) Completeness.—In the second place, proof aims at completeness. If there are five vital elements in the situation, it will not do to prove only two or even three. It is necessary to prove all five. For example, if we wish to prove that Mr. X’s dog bit the president’s dog so that the president’s dog died, it is necessary to prove (1) that one dog did actually bite another, (2) that the dog which did the biting was Mr. X’s dog, and (3) that the dog which was bitten was the president’s dog, and (4) that the bitten dog died, and (5) died in consequence of the bite. In a case of

² Cf. the case of *N*-rays, referred to above (p. 192).

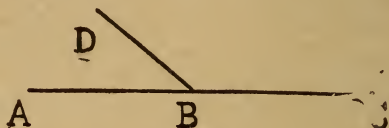
this kind which the writer has in mind, (3) and (5) were never proved, though there was "strong presumptive evidence." In actual fact, however, the president's dog was not bitten, and is still alive.

Our construction of the mental model must include *all* the relevant features. Thus, in the attempted "proof" that the universe is equal in size to a lump of sugar, because each is divisible to infinity, and infinities are equal—it is forgotten that division in no way increases or diminishes the *size* of the object divided, so that if the sizes were strikingly different before division, they will remain strikingly different after division, however much infinities may be *numerically* equal. The omission would be at once observed if it had been urged that each object could be divided into two halves or four quarters, and that two halves = two halves, or four quarters = four quarters, *etc.* But once the word "infinity" is used, the vital omission seems to escape our attention. Our mental model, then, must be not only objective, but also complete.

How Far Realisable? (A) With Mind-Made Entities.—How far can this aim be realised? Let us consider first the case of mind-made structures. Mathematical examples have been given above, and mathematical proof is sometimes taken as a type of what demonstration should be, strict, rigorous, exact. We proceed by constructing a mental model, composed of dots, lines, or other quantitative or spatial symbols, and seem by these means to obtain an insight into the interrelation of the parts of our mental model—an insight which is usually satisfactory in point of both objectivity and completeness. In the world of mathematical entities, where our mental model and the subject we are studying coincide, there can be no doubt of the objective reference of our procedure. We construct the object itself, and in so doing come to realise its various implications, in a way which we find adequate, though not, perhaps, incapable of improvement.

In respect of completeness, however, a certain doubt may be felt. It is well known that the proofs of many of the theorems in Bk. III of Euclid employ the full definition of the circle, in cases where a knowledge of conic sections shows that something less than the full definition of the circle would have been sufficient. That is to say, certain of these proof-models are *too* complete, in that they use too

much argument and evidence, including certain elements which, strictly taken, are irrelevant. Further, the existence of "alternative" proofs raises a certain doubt. Are such proofs equally objective, and equally complete? Compare, *e. g.*, the modern proof of Euclid 1. 13 with the original proof. The modern proof takes a straight line ABC , and points out



that the angle $ABC = \text{two right angles}$. By taking a straight line BD , pivoting on B , and moving from a position coinciding with BA to a position coinciding with BC , it is easily realised that the space ABC remains equal to two right angles, however it may be divided up by the successive positions of BD . This is far more direct than the Euclidean proof, which makes use of the addition of equals to equals, *etc.*, and does not give so much insight into the spatial relation involved as is given in the modern proof. The proof which is more direct appears to be more objective and more complete.

Let us take another example. How do we prove that "The supply of game for London is steadily going up . . ." means "The game is up . . ." ? We prove it by constructing the plan of the cipher—every third word—and then reading off the first, fourth, seventh, *etc.*, words, and finding that these make sense, and a sense which is strictly appropriate to the whole situation. The proof consists in constructing a mental model which really gives insight into the relations involved, and is in fact the model in accordance with which the cipher was originally constructed. The clinching element about the proof is that it *works*. Its details coincide with the details of the example, just as in Euclid's superposition method the triangle ABC coincides with the triangle DEF , and it makes sense, *i. e.*, perfectly fits the circumstances. Such a proof is both objective and complete, for the mental model exactly coincides with the object in question. In geometry, however, where we are studying the nature of space, and this is not entirely mind-made even though we move in an almost closed circle of definitions, postulates, and axioms,—*i. e.*, mental models—our proof was not so

wholly objective and complete. In dealing, then, with mind-made entities, we can say that, so far as they are truly mind-made, our proofs can be both objective and complete, but that so far as they are not fully mind-made, but deal with such an entity as the nature of space, they are not fully objective and not fully complete, but admit of scientific progress in both these directions.

(B) **With Natural Phenomena.**—In dealing with objects other than mind-made entities, we use, as we have already seen, mental models for most of our intellectual operations. Proof is no exception is this general rule, and proof moves wholly within the realm of mental models, especially of a mathematical type. We realise this especially in the case of physical science. Such proofs deal with objects only in respect of their mathematical properties—*i. e.*, only so far as they coincide with ideal units, ideally straight lines, *etc.* Thus we prove that if a ladder slides down a wall, the path described by someone who is in the middle of the ladder will be the arc of a circle. But this is strictly true only if we suppose the ladder to be fairly represented by a mathematically straight line, the side of the house and the surface of the ground by a mathematically exact right angle, and the person in question to be occupying the mathematical center of the ladder. It is well known that a certain allowance has to be made, in practise, for some divergence from the exactitude of the mathematical model, and that such proofs are thus not entirely objective and not entirely complete.

Let us take another example. In laboratory psychology there is an experiment with free associations which bears upon criminology. A student commits one of two artificial "crimes," the conditions of which are established beforehand by the experimenter. The experimenter does not know which of the two he has committed, but proceeds to test him by calling for associations in connection with a list of stimulus-words, some of which bear upon crime *A*, and some on crime *B*. The reaction-time for each association is taken, and if the reaction-time for associations connected with crime *A* is noticeably longer, on the average, than the reaction-time for associations connected with crime *B*, the student is pronounced guilty of crime *A*. The proof consists in constructing the two possible association-situations, and seeing which gives the longer reaction-time. There is an average reaction-

time for every individual, and a noticeable departure from that reaction-time must have a special reason. For laboratory purposes, such a proof is regarded as sufficient.³

In actual laboratory practise, this mental model works fairly well, and in medical practise Psycho-analysts use it as a regular method of investigation. But it is neither perfectly objective nor perfectly complete. There are nearly always lengthened reaction-times on the side of the crime which has not been committed—though these are not, as a rule, numerous—and the reactions to words of criminal import are not always lengthy, but may be within the limits of probable error. There is thus room for improvement in the technique of the method, and in general we may say that wherever, as in the case of such natural phenomena, there is a gap between the mental model which we employ and the facts which we are attempting to study by its means, our proof falls short, precisely to that extent and for that reason, of objectivity and completeness.

Validity of Proof.—In the ordinary use of terminology, an argument or mental model *must* be valid, before it is dignified by the name of “proof.” If it is not valid, if it falls short of proof, it is given some other name, such as “presumptive evidence.” We shall accept this terminology, and shall refuse to regard as proofs, mental models which are invalid. All proofs, then, as such are valid. But, as we have seen, there are degrees of validity. The history of a science such as mathematics sufficiently shows that a number of alternative proofs of a conclusion are equally possible, but that some of them enter more directly into the nature of the relations studied—*i. e.*, are, as we have seen, more objective and more complete than others. Validity is thus seen to be a matter of objectivity and completeness, and the degree to which a proof is valid can thus be judged adequately only in the light of further progress in scientific knowledge. Evidence may be sufficient to prove our point without exhausting the possibilities of proof. For example, circumstantial evidence is often sufficient to prove a man guilty in the courts. But few authorities would regard circumstantial evidence as the most satisfactory form of proof.

³ The method is sketched very briefly. For further information, consult A. A. Brill, *Psychoanalysis*, Ernest Jones, *Psychoanalysis*, and the papers by Jung and Freud in the *American Journal of Psychology* for 1910.

It is a little indirect and external, and evidence of this kind, which looked like overwhelming proof, has occasionally been overthrown by evidence of a more direct character. Proof, then, is valid so far as it gives genuine insight into the relations in question, *i. e.*, so far as our mental model coincides in its main outlines with the structure of the object with which we are dealing. Except in the case of purely mind-made entities such as ciphers, it is never final, but is progressive, and the degree of its validity can only be judged in the light of further scientific advance.

Types of Proof.—The most frequent model used for purposes of proof is undoubtedly the mathematical type of model. In physical science the importance of such a type of proof has long been recognised as supreme, and since the days of Plato the mathematical model has been regarded as constituting almost the ideal kind of proof. But taken strictly, there are at least as many possible types of proof as there are possible subjects of exposition, and for some of these the mathematical type of proof would be regarded as merely preliminary, while for others it would be wholly inadequate. For example, in ethical and religious questions, a mathematical type of proof may well be used in marshaling evidence and arranging one's data, but such a method is merely preliminary.⁴ It is of assistance in preparing the ground, but in dealing with an ethical question, what we desire is an insight which is ethical. So too in dealing with historical questions, or questions of musical technique, mathematical types of proof can at best play only a very subordinate part. On the whole, then, there are so many types of model which can be regarded as possible, that it is unprofitable to attempt to enumerate and classify them.

It is usual, however, to distinguish two typical forms of proof which differ, not in respect of the kind of model used, but rather in the way in which this is applied and in the kind of insight to which it leads. These are known as (1) direct proof and (2) indirect proof. All the instances previously studied in the chapter would be considered cases of direct proof. Direct proof attempts to construct such a mental model of the situation with which we are dealing,

⁴ For the scientific application of mathematical models to religious questions, consult the *Journal of Religious Psychology*, and on ethical questions, the *International Journal of Ethics*.

that it shall be possible to attain to a simple, straightforward, and direct insight into the relations involved, as when we superimpose one triangle directly upon another, or when we reconstruct a situation which is in question, by appealing to the evidence of trust-worthy eye-witnesses. Indirect proof is like the *reductio ad absurdum* in Euclid. It attempts to prove that *A* must be *B* on the ground that the contradictory supposition leads to absurdities.

Let us consider an example of indirect proof. To prove that *A* did not personally murder *B*, in spite of his known motive to do so, and in spite of circumstantial evidence against him, it is enough to prove that *A* was in another town at the exact time when the murder was committed. The *alibi* is a convincing form of indirect proof. We construct a mental model of the situation, and find that it cannot possibly be made to square with the supposition of *A*'s personal guilt. That is to say, from direct insight into the requirements of the situation we see that another suggested mental model will not fit. Into this incompatibility also we have an insight which is direct. An indirect proof of any statement thus consists of a direct refutation of the contradictory opposite. Hence it has been suggested by Herbert Spencer that a criterion of truth is the inconceivability of the opposite, and in practise there is no doubt that indirect proofs may be of great assistance in bolstering up an attempt at direct proof which is not perfectly convincing. But what is convincing about the so-called "indirect" proof, is not its *indirectness*, but its *direct* side. We believe that *A* could not possibly have personally murdered *B*, because we have direct proof that he was in another town, and because we can see *directly* that it was necessary for him to have been at the scene of the murder if he is to be regarded as guilty of the charge. We compare these two mental models, and see *directly* that they are incompatible. The nature of proof, then, is fundamentally to be direct, and so far as it falls short of directness, so far it falls short of convincing us of its validity. Logically, then, proof is always direct.

Summary.—We prove by constructing a mental model of the situation in question, in such a way as to make clear the inter-relation of elements in the model which we have constructed. Reflection upon our own construction leads to an insight which is direct and convincing. The aim of such

mental models is objectivity and completeness, and in cases where our construction gives us the object itself, this aim can be attained. In the case of natural phenomena, however, where there is a gap between the mental model and the phenomenon in question, we can only approximate to complete objectivity. While it is usual to distinguish an "indirect" form of proof, the nature of proof is essentially direct, and the degree of its validity can be determined only in the light of further advance in knowledge. As knowledge is never complete, but progressive, so proof is never absolutely final, but advances with advancing science.

FOR FURTHER READING

H. Lotze, *Logic*, Bk. II, chapter iv. Chr. Sigwart, *Logic*, Vol. II, pp. 192-210. W. Wundt, *Logik*, (3rd Edit.), Vol. II, pp. 65-85.

EXERCISES

How could you prove: (a) That you really are where you think you are. (b) That Napoleon ever lived. (c) That $459 - 387 = 72$. (d) That a bird in the hand is worth two in the bush. (e) That a university degree is a desirable asset. (f) That Beethoven was one of the greatest musicians who lived within the last two hundred years. (g) That lying and stealing are wrong. (h) That life is worth living. (i) That corn will not grow if planted early in the spring?

CHAPTER XXX

FALLACIES

Fallacies are older than logic. Indeed, one of the chief motives which first led to logical study was the reaction from sophistry or the deliberate use of fallacies to deceive and entangle others. In consequence of this interconnection between the search after truth and the avoidance of falsity, some study of the nature of fallacy has always formed an integral part of scientific method. No one entirely escapes falling into these errors. Even logicians of the caliber of John Stuart Mill have made mistakes in this way which others have pointed out, but which they themselves were never able to see. At the present day, fallacious arguments are not so frequently used with deliberate intent to deceive as was perhaps the case in ancient Athens, and it is more a question of putting ourselves upon our guard, so as not to fall into these traps which await each one of us, and so deceive ourselves.

Definition of Fallacy.—Fallacies are sometimes defined as failures to prove—*i. e.*, as though it is in relation to attempts at *proof* that they are especially noticeable. It is certainly true that a fallacy proves nothing, and that if the aim of the thinker who fell into the fallacy was to prove something, he has failed of his aim. But this definition is not wide enough for scientific accuracy. Many a fallacy into which we frequently fall can hardly be brought under the head of proof. For instance, there may be failures to judge or to infer correctly, failures in analysis and synthesis, in abstraction and determination, failures in induction and deduction. And though all of these *may* indeed be used as methods of proof, they are far more frequently used as methods of investigation. It is thus possible to fall into fallacies in respect of investigation, as well as in exposition. So too in exposition there are fallacious attempts at definition and classification, as well as at proof. There is no limit to the opportunities for error, and they are quite certainly not restricted to the

field of attempted proof. We shall therefore define fallacies as *mistakes in thinking*—using the term *thinking* in its widest possible sense.

Occasions of Fallacy.—The pursuit of truth takes place, as we have seen, by the construction of mental models. Fallacies arise when these mental models are used in some way which is incorrect. There are three main possibilities of such incorrect use:—(1) in relation to the data which furnish our starting-point—the mental model which we construct may misrepresent the concrete situation which we are attempting to understand, as when the paranoiac interprets the most innocent actions of the persons around him as the deliberate designs of conspirators. (2) In the second place, there may be no mistake about the data as such, or about their relation to the mental model, but the model itself may contain some logical flaw, such as an inconsistency. This is frequently the case with our typical rules in ethics and esthetics. Unless formed with the utmost care, these rules often contain certain inconsistencies which only become apparent in the course of time, as their consequences develop and lead, perhaps, to results the opposite of what we had intended.¹ (3) In the third place, the application of our mental model to concrete facts may be careless and so lead to mistakes, as when we attempt to apply any system of theoretical principles in practice, or to carry out general orders in detail. The application of *any* general principle is full of dangers of this type.

Fallacies of this general kind may arise either in investigation or in exposition. In exposition, however, there is an additional occasion of error. This arises from the fact that in exposition there are at least two parties, the writer or speaker on the one hand, and the reader or hearer on the other. There is thus a certain duality of outlook which leads, perhaps inevitably, to certain mistakes in understanding. The speaker has *his* set of mental models, and the hearer has *his*. These two sets are bound to be partly different, in view of the differences in education and in habit of mind. The hearer translates what he hears, into his own set of mental models, and in so doing can hardly escape a large number of errors. We see this most clearly, perhaps, in the case of foreigners.

¹ For a number of instances of rules taken even from physical science, cf. F. H. Bradley, *Appearance and Reality*, the first half of the book.

When we hear an address in French or German, we translate it into our own ideas or mental models, and there is, as we all know, much which cannot possibly be translated with absolute correctness. And if, perhaps, we do not understand French or German correctly, and the foreigner in question can not understand *us* correctly either, our attempts at conversation are pitiable indeed.

Such fallacies arise from a difference of mental model. What *A* means in one sense, *B* may translate into an entirely different set of mental models. This can take place in a single language just as well as in two, as will be seen if we consider one or two instances.

Where do you live, Pat? With Mike.

Where does Mike live? With me.

But where do you and Pat both live? Sure, isn't it *together* that I'm telling you we live?

The difficulty arises here from perfectly honest misunderstanding. The mental model of the questioner is spatial. He wishes to know Pat's street address, whereas Pat's mental model is social, and has not the remotest glimmering of a connection with spatial questions. Such mistakes are extremely common in every walk of life, as well as in scientific exposition. We may, perhaps, notice one more example.

She: Do you admire me for my intellect, or for my beauty?

He: Not for your intellect.

She: Flatterer!

In this case *She* assumes as an explanation of their situation the mental model of admirer and admired, while *He* assumes the mental model of borer and bored. In this case the divergence of mental models is very thinly disguised, and may well be deliberate on both sides, as is so frequently the case on occasions which call for the employment of social "tact."

Such cases of a confusion of two logically distinct mental models may even happen to a single individual. The best known instance is Mill's famous fallacy:—

The only proof that an object is visible is that people actually see it. The only proof that a sound is audible is that people actually hear it. And so of the other sources of our experience. In like manner, the sole evidence that anything is desirable, is that people actually desire it. People do actually desire

Pleasure. Therefore, Pleasure is desirable, *or Good, and in fact the Chief Good.*

The argument proves that pleasure is something which people can desire, but Mill takes it as proving that it is a "good" or something which they *ought* to desire. The confusion is thus between a psychological and an ethical mental model. Such mistakes are very common in attempts to prove ethical, esthetical, or religious beliefs in terms of models which are psychological, economic, or biological—*i. e.*, other than ethical, esthetical, or religious.²

Yet another occasion of fallacious thinking arises from the way in which the written or spoken language may suffer from slips in the mechanism of expression, so that one model is intended, but another suggested. This is extremely common. *E. g.*, from a New York paper:

WANTED. A groom to look after two horses of a pious turn of mind.

A second-hand morris chair for a bachelor with richly carved claw-feet.

In every-day cases like these, we can usually distinguish what was intended from what is suggested. But any student who has done much translation from one language into another will know that there are many ambiguities, arising probably from some slip in the mechanism of expression, where no ingenuity can succeed in discovering what the original author may have meant. This is particularly well known in the case of the Greek and Latin classics, where a most elaborate technique has been developed for dealing with just such errors. In many cases, however, the text proves to be hopelessly corrupt, and the modern editor resorts to emendations of his own—*i. e.*, reconstructs the passage in accordance with a mental model which appears to *him* to satisfy the requirements of the situation. The fallaciousness of such emendations, however, is universally admitted.

From these considerations, we realise that the occasions of fallacies are to be sought in the relation of our mental models to the facts of experience. Our constructions may differ from the facts with which we start, or from the facts to which we wish to apply them, or from other possible models based upon the same data.

² Cf. Locke's "proof" that it is impossible to be a sincere Atheist, in the *Essay*, Bk. IV, chapter x.

Characteristics of Fallacy (A) Subjectivity.—However fallacies may arise, there are, however, certain characteristics which they one and all exhibit. In the first place, fallacy is subjective. All thought and all reasoning, whether for purposes of investigation or for purposes of exposition, ostensibly aims at truth— at bringing us into connection with objective facts and objective laws. It is because a thought somehow fails of establishing this connection that it is called a fallacy. We are left on the hither side of the fence, and take our own ideas, our mental models, for the realities.³ At times, we even take the symbolic expression, the word itself, for the reality, and try to substitute for insight into the reality a futile discussion based upon the philological characteristics of the word. Thus a student, asked in an examination in formal logic to define the technical term “contrapositive,”—which is one of the forms of “immediate inference”—answered that it was something which was (1) not positive, for it was opposed to the positive (*contra* in Latin means “against”), but also (2) not entirely negative. For example, “counterfeit money” was opposed to the positive, for it was false currency, but was also not entirely negative, for you could, perhaps, succeed in passing it!

As a general rule, then, we fall into fallacies, when this takes place, by getting lost in the mechanism of our own thinking, whether this is due to the complication of our mental models, or arises from substituting the word for the thing. The mental model comes between us and the reality, and our thought remains satisfied with a superficial interpretation, which seems good to *us*, at least for the time being, but will not withstand a serious comparison with the objective facts. Thus the hasty classical student translates the famous line *Frigidus in pratis cantando rumpitur anguis* as “The cold meadow-snake bursts into song,” (instead of “is torn asunder by magic charms”), or. . . . *et odora canum vis* as “and a powerful smell of dogs,” (instead of “and a keen-scented pack of hounds”),—and is thoroughly satisfied with his entirely original rendering. So too many novelists are satisfied that they are thoroughly in touch with real life, when as a matter of fact they are revelling in a world of mental models which are mental fictions. Subjectivity, then,

³ Cf., in this connection, the first few pages of Plato's *Republic*, Bk. VII.

or failure to get into touch with objective facts, is one of the chief characteristics of fallacious thought.

(B) **Incompleteness.**—A second characteristic of fallacious thinking, is its incompleteness. We leap at conclusions which are false, only because we do not pay sufficient attention to the evidence before us. If we always adhered to careful methods of analysis and synthesis, mistakes would more rarely arise. But we pass over something without noticing it, and the consequences of such a slip, slight perhaps in itself, may be serious. Thus, to suppose that the country is prosperous because many business men are making money, and all our acquaintances happen to be doing well, is a mistake which arises from incomplete observation. Other cases are not so serious. When the Home Guard Private asked, "If we join the National Guards, what will be our relations with the other units,"? it was sufficiently obvious that he meant, would his battalion—the parent organisation—be numbered 1 or 13. It was perhaps deliberate incompleteness of observation which made the Major (who didn't know) answer, "Friendly—at least I hope so." So too with the child's reasoning, "A penny is a copper (coin), and a "copper" is a policeman, and a policeman is an officer (of the law), and an officer (of the naval or military forces) is a gentleman; therefore a penny is a gentleman." Each step in continuing such an argument can be taken only by one who is wilfully blind to many elements in each transition. Incompleteness, then, is a second characteristic of fallacious thinking.

Scope of Fallacies.—There is no limit to the scope of fallacious thought. In the sphere of mind-made entities, mistakes creep in almost as readily as when we are dealing with natural phenomena. By a slight mis-drawing of the figure, which passed unobserved, it has seemed possible to demonstrate that one right angle is equal to, and greater than, another right angle, or that parallel straight lines meet before reaching infinity. So too in ethical, esthetical, and religious thought, the confusion of mind in which we so easily involve ourselves is too well known to require illustration. So also in attempting to solve unfamiliar problems, in mathematical as well as in every-day thinking, we often use the method of *trial-and-error*. We go astray a few times before striking into the right path. Still, in the end, we

can escape error almost completely in this field. But in the case of natural phenomena, complete objectivity, as we have seen, appears to be out of our power. Successive generations can approximate to a more accurate comprehension of the workings of nature, but a full comprehension is denied us. We use models which never quite fit the concrete circumstances, and our empirical interpretations are thus necessarily infected with error. To some slight extent our best and finest efforts at understanding the world in which we live are mistaken; and there is only one way of avoiding fallacy in this field—*viz.*, by preserving a slightly sceptical attitude of mind towards all claims of finality. To recognise the trap is to avoid falling into it, and while our thought in this field is necessarily imperfect, it is not necessarily fallacious. It would be fallacious only if we thought we *knew* in cases where we have only presumptive evidence. There is no fallacy, so long as we maintain the Socratic attitude, and at least know that we don't know. At the same time, we should transcend the Socratic position in believing that the broad basis of experience, upon which our modern science rests, enables us to approximate to a knowledge which for practical purposes is becoming progressively more adequate.

Source of Fallacy.—From the viewpoint of pure logic, there is no such thing as error. When we think logically, we think truly, and it is only so far as we fail to follow the rules of pure logic that we deviate into fallacies. A purely rational being never errs. True, perhaps,—but then, do we know any purely rational beings? Living, as we do, in a world which we experience through senses which are easily confused and deceived, and with a memory which we trust, in spite of its known treachery, only because we have nothing better in which to trust, and with powers of self-deception which frame, as valid and logical, reasonings which are mere distorted reflections of instinctive wants, or of social conventions which have long since lost what little semblance of reason they may once have possessed—is it any wonder that we fall, time and again, into the same old fallacies, as well as constantly blundering into new ones?

We have a dual nature. On the one hand, we have the demands of a logical reason, voiced in the ideal conceptions of truth, goodness, beauty, and the like. On the other hand

we have the mechanism of our nervous system, with its sense-organs at one end and its muscles at the other, an instrument devised for practise rather than theory, cradled in instinct and educated in custom, and inherently incapable of satisfying the ideal demands of pure reason. If with such an instrument we believe we can fulfil the demands of transcendent thought, we fall necessarily into fallacy. The only escape from this ever-present source of error is to recognise, once and for all, that perfect satisfaction of these ideal demands is out of the question for beings whose sole mechanism for fulfilling such demands is a central nervous system developed through the dark ages of animal evolution.

The utmost we can do is so to organise our experience as to approximate to making sense of it—to bring our ideals into connection with the facts, and to elevate the brute facts in the light of our ideals, as far as this may be possible, and thus create a science and a mode of life which shall combine actual experience and ideal desire, and gradually and progressively approach the haven where we fain would be. If we fully recognise this, the chief source of fallacy will be removed. If we know, not only what we want, but also what we can get, we are not likely to confuse the two. And if, avoiding that confusion, we go to work to create that science and that life which are possible for us, substituting breadth of experience where depth of insight seems denied us, we shall realise the fruits of the Socratic spirit, and shall act out the highest life which is in our power—a life self-determined, free, and raised above the deeper sources of self-deception.

Types of Fallacy.—There are no special types of fallacy. All fallacies partake of a single form—*viz.*, confusing mental models with the more concrete realities of experience,—and all attempts at enumerating and classifying typical forms of fallacious thought either (1) re-state the general nature of fallacy, or (2) mention some special occasion of possible error. But these are too many to be enumerated. For example, the best known type of fallacy is what is called *Petitio Principii*, or begging the question. It is usually illustrated by such examples as circular definition, or as the explanation of some event in terms of itself. *E. g.*, “A cause is that which produces an effect, and an effect is that which is produced by a cause,” “Wood is the ligneous part of trees,” “The poppy (in

medicine) sends people to sleep, *quia est in eo virtus dormitiva*," "We are able to remember what has happened to us because we possess the faculty of Memory," etc. A minor type of such false assumption is known as the "fallacy of double question"—*e. g.*, "Have you decided to settle down to a decent kind of life at last?", "What have you done to your coat?" A second well known type of fallacy is called *Ignoratio Elenchi* or irrelevant proof. This is illustrated by brow-beating a witness, attacking the personal character of an opponent, or raising some national issue, instead of arguing on the facts of the case, and generally by appealing to prejudice, hope, and fear, rather than to reason. But it is easy to see that every instance of false assumption is also an instance of irrelevant proof, and that every instance of irrelevant proof is an instance of false assumption. In fact, we have here, not two typical forms of fallacy, but two statements of the essential nature of all fallacious thought. To think that an assumption of ours amounts to proof is to confuse a mental model with the reality, and to appeal to emotion or prejudice rather than to reason, is to attempt to substitute a subjective mental model for an objective understanding of the facts. Such attempts are clearly re-statements of the essential nature of fallacy as such.

So too the celebrated division of fallacies into two classes, (1) *in dictione*, and (2) *extra dictionem*—*i. e.*, fallacies in language rather than in thought, and fallacies in thought rather than in language, respectively—breaks down in the face of serious criticism. In the first place, the distinction is thoroughly artificial—for as thought expresses itself in language, and language is a mere vehicle or mental model for expressing thought, *all* mistakes in language are due to mistakes of thinking. Thus the typical "fallacy of accent"—*e. g.*, "Saddle me the ass. And they saddled *him*"—could not possibly arise unless there were some inattention of thought and thus some failure to grasp the meaning. So too in the comic opera, when Patience, who "cannot tell what love may be," mentions that she once had a beloved playmate, "and, by the way, *he* was a little *boy*," the Chorus immediately reply that they "thought as much—*he was* a little *boy*." Patience rejoins, "Remember, pray, he was a *little boy*." The variations of accent follow the variations of meaning, and it is impossible to separate the words from the thought. In the second place,

if we admit that linguistic usage *is* at times misleading, as in the famous oracle, "Pyrrhus, I say, the Romans can subdue," such ambiguities of accident or syntax are but single occasions of error, and are as nothing when compared with the vast and unclassified field of such occasions. If we wish to avoid a fallacious and superficial clearness, we shall refuse to attempt a classification of these special occasions of error, and shall assert that all fallacies belong to a single type—*viz.*, the confusion of mental models with realities or with other mental models.

FOR FURTHER READING

W. R. Boyce Gibson, *The Problem of Logic*, chapter xxxlii. H. Plato, *Euthydemus*. Lotze, *Logic*, Bk. II, chapter vi. J. G. Hibben, *Logic*, Part II, chapter xvi.

EXERCISES

Are the following arguments fallacious, and, if so, in what does the fallacy consist: (1) "Who rules o'er" freemen should himself be free"? You might as well say, Who drives fat oxen should himself be fat! (2) It is a mistake to say that the best judges in matters of art are always in a minority. For, consider—suppose it true that the minority are always the best judges, and carry it to extremes. The smallest minority consists of one man. If the principle is true, then each man will himself be the best judge, and there will be as many best judges as there are individuals who differ from others, and thus constitute extreme minorities. But this is absurd. Therefore, the majority are always the best judges. (3) Let $x = a$, then $ax = a^2$, and $ax - x^2 = a^2 - x^2$ *i. e.*, $x(a - x) = (a + x)(a - x)$, and by cancelling, $x = a + x$, *i. e.*, $x = 2x$, or $1 = 2$. (4) I am a Chian, and no Chian can open his lips without telling a lie. Therefore I lied when I said I was a Chian, etc., so that I am not a Chian—in which case, perhaps, I told the truth, and thus am a liar after all. (5) Other people cannot be as sensitive as I am; for they do not make the same fuss about their feelings as I do. (6) Mr. X is a sound man for Senator, for he made an excellent after-dinner speech the other evening. (7) I could be a great artist, if it were not for my environment; for I feel it within me. (8) Mr. Z is not to be trusted as mayor for his table manners leave much to be desired.

CHAPTER XXXI

THE SYSTEM OF THE SCIENCES

In periods of scientific development, a tendency arises in the direction of applying scientific method in somewhat narrow channels, and perhaps—in view of the vast body of scientific knowledge and the exacting requirements of modern technique¹—such specialisation is inevitable. Inevitable or not, however, it is certainly the tendency, and the modern student, after years of study, sometimes complains of bewilderment. He cannot see the wood for the trees, and feels a need for breadth, as well as depth, of vision. He wishes to form a mental picture of experience as a whole, and not only of what he sees through his microscope or in his test-tube. In answer to this dissatisfaction and vaguely formulated demand, there have arisen attempts to systematise the results and principles of the various departmental sciences, and thus to give a single world-picture which shall be just to all the chief discoveries of science, and shall at the same time satisfy the craving of the mind for unity and totality. The best known examples of such attempts are found in the “Synthetic Philosophy” of Herbert Spencer, and—to a lesser extent—in the more exact but not less unwieldy tomes of Wilhelm Wundt. On a minor scale, however, most modern writers on logic attempt to draw together the various lines of inquiry and envisage them as a whole.

Nature of Such Systematisation.—Such systematisation is, in the first place, a last and most gigantic attempt at analysis and synthesis. We wish to take *all* knowledge for our province and put it together. That is synthesis. We wish also to put it together in a way so articulate and organised that we can see the inter-relation of the parts in the light of the whole. That is (partly) analysis. We wish our system to be both analytic and synthetic.

¹ This is true not only of laboratory sciences like physics and chemistry, but also of the social and linguistic sciences. History and philology have their technique as well as microscopy and histology.

In the second place, such systems are both abstract and determinate. They are abstract. It is impossible for one man to carry in his head at one and the same time all the detail of science, and consequently the world-picture which we desiderate, will have to be largely in outline. That is to say, it will necessarily be abstract. Medieval metaphysics, which largely follows the conception of Aristotle,² is highly abstract. According to this conception, we pass from the more concrete and detailed knowledge to the higher or "first" principles, simply by leaving out the detail and retaining the bare outline. The highest and intellectually most abstract of all is the concept of Being, and the chief function of a system of first principles is, from this viewpoint, to study the nature of Being *qua* Being—apart from its specific differentiations in the wealth of detail which we find in nature. A man, month, mountain, nation, wind, all *are*, or have Being. But they have different kinds of Being. The kind of Being which a month has is very different from the kind of Being which a man has, or even which a mountain has. The science of *ontology*, however, was intended to abstract from all these differences, and deal with the concept of bare Being. It is easy to see that the final world-picture in terms of pure Being, would turn out to be the barest of bare skeletons, with all the life and color gone.

In more modern times, however, and especially since the work of Hegel, thinkers have tended to regard the picture of the whole as more determinate and concrete, and to view the isolated fragments of the system—particular elements of experience—as thin and poor in content, in short as abstract. For the modern viewpoint, a concept is not abstract *qua* intellectual, and concrete *qua* sensory, but is abstract *qua* fragmentary, and concrete as seen in its place in a totality. For this view the concept of Being is not the poorest and most empty of content, but the richest and fullest of all contents. It contains within itself the principle of determination of all the specific forms of Being, and instead of being an abstract skeleton, made up of only the single element common to all entities, is the richest of all beings, containing as it does the infinite variety of nature, and *all* possible, as well as all actual, details. It is produced by addition rather than by subtraction, and is

² Aristotle's view is developed in the *Metaphysics*. Cf. in particular, Met. I, 1.

the sum-total of Reality, the Absolute or *ens realissimum*.³ It is thus the most concrete and determinate of concepts, though it is still regarded as partly abstract.⁴

In the third place, such a system is the final work of induction and deduction. It is to be the final mental model of the universe, and should sum up in itself all the preceding labor of analysis and synthesis, abstraction and determination, by proceeding to determine, as far as possible, the law of the Whole. It is usual to regard this law as the law of Reason, a principle of organisation, by means of which the whole universe is regarded as a vast individual, containing within itself both identity and difference.⁵

Finally, it is possible also to regard such a system as a single gigantic definition or as a final classification, and this view has exercised considerable influence upon the work of Spencer and Wundt. Such attempts at classification have been very numerous, and we shall consider some of the best known types later. In general, then, we can say that the nature of such systematisation is to complete, at least in general outline, the work of investigation and exposition, to round it off in some way, so that the specialist, at work upon his particular portion of the whole, may also form a mental picture of the whole, and may thus realise his unity and fellowship with his co-workers in other parts of the field.

Aim of Such Systematisation (A) Objectivity.—The aim of such systematisation is not, however, merely to frame a single model of the whole, so that the specialist may not feel lost or cut off from his fellows. We aim at something more than a cure for intellectual *Heimweh*. In actual fact, almost everyone frames some sort of idea of experience as a whole, of the meaning and value of life, and of the place of man in the universe—*i. e.*, reacts in some way to what have been called the Great Problems. These reactions, however, tend to be somewhat arbitrary and subjective, and reflect a somewhat narrow and eclectic viewpoint. Optimism and pessimism, for instance, are ordinarily somewhat shallow, and have little hold upon objective facts. They are usually prejudices, rather than scientifically tested models, and the aim of scientific

³ Cf. G. Simmel, *Hauptprobleme der Philosophie*, chapter 1, F. H. Bradley, *Appearance and Reality*, Part II.

⁴ Cf. H. Joachim, *The Nature of Truth*, last chapter.

⁵ Cf. F. H. Bradley, *Principles of Logic*, pp. 449–450.

exposition in systematising, precisely as in defining or classifying, is, before everything else, to be objective.⁶

It is not merely to satisfy our craving for unity and totality of outlook, that we systematise. For that craving can be satisfied by almost *any* sort of mental model. Every religion, every code of thought current in artistic, commercial, charitable, and family circles, has its own solution of this problem, and such solutions are found fairly satisfactory, so far as the needs of such circles go. But the scientifically minded man wants something more than the mere satisfaction of a subjective desire—he wants to know the facts. He wishes his system to be satisfactory, not merely to himself, in a subjective way, as a kind of registering of his private and unstandardised reaction to the universe as he happens to experience it. He wishes his system to be objectively valid, and to be true of experience as a whole. For instance, many people regard the whole universe as revolving around themselves or the interests of their immediate friends or profession. This is a narrow and prejudiced view, and the aim of scientific exposition is at something more all-inclusive, and more definitely in contact with objective facts and objective laws. The first aim, then, of such attempted systematisation, is objectivity.

(B) **Completeness.**—In the second place we aim, here if anywhere, at completeness. A partial or one-sided view is here wholly out of place. *Aut totum, aut nihil*. The materialistic view of the universe, for instance, is one-sided and incomplete. Viewing all reality as matter in motion, and all science as specialised effort to solve special problems of moving matter, it tends to leave out all the characteristic work of the mental and moral sciences, and is of very little use as a working hypothesis in the historical and philological sciences. It is incomplete, and thus, as a view of the whole, is a travesty of the facts.⁷ So too the study of truth-values, such as we have in logic and in the natural sciences, is one-sided, unless attention is paid also to the ethical, esthetical, and religious valuing, and a purely theoretical view of the whole is unjust to three-fourths of life—just as perhaps a purely practical view of the whole, or a view purely esthetical, is equally incom-

⁶ Cf. A. O. Lovejoy's presidential address to the American Philosophical Association, *Philosophical Review*, Vol. XXVI, 1917, pp. 123-163.

⁷ For a clear exposition of materialism, see Büchner, *Matter and Energy*.

plete.⁸ In attempting to set up a mental model which shall represent the universe as a whole, it is necessary to be at least as complete as is possible, and to leave unrepresented no class of experiences, no view of the facts, however distorted. The idea of the whole must be *all-embracing*—*i. e.*, must be complete.

How Far Realisable?—How far can we put together the various lines of scientific effort, and systematise them so as to present a world-picture which shall be both objective and complete? It is not difficult to see that this cannot be wholly accomplished. In the first place, our sciences are not wholly objective. They consist of mental models which do not perfectly correspond to the facts, but are in a process of transformation which renders them ever more and more acceptable from an objective point of view. Where complete objectivity is not to be found in the data to be synthesised, complete objectivity can hardly be expected in the total picture. In the second place, no single science is anything but incomplete, and it is not held that the subjects of scientific inquiry will ever be completely understood. Here also we have only approximation towards our goal, and here also we must admit that if the data are incomplete, the whole which is to be constructed out of such data must itself be at least equally incomplete. A synthesis of our various lines of scientific inquiry, then, cannot be wholly objective and cannot be wholly complete, in the sense of giving us a final world-picture. This has become so well known, that at the present day the idea of constructing such a world-picture has been abandoned, and in its place it is proposed merely to attempt to put together our mental models and instruments of investigation in a way which shall be just to *them*—*i. e.*, to relate the sciences, imperfect as they are, to one another, and to discover, if possible, the relation of historical inquiry to philosophical or to palaeontological research, or the relation of histology to botany and zoology, or the relation of psychology to the whole field of scientific inquiry, *etc.* In a word, the aim is no longer to present a final picture of the world as it *is*—for no one man and no group of men has the requisite knowledge—but rather to systematise the sciences as we have them at the present day, and thus to understand the interrelation of our own

⁸ Cf. Varisco, *The Great Problems*, pp. 26–27, 286 ff., Appendix V.

methods and mental models. As this inquiry is definitely and explicitly confined to the world of mind-made entities, we can approach it with more confidence. In principle, at any rate, it should be capable of realisation.

Types of Such Systematisation.—In the history of science, there have been many attempts at such systematisation, at embracing the work of science as a whole and viewing it from a single standpoint. One of the most famous is the system of Plato. He has (1) a general view of the whole field of human knowledge, and (2) a special view of the field of scientific inquiry. The general view is known as the “four stages of intelligence,” and is symbolised in the accompanying diagram. In the lowest stage of intelligence, we have the attitude of uncritical acceptance of any and every view, devoid of the faintest vestiges of scientific method. In the second stage we have the attitude of practical common-sense, which tests theories only in the light of their immediately practical workings. In the third stage we have what we should call the field of the departmental sciences. Things which we can touch and see are here dealt with only so far as they throw light upon laws in some department of scientific inquiry, and the interest in this field is an interest in law, rather than in things or opinions. In the fourth and final stage, an attempt is made to transcend the limitations of the departmental sciences and construct an undepartmentalised view of reality as a whole, by the use of pure reason. This is the field of metaphysics.

So much for the general view of human knowledge. The third stage, which contains the field of science, is divided up in accordance with the principle of passing from the simple to the complex, and turning the soul from the changeable to the permanent and eternal. First we have arithmetic, then plane geometry, solid geometry, astronomy, and harmony, and finally a study of the mutual association and relationship of these sciences, which teaches us the ties which bind them together—what we might, perhaps, call epistemology. This bridges the way to the science of sciences, dialectic, which corresponds, in a rough and approximate way, to what we have recognised as the field of transcendent judgments.⁹

The number of sciences thus recognised by Plato is extremely small—“imperfect” and empirical sciences are expressly

⁹ Cf. Plato's *Republic*, from the last few pages of Bk. VI to half way through Bk. VII.

Fourth Stage

Dialectic or Metaphysics

Third Stage

Science

Epistemology (?)

Harmony

Astronomy

Solid Geometry

Plane Geometry

Arithmetic

Second Stage

Common Sense

Lowest Stage

excluded from his scheme—and all belong to a single group, the mathematical group, which leads on to epistemology and metaphysics. The philological, social, and historical sciences find no place in the scheme,¹⁰ and the sciences which he does admit, are admitted only on the ground that they are busied with the eternal rather than with the empirical, and are thus adapted to turn the eye of the soul away from the world of sense-perception, and to develop our powers of “pure” reasoning.

This view has been, and still is, extremely important in its influence upon religious minds, and upon the minds of those who are especially interested in the ideal development of

¹⁰ Plato does, perhaps, recognise political science, but this represents the practical application of dialectic, and hardly finds a place in the third stage of intelligence.

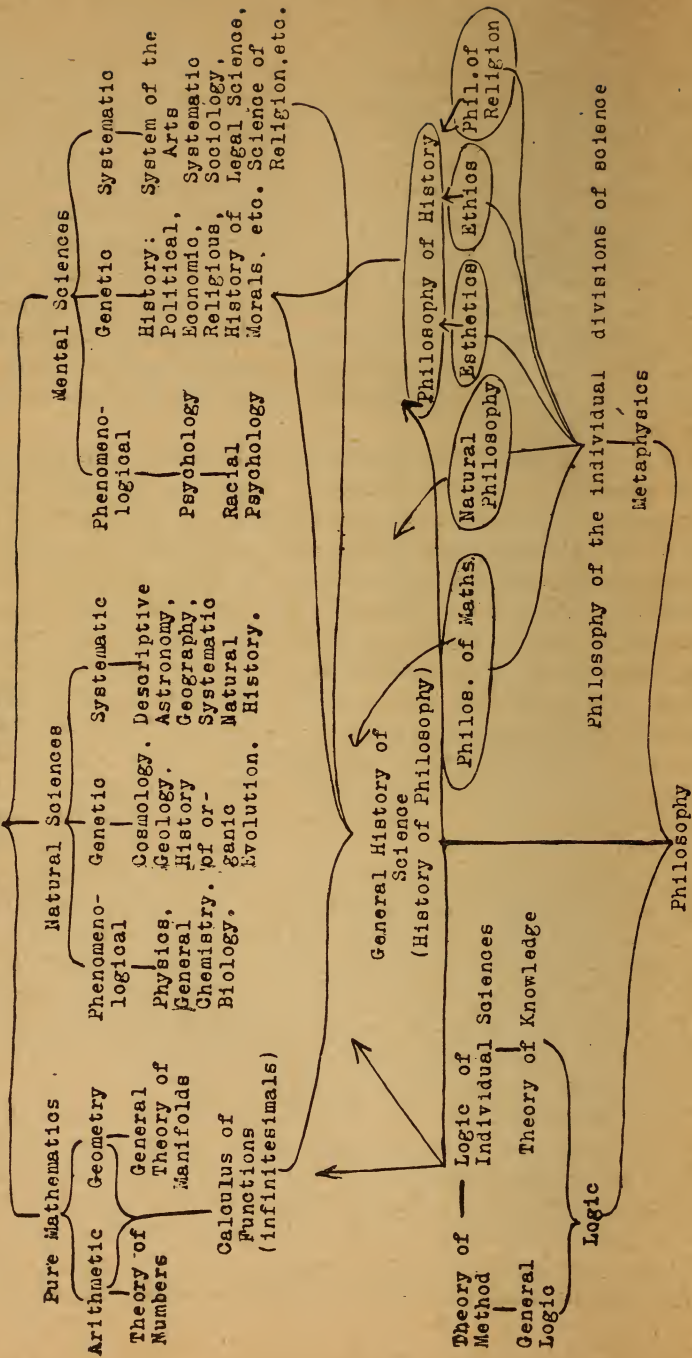
humanity. But from the point of view of the scientist who wishes to be just to *all* lines of scientific inquiry, it is clearly unsatisfactory. It is neither objective nor complete, and the reason for its failure to solve our problem is to be sought in the fact that the viewpoint, from which the different lines of inquiry are united, is external, and belongs to a theory of the ideal nature of man, rather than to scientific research itself.

As representative of more modern attempts to solve this problem, it will be, perhaps, sufficient to consider the scheme of Wundt. As will be seen from the accompanying diagram, it attempts to be both objective and complete, and to give a mental picture of the interrelation of all lines of actual scientific inquiry.

Validity of Such Systematisation.—Complete validity in such attempts at systematisation is hardly to be looked for. The nature of the attempt to systematise is largely descriptive. And to describe accurately and completely the relation of the various sciences to one another is hardly possible. The reason for this is to be sought in the fact that there are, at the present day, no final or hard and fast lines of distinction. The botanist has to be something of a chemist and mathematician. He frequently has to know something of geography, geology, and zoology, something of the general theory of evolution, and sometimes even of psychology and logic. So too the psychologist requires knowledge of biology, of physics, and of sociology, as well as of mathematics. Every specialist uses, in actual practise, whatever he finds necessary to the solution of his special problem, regardless of its place in the "system," just as the comparative anatomist pays little or no attention to the system which we find in zoology. Where the work of scientists thus cuts across any lines which the systematiser may draw, it is plain that description of the work of science can hardly be expressed in any of the typical systematic forms, if we wish to be objective in our description. Complete validity, then, is out of the question.

What the systematiser *can* aim at, however, is a reasonable completeness, and at some principle of organisation which makes his system convenient for purposes of reference. Such a system as Wundt's certainly fulfils this aim, and is so far to be regarded as valid. But it remains sufficiently obvious that *any* such system must be arbitrary and from many viewpoints unsatisfactory; and if we keep in mind the concrete

Individual Sciences



needs of scientific research and scientific exposition, we shall see that no one system can possibly be regarded as final, even for purposes of reference. For example: for certain purposes a group of Medical Sciences is advisable, and for certain other purposes a group of Social Sciences. Yet neither of these demands receives satisfaction from such a system as that of Wundt. All that we can hope for, then, in this field, is a system which shall present us with a general view of the work of science, in a way which is fairly accurate and reasonably complete. Complete validity, however, even in the case of *this* purpose, is not to be expected.

Summary.—So far we have seen that, in answer to certain needs arising in an age of over-specialisation, attempts are made so to systematise the special sciences as to present us with a general world-picture. This attempt is eventually given up as unscientific at the present stage of knowledge, and in its place we try to arrange the lines of actual scientific inquiry in such a way as to obtain a general view of the work of science. This too is found to be unsatisfactory in point of objectivity and completeness, and the only purpose which it can reasonably be expected to serve is, convenience for reference. This aim also cannot be completely accomplished, but systems such as that of Wundt are at least helpful and suggestive.

FOR FURTHER READING

J. G. Hibben, *Logic*, Part II, chapter xvii. Herbert Spencer, *Classification of the Sciences*. W. Wundt, *Logik*, (3rd Edit.), Vol. II, pp 85–100.

CHAPTER XXXII

THEORY OF SCIENTIFIC METHOD

The Problem.—So far we have discussed in detail the chief characteristics of scientific method, whether for purposes of investigation, or for purposes of exposition. It now remains to put together what we have discovered, and to formulate a general theory of scientific method, comparable to the theory of judgment and the theory of inference discussed above. The general nature of scientific method plainly consists in framing some mental model or hypothesis in reference to a given situation, and then attempting to understand the situation in terms of the mental model, or of some modification of that model introduced after further reference to the given situation. All that we really understand or are able to take into our mental grasp is, in the last analysis, the structure of the mental model itself; but we can make this progressively more adequate by continued reference to the concrete situation. This "reference" takes place by means of sense-perception. There are thus two factors in the use of scientific method, (1) the sensory, by which we are able to keep in some sort of touch with the natural environment, and (2) the intellectual, by means of which we construct and modify our mental models. To frame a "theory" of scientific method, it will be necessary to treat each of these factors separately, before taking them together, and to treat them in two ways. In the first place we shall state what part they actually do play, and by this description answer the question of fact. In the second place we shall examine briefly the objectivity and completeness of the results so attained, and by this criticism answer the question of validity.

The Sensory Element in Scientific Method. (A) Mind-Made Entities.—In the case of mind-made entities, sense-perception plays a very minor role, but still a role which is both appreciable and necessary. In experimenting with a jig-saw puzzle or a cipher, the sense of sight plays an appreciable part, and the sense of touch may also come into play.

In solving mathematical or ethical problems, many sensuous elements appear to be indispensable, though they are, of course, in no sense final. Elements other than sensory play the chief part, but here also, as in the case of natural phenomena, sensuous perception is a condition without which we could have no scientific knowledge, and its function is, to keep us in touch with the concrete situation, whatever that may be.

(B) **Natural Phenomena.**—In dealing with natural phenomena, it is only by means of sense-perception that we become aware of them. It is only by means of our senses, sight, hearing, and the rest, that the physical world is *given* to us in the form of concrete situations. We build houses out of materials which we can touch, see, and handle. The earth on which we live, the rain and sun, the changing seasons—even our books, music, and art—all these are given to us, in the first place, by way of sensuous perception. And yet, sensation plays no final part. The world of physical science is very different from the sensations of color, sound, and contact, with which our sense-organs respond to stimulation. Sensation is only one element in scientific method, and its function is to furnish us with a starting-point, a concrete situation which we can proceed to analyse and synthesise, *etc.*, until our intellectual aspirations are satisfied, so far as this is possible.

The Intellectual Element. (A) **Mind-Made Entities.**—In scientific method, the part played by intellectual elements is far more in evidence. The whole matter of constructing mental models, and deducing consequences from the general plan of such models, is a matter for the intellect. The way in which these models are constructed, however, has perhaps not been made sufficiently clear. They are constructed, in every case, by applying the intellectual standards of identity, difference, and organisation, internal and external, so far as such application proves to be possible. The case of analysis and synthesis has been dealt with, at least in principle, in an earlier chapter.¹ In the case of abstraction, it is still more obvious that in singling out for special attention some one element or aspect of the situation which results from analysis, we are applying the standard of identity, and that in excluding from consideration every element or aspect other than the one espe-

¹ Chapter xvi, where analysis and synthesis are dealt with, in principle, under the heading of "analytical expansion" and "systematic constructiveness," respectively.

cially selected, we are applying also the standard of difference. So also in the case of determination. We determine a single element or aspect by placing it experimentally in a number of contexts, each of which is different, and each of which adds a new determination to the single element with which we started. In this case, it is sufficiently obvious that the standards of identity, difference, and organisation are being used. Finally, in the case of induction and deduction—elaborate methods which use every resource of the preceding methods in establishing some law—it should be sufficiently plain that the same standards are being used, though in a way which is more complex, as the situation and the methods employed are, as a general rule, more complicated. So too definition usually involves some kind of statement of what the object defined *is*, and some distinction of it from objects which are like it but are regarded as different; and classification is very definitely an organisation or system which contains a number of differentiated elements, each of which may be regarded as an identity. So too proof proceeds by constructing a mental model which represents the situation to be proved. This, like all mental models, is a little system built up out of elements which are mental counters, differentiated identities, and the same standards govern our constructions in the case of proof as in the previously mentioned cases.

(B) **Natural Phenomena.**—With natural phenomena we deal, as we have already seen, indirectly. We construct mental models in the form of hypotheses, and by trying out one of these after another come as close to understanding the nature of such phenomena as we can. In constructing these mental models we use the same intellectual standards of identity, difference, and organisation, to which we have already referred in the case of mind-made entities, and in general, the part played by intellectual elements, considered by themselves, is approximately the same in the two cases. It is, in fact, only in relation to sensory elements that a difference can be established. This difference consists in the fact that problems concerned solely with mind-made entities can as a rule be completely solved, while problems concerned with natural phenomena cannot be completely solved. That is to say, our mental model may *be* the mind-made entity itself—the object studied, and the model in terms of which we approach the study of it may completely coincide. But in the case of nat-

ural phenomena, such coincidence is an *ideal* towards which we can progressively approximate—but some gap always remains. So far, then, as the construction of mental models as such is concerned, there is no difference between the two cases. In dealing with natural phenomena, as in dealing with mind-made entities, we construct our experimental models in terms of the intellectual standards of identity, difference, and organisation.

Summary.—Thus we see that, so far as the *description* of scientific procedure is concerned, the theory of scientific method resembles the theory of judgment and the theory of inference. Sense-perception furnishes us with the starting-point for our intellectual operations, and the intellectual operations consist in taking to pieces and rearranging the material given in the concrete situation, in such a way that we obtain insight into its laws and principles. This taking to pieces and rearranging takes place by the experimental construction of mental models which we then proceed to test and verify by reference to the datum. Compared with judgment, the field of scientific method corresponds approximately to the field of symbolic judgment, and the typical example of both is the algebraical solution of some concrete problem by means of simultaneous equations. In symbolic judgment, however, the field seems perhaps a little wider, while the mental models or symbols of which science makes use seem more restricted to quantitative and causal models. The difference is, however, only apparent. For though science does make use chiefly of these two types of symbol for interpreting our experience, it can and does use other models also, and in principle is precisely as unrestricted as is symbolic judgment.²

Compared with inference, again, there is but little difference. Both proceed by analysis and synthesis, by taking apart the given situation and reconstructing it in the light of intellectual standards, and both use approximately the same methods and the same standards. But the field of inference is perhaps slightly wider than the field of scientific method, for it

² Scientific method in practise is restricted to the field of theoretical values, and takes no account of ethical, esthetical and religious values as such—*i. e.*, *qua* ethical, esthetical, and religious. This is because these values do not lend themselves to explanation in terms of either mathematical or causal models. But in principle they may some day be a part of science.

covers the ground of judgments of experience also,³ and even—to some slight extent—the ground of transcendent judgments. Scientific method, however, is rigidly restricted to the symbolic reconstruction of its data, and sharply distinguishes itself from any attempt to transcend the field of possible experience. It is thus a specialised and concentrated application of inference to a somewhat narrow part of the whole possible field.

Validity of Scientific Method. (A) Mind-Made Entities.

—In the light of our previous chapters, we can state briefly the conditions of the validity of scientific method. As we have seen, it is possible for scientists to make mistakes. Analysis and synthesis, abstraction and determination, induction and deduction, of themselves are not infallible; and as we pointed out in the chapter on Fallacies, it is possible to go astray even in dealing with mind-made entities. Owing to accidents of educational environment, different people tend to use slightly differing sets of mental models, and it is hard for *A* to understand exactly the mental models of *B*. There is thus in all exposition a certain amount of marginal error, which can, of course, by careful attention be reduced to a minimum. But when all is said and done, some slight difference between the symbolic tools or models used by *A* and the mental instruments used by *B* tends, in practise, to remain, and is thus a constant source of error. But the chief source of error is undoubtedly to be sought in the relation of the intellectual to the sensory element. As we saw in the case of the symbolic judgment, even in dealing with text-book problems—which are certainly mind-made entities—it is very easy, in the preliminary analysis and synthesis which gives us the *x* and *y* equations, to omit or add something which, slight though it may be in itself, yet vitiates in some degree all subsequent inferences and conclusions. So also in the verification, when we come back from our deductions and compare these detailed consequences of our mental model with the details of the concrete situation, it is very easy to overlook a few obstinate facts in favor of a fas-

³ Scientific method also contains generalisations from experience, arrived at by abstraction and determination, and thus to a slight extent, enters the field of experiential judgment also. But its chief work lies more in the field of symbolic judgment.

cinating theory.⁴ This is especially likely to happen where the original situation is complex. These occasions of error are always with us, and invalidate a large percentage of our attempts to apply scientific method.

Still, in dealing with mind-made entities, there is no doubt that we *can* be entirely successful. Our procedure can be both objective and complete. The conditions of its validity are simple, and are the same as the conditions already noticed as indispensable for validity in judgment and in inference. The sensory apprehension must be direct, and the intellectual construction must obey the intellectual norms of identity, difference, and organisation, and finally, the sensory and intellectual elements must be brought together correctly. That this *can* be accomplished, our success in solving mathematical problems, in solving ciphers, and—in many cases—in solving ethical, esthetical, and even religious problems, sufficiently attests. In dealing with entities which are strictly mind-made, complete validity is possible.

(B) **Natural Phenomena.**—In dealing with natural phenomena, we have already seen that complete and final validity is out of the question. It is, however, possible that our methods—*e. g.*, the mathematical treatment of physical, psychological, and even sociological problems—may be valid as far as they go. On what does their validity depend? It depends upon the same conditions as we discovered in discussing judgment and inference. So far as the sensory apprehension is direct, it is ultimate and must be accepted. So far as the mental models are framed strictly in accordance with intellectual standards, they also are ultimate, and must be accepted. Finally, so far as both sense and intellect are correctly brought to bear upon one and the same problem, it can so far be solved, and solved in a way which is valid—though perhaps it remains incomplete. The only adequate test of validity in these cases, where our sensuous apprehension is not perfectly direct, and our intellectual efforts seem confined to the indirect trial-and-error method which tests one mental model after another until it approximates to discovering something which will “work”—in such cases the only adequate test of validity is the

⁴ One of the best known instances in recent years is the late Dr. Verrall's interpretation of the *Agamemnon* of Aeschylus. Substantially all the critics are agreed that it is “brilliant, but not convincing.” The Baconian view of Shakespeare is another famous instance.

advance of science itself. If our methods not only provide us with a fair solution of their special problem, but are also found fruitful in other fields, and of assistance in helping on the advance of science, they may be looked upon as valid.

Conclusion.—If we look back over the whole course of our inquiries, we find that the endeavor of man to solve his problems by the use of logical thought, rather than by trusting to instinct and feeling, is of a very definite and pronounced character. In judgment, we take apart the given, sensory flow of experience, split it up into elements which are cut off and fixed by the mind, and form differentiated identities out of which we proceed to build up an intellectualised model of the situation with which we are dealing. In inference, we analyse and expand, constructing out of precisely similar intellectualised elements whole edifices of thought which extend the intellectually reliable aspects of our experience almost without limit. Finally, in scientific method, we reduce this framing of mental models to a system, to the methodical application of certain tested and approved types of mental model, especially of a mathematical character, in a way which leads to the gradual but sure advance of scientific knowledge. That is to say, the character of logical thought consists in substituting, for the vague continuity of sensuous feeling, the highly artificial but thoroughly determinate and exact mental counters known as concepts.

With sensation and feeling, we cannot rise above the concrete situation. We live these, we experience these, we *are* these. Sensation, feeling, impulse—these are the stuff of which life, as we are conscious of living, is made up. These are what are half-revealed by introspection, by looking within and attempting—vainly, as it seems⁵—to place our finger upon the pulse of consciousness. They are parts of the stream of consciousness, and constitute our life. Our life—but not our *knowledge*. We *live* them, but cannot contemplate and understand them. They evade our mental grasp, and leave us vainly trying to say many things. We grasp at consciousness, and are left with—a psychological theory. We try to apprehend the nature of thought, and are left with—a theory of logic. The nature of understanding is, as we say, not intuitive but discursive. It is indirect, and constructs intellectualised enti-

⁵ Cf. W. B. Pillsbury, *Fundamentals of Psychology*, p. 7.

ties which are always somewhat different from the realities which they are supposed to represent. If we try to understand, we inevitably construct more or less plausible *theories*, and substitute for the reality the mental construction which seems to explain it best. What we understand is thus always primarily what our intellect has itself introduced into the phenomena studied. The structure built up in the sciences is a mind-made structure, and is intelligible precisely in so far as it is of the nature of intellect. It is of the mind, mental, and has, in relation to the phenomena which we experience, an application which is only secondary and indirect.

At the same time, there is no doubt that logical thought is successful. One steamer leaving the wharf a day behind a slower steamer may seem to have little to do with algebra, and the distant smoke on the horizon may seem perhaps to have more connection with poetry than with trigonometry. And yet, trigonometry and algebra can give us exact information as to the distance of that smoke on the horizon and the hour at which we shall pass the slower steamer, and that information will be found to work. Without such mental models, our steamer and railroad schedules would be impossible, and the whole series of conventions and symbols upon which our modern civilisation rests would vanish into thin air. Logical thought works. It is tested and found valid at every moment of our every-day life, as well as in the laboratories of scientists.

How are we to explain this validity? How is it that a mind-made model will tell us more about the structure of reality than our most intimate psychological experiences? We can only hint at an answer to this inevitable problem. We can account for the success of our mental models only upon the assumption that these represent—at least approximately—the intelligible structure of the concrete facts of experience, that the phenomena of nature have intelligible laws, that the universe is essentially rational or even mind-made, and that its rationality is akin to the rationality which we discover in our own logical thinking. It is, in fine, only so far as our reason is identical in principle with the reason embodied in concrete facts, that the unfolding of our own mental constructions can bring us into touch with the nature of the universe. The further study of this problem and of this answer belongs, not to logic, but to metaphysics.

FOR FURTHER READING

B. Bosanquet, *Logic*, Bk. II, chapter vii. F. H. Bradley, *Principles of Logic*, Bk. III, Part II, chapters iii-iv. H. Lotze, *Logic*, Bk. III, chapter v. Chr. Sigwart, *Logic*, Vol. II, pp. 548-557.

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